For questions: at52@rice.edu April 3

EXERCISE SHEET : CALCULATING INTEGRