LEANING TOWARDS HIGHER RANK

Martin Bobb (Max Planck Institute in Leipzig)

Title: Affine laminations and coaffine representations

Abstract: We consider certain representations of a surface group into $\mathrm{SL}(4,\mathbb{R})$ called convex cocompact coaffine representations. These representations act geometrically on a 3-dimensional convex body in projective space, and are part of a broader (and more difficult) landscape of such geometric actions. From classical cases, we would anticipate an analytic object called a transverse measured lamination to capture the geometry of these representations, however paradoxical examples reveal that a generalization is necessary. We will discuss a nice resolution to these difficulties, and describe a space of affine measured laminations which parametrize the space of convex cocompact coaffine representations. Along the way we make an interesting connection to the dynamics of affine interval exchange transformations. Joint work with James Farre.

Samuel Bronstein (Max Planck Institute in Leipzig)

Title: A family of Anosov representations in $\mathrm{SL}(3,\mathbb{R})$

Abstract: We consider a family of representations from surface groups in $\mathrm{SL}(3,\mathbb{R})$, which contains the nonprincipal Fuchsian representations, and is parametrized via the Non Abelian Hodge Correspondence by holomorphic half-cubic differentials. Using the particular structure of these Higgs bundles, we show that those representations are Anosov, and give a foliation of a domain of discontinuity in the space of flags of Euclidean 3-space. This is joint work with Colin Davalo.

James Farre (Max Planck Institute in Leipzig)

Title: Horocycles, Lipschitz maps, and laminations

Abstract: The geodesic flow on the unit tangent bundle of a hyperbolic surface is a prototypical example of an Anosov flow, exhibiting rich, structurally stable dynamics. While invariant measures and closed sets for the geodesic flow can be quite complicated, the closely related horocycle flow exhibits striking rigidity. For example, on a closed surface, every horocycle is dense and equidistributed in its

unit tangent bundle; this is very far from being true for geodesics. Despite this rigidity, the behavior of horocycles remains mysterious in the setting of hyperbolic surfaces with infinite topology. In this talk, I will explore the dynamics of horocyclic trajectories for a particular class of infinite-area regular covers of closed hyperbolic surfaces. Surprisingly, these trajectories are intimately connected to an exceptional set for the geodesic flow: the distance-minimizing geodesic lamination. Both objects arise naturally in a geometric optimization problem: what does an optimal Lipschitz map to the circle in a given homotopy class look like? This is joint work with Or Landesberg and Yair Minsky.

Xenia Flamm (IHES)

Title: Non-Archimedean Hilbert geometry

Abstract: The Hilbert metric is a distance defined on properly convex subsets of real projective space. It generalizes the hyperbolic metric in Klein's model of hyperbolic space, where the convex set is the unit ball. The study of degenerations in Hilbert geometry leads to replacing the reals by a non-Archimedean ordered valued field. The aim of this talk is to introduce convex sets over such fields, describe their Hilbert metrics through several examples and view how they arise as limits of classical Hilbert geometries. This is joint work with Anne Parreau.

Viola Giovannini (Université du Luxembourg)

Title: Renormalized volume for convex co-compact hyperbolic 3-manifolds with compressible boundary

Abstract: Given a hyperbolizable 3-manifold N, the renormalized volume is a real-analytic function on the space of convex co-compact hyperbolic structures on the interior of N, which always have infinite hyperbolic volume. When the boundary of N is incompressible the renormalized volume is always non-negative, otherwise it has infimum $-\infty$. We present some results on the behavior of the renormalized volume for manifolds with compressible boundary, and a new adapted version for this setting.

Qiongling Li (Nankai University)

Title: Harmonic metrics of Higgs bundles over non-compact surfaces

Abstract: In this talk, I will discuss existence and uniqueness results for harmonic metrics on certain Higgs bundles over non-compact surfaces. The cases I will cover include cyclic Higgs bundles, generically regular semisimple Higgs bundles, Higgs bundles in the Hitchin section, and generically regular nilpotent Higgs bundles. This is based on joint works with Takuro Mochizuki (Kyoto University) and Song Dai (Tianjin University).

John Loftin (Rutgers University)

Title: Equivariant conformal harmonic maps from surfaces into rank 2 real buildings

Abstract: In joint work with Andrea Tamburelli and Mike Wolf, we produce many equivariant minimal embeddings f of the universal cover of a closed oriented surface S of genus at least 2 into the asymptotic cone B of the symmetric space $X = \mathrm{SL}(3,\mathbb{R})/\mathrm{SO}(3)$. More specifically, we use a ray of cubic differentials $\{tQ\}$, related Higgs bundles and the nonlinear Hodge correspondence to produce a representation of $\pi_1 S$ into $\mathrm{SL}(3,\mathbb{R})$ and an equivariant conformal harmonic map into X. Then take the limit $t \to \infty$. The geometry of the image of f can be read off explicitly from a flat structure induced by the cubic differential.
More generally, we can consider B to be a real building of rank 2. We address the question of uniqueness of equivariant conformal harmonic maps into a real building B of rank 2. In the case of $\mathrm{SL}(3,\mathbb{R})$ and an action of the fundamental group of a surface into the isometries of B, we can show uniqueness for generic equivariant conformal harmonic maps. We discuss in some detail the geometry of the spaces involved and make a conjecture about the general case. This is joint work in progress with Tamburelli and Wolf.

