Single Variable Calculus I
Math 101, Spring 2020

This exam has 10 problems worth 100 points distributed over 8 pages, including this one.

Instructions: You may work on this exam for at most three hours. Time is supposed to start when you turn this page. You may not consult any notes or books during the exam, and no calculators are allowed. Show all of your work on each problem and carefully justify all answers. Points will be deducted for irrelevant, incoherent or incorrect statements, and no points will be awarded for illegible work. If you have a tablet or a printer, please write your answers in the spaces provided. Otherwise, make sure to clearly indicate which question you are answering in each page and include the signed honor pledge at the top of the first page.

Deadline for submission: Please upload on canvas a pdf file with your answers by May 1, at 11:59pm.

Name:

Honor Pledge: *On my honor, I have neither given nor received any unauthorized aid on this exam.*

Signature:

Question	Points	Score
1	10	
2	21	
3	6	
4	6	
5	15	
6	8	
7	5	
8	8	
9	8	
10	13	
Total:	100	

1. (10 points) Answer true or false and circle it. No explanation is necessary.

(a) Let $f(x)$ and $g(x)$ be continuous functions. Then

$$\int_0^1 f(x)g(x)\ dx = \left(\int_0^1 f(x)\ dx\right)\left(\int_0^1 g(x)\ dx\right).$$

TRUE *FALSE*

(b) Every differentiable function is continuous.

TRUE *FALSE*

(c) Let x_0 be an inflection point of a function $f(x)$. Then $f'(x_0) > 0$.

TRUE *FALSE*

(d) Let $f(x)$ be a continuous function such that $\lim_{x\to+\infty} f(x) = +\infty$ and $\lim_{x\to-\infty} f(x) = -\infty$. Then there is a point x_0 such that $f(x_0) = 0$.

TRUE *FALSE*

(e) The function $F(x) = \int_0^{x^2} e^{-t^2}\ dt$ is an antiderivative of the function $f(x) = e^{-x^2}$.

TRUE *FALSE*

2. Compute an antiderivative of the following functions:
   (a) (7 points) $f(x) = \frac{1}{x\sqrt{1-\ln^2(x)}}$

   (b) (7 points) $g(x) = \cos(x)\sqrt{\sin(x)}$

   (c) (7 points) $h(x) = \frac{1}{\sqrt{x}+x\sqrt{x}}$

3. (6 points) Show that the functions $f(x) = \arctan(x)$ and $g(x) = -\arctan(\frac{1}{x})$ differ by a constant. Then find such a constant.

4. (6 points) It is given that the function $f(x) = 2\arctan(x) - x^3 + 2x^2 - 3x + \ln(x)$ has a critical point at $x = 1$. Is it a local maximum, a local minimum or neither?

5. Compute the following limits:

   (a) (5 points)

$$\lim_{x \to 0} \frac{\sqrt{x+1} - \sqrt{1-x}}{x}$$

   (b) (5 points)

$$\lim_{x \to -\infty} \frac{x^3 + 5x - 1}{1 - x^3}$$

   (c) (5 points)

$$\lim_{x \to 0} \frac{\arctan(x) - x}{x^2}$$

6. (8 points) Find the equation of the line tangent to the curve $y + \ln(y) + xy^2 = 1$ at the point $y = 1$.

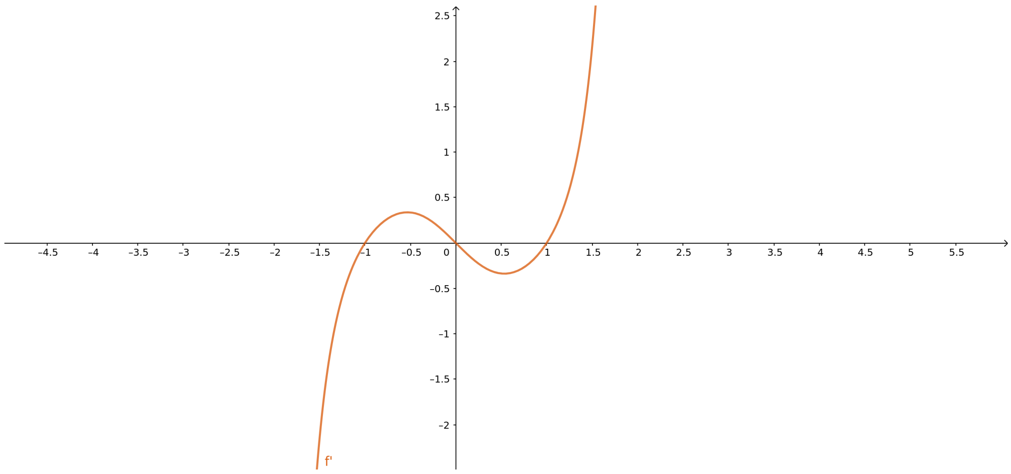
7. (5 points) Find the critical points of the function

$$f(x) = \int_0^{x^3-3x} \frac{e^t}{t}\, dt$$

8. (8 points) Find the area of the region between the graphs of the functions $f(x) = x^2$ and $g(x) = 3x + 4$.

9. (8 points) The shape of a football can be approximated by rotating the graph of the function $f(x) = \sin(x)$ for $x \in [0, \pi]$ around the x-axis. What is the approximate volume of a football? (*Hint: it may be useful to know that* $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$)

10. Below is the graph of a function $g(x)$. Consider $f(x) = \int_0^x g(t)\, dt$.



(a) (6 points) List the critical points of $f(x)$ and indicate if they are local maxima, local minima or neither.

(b) (2 points) Show on the graph above where the inflection points of $f(x)$ are located.

(c) (5 points) Draw an approximate graph of $f(x)$.