Sara Maloni (University of Virginia)

Title: Topology of the space of d-pleated surfaces

Abstract: In this talk, we will discuss joint work with Giuseppe Martone, Filippo Mazzoli and Tengren Zhang about the space of d-pleated surfaces and its topology. d-pleated surfaces can be considered as a higher rank generalization of the classical notion of classical pleated surfaces in three-dimensional hyperbolic space, or, equivalently, as the space of representations obtained by bending a Hitchin representation along a geodesic lamination. We will give a description of the global topology of the

space of d-pleated surfaces and prove that every connected component of the character variety contains exactly one connected component of the space of d-pleated surfaces.

Filippo Mazzoli (University of California Riverside)

Title: Constant mean curvature foliations of almost-Fuchsian manifolds

Abstract: Quasi-Fuchsian groups have been objects of extensive study since the 1890s. By naturally acting on the 3-dimensional hyperbolic space, they describe a wide class of complete, infinite volume, hyperbolic 3-manifolds, and their properties play a crucial role in Thurston's hyperbolization theorem and, more generally, in the study of the geometry and topology of 3-manifolds. Following Uhlenbeck, we say that a quasi-Fuchsian manifold is almost-Fuchsian if it contains an incompressible minimal surface with principal curvatures between -1 and 1. A conjecture by Thurston asserts that any almost-Fuchsian manifold admits a foliation by constant mean curvature (CMC) surfaces. In this talk, I will describe a result from an upcoming joint work with Nguyen, Seppi, and Schlenker, where we describe explicit conditions of the first and second fundamental forms of the minimal surface of an almost-Fuchsian manifold that guarantee the existence of a CMC foliation.

Charles Ouyang (Washington University)

Title: New Minimal Lagrangians in $\mathbb{CP}^2$

Abstract: Abstract: Minimal Lagrangian tori in $\mathbb{CP}^2$ are the expected local model for particular point singularities of Calabi-Yau 3-folds, and numerous examples have been constructed. In stark contrast, very little is known about higher genus examples, with the only ones to date due to Haskins-Kapouleas and only in odd genus. Using loop group methods, we construct new examples of minimal Lagrangian surfaces of genus $(k-1)(k-2)/2$ for large k, hence providing the first examples in even genus. Time permitting, we show our surfaces lift to embedded special Legendrian surfaces in the 5-sphere. These are the first embedded examples. This is joint work with Sebastian Heller and Franz Pedit.

Hugo Parlier (Université de Fribourg)

Title: Criss-crossing curves

Abstract: The crossing lemma for simple graphs gives a lower bound on the necessary number of crossings of any drawing of a graph in the plane in terms of its number of edges and vertices. Viewed through the lens of topology, this leads to other questions about arcs and curves on surfaces.
In joint work with Alfredo Hubard, we provide estimates on the necessary number of intersections of any realization of m distinct homotopy classes of curves on a (fixed) surface. These estimates allow us to answer questions raised by Pach, Tardos, and Toth concerning a version of the crossing lemma for graph drawings with non-homotopic edges. Our approach uses the geometry of hyperbolic surfaces in an essential way.

Stefano Riolo (Universitá di Bologna)

Title: On the topology of convex projective manifolds

Abstract: Properly convex, real projective manifolds form an interesting class of aspherical manifolds, including hyperbolic manifolds. In a joint work with A. Seppi and L. Slavich, we provide some topological obstructions that imply, among other things, that their fundamental group is not quasi-isometric to any complex hyperbolic lattice. Time permitting, a conjectural solution to the geography problem in dimension four will be sketched.

Nicholas Rungi (Universitá di Torino)

Title: Para-complex geometry and cyclic Higgs bundles

Abstract: In the early 2000s, Labourie and Loftin independently showed that the Hitchin component for $\mathrm{SL}(3,\mathbb{R})$ admits the structure of a holomorphic vector bundle over the Teichmüller space of the surface. The proofs rely on the existence of a special equivariant immersion, known as hyperbolic affine sphere, associated with the Hitchin representation. In the more general setting of representations into $\mathrm{SL}(2m+1,\mathbb{R})$, however, very little is known about the existence of such equivariant immersions. The aim of this seminar is to present a new approach, based on para-complex (or split-complex) geometry, which allows one to prove the existence of equivariant surfaces in a specific homogeneous space, generalizing the case of hyperbolic affine spheres for $m = 1$. After introducing the homogeneous space in question, we will explain how the use of stable cyclic Higgs bundles (not necessarily in the Hitchin component) plays a fundamental role in the construction of such immersions. Time permitting,

we will also discuss new geometric implications that arise from this para-complex geometric framework. This is joint work with A. Tamburelli.

Nathaniel Sagman (Université du Luxembourg)

Title: Complex harmonic maps and Goldman's symplectic form

Abstract: Beginning with the early work of Hitchin, harmonic maps to Riemannian symmetric spaces have played an essential role in higher Teichmuller theory. One striking application is due to Labourie, who made use of harmonic maps to parametrize each rank 2 Hitchin component as a holomorphic vector bundle over Teichmuller space. In this talk, I will introduce complex harmonic maps to holomorphic Riemannian symmetric spaces, which "complexify" the ordinary theory of harmonic maps. In particular, I will explain how we are using complex harmonic maps to prove that on every rank 2 Hitchin component, Labourie's complex structure is compatible with Goldman's symplectic form, in the sense that they together define a pseudo-Kahler structure. This is all joint work with Christian El Emam, some already in print and some in progress.

Leone Slavich (Universitá di Pavia)

Title: Convex projective structures on locally symmetric spaces

Abstract: We will characterise the compact locally symmetric spaces which admit a finite index cover with a (properly) convex real projective structure, and show how these convex projective structures arise. The main result can be synthetised by saying that possessing a virtual convex projective structure only depends on the geometry of the locally symmetric space, and that the geometries in which such structures arise are those which admit a convex projective model akin to the Beltrami-Klein model of real hyperbolic space. Joint work with Stefano Riolo and Andrea Seppi.

Rym Smai (Université Côte d'Azur)

Title:

Abstract:

Peter Smillie (Max Planck Institute in Leipzig)

Title: The Schoen conjecture in higher rank

Abstract: In 2015, Markovic showed that every quasisymmetric map from $\mathbb{RP}^1$ to $\mathbb{RP}^1$ is the asymptotic boundary map of a unique harmonic quasi-isometry from $\mathbb{H}^2$ to $\mathbb{H}^2$, resolving a conjecture of Schoen. Joint with Max Riestenberg, we show that this is a special case of a more general extension theorem by harmonic maps, in which the target may be higher rank. As a consequence, we construct a universal Hitchin component for $\mathrm{SL}(n, \mathbb{R})$ generalizing universal Teichmüller space when $n = 2$.

Jérémy Toulisse (Université Côte d'Azur)

Title: A W-volume in anti-de Sitter space

Abstract: Motivated by work of Krasnov and Schlenker on the renormalized volume of hyperbolic 3-manifolds, we construct a Lorentzian version of the theory. This defines a (possibly infinite) invariant of positive curves in flag varieties. This is joint work with François Labourie and Yilin Wang.

Enrico Trebeschi (Université Côte d'Azur)

Title: Generalized convexity in anti-de Sitter space

Abstract: The *Anti-de Sitter* space $\mathbb{H}^{n,1}$ generalizes the hyperbolic space in the Lorentzian setting signature. It has been studied in the framework of (higher higher) Teichmüller theory because it is a pseudo-Riemannian symmetric space of the group $\mathrm{O}(n, 2)$. The study of *constant mean curvature* (CMC) hypersurfaces in Anti-de Sitter have been studied froma a qualitative point of view by several authors ([BBZ07, ABBZ12, BS10, Tam19, Tre24]). The dependency of the geometry of surfaces constant vanishing mean curvature, called *maximal*, in $\mathbb{H}^{2,1}$ and their convex hull has been enlighted by work of [BS10], and quantified in [Sep19].
In this talk, I will explain how I generalized the work of [Sep19] in higher dimension, comparing the extrinsic geometry of maximal hypersurfaces with the *width*, namely the timelike diameter, of their convex hull. In my thesis, I generalized the notion of convex hull to be suitable to the study of hypersurfaces with non-zero constant mean curvature: in the same flavour, the width of the $H-$*shifted convex hull* controls the geometry of the corresponding CMC hypersurface. Time allowing, I will discuss some application of these generalizations to the study of intrinsic geometry of CMC hypersurfaces and to classical Teichmüller theory.

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Michael Wolf (Georgia Institute of Technology)

Title: Ray structures on Teichmüller space

Abstract: We depict harmonic map ray structures on Teichmüller space as a geometric transition between Teichmüller ray structures and Thurston geodesic ray structures. As an application, while there may be many Thurston metric geodesics between a pair of points in Teichmüller space, we find that by imposing an additional energy minimization constraint on the geodesics, thought of as limits of harmonic map rays, we select a unique Thurston geodesic through those points. There are applications to the envelopes of Thurston geodesics between a pair of points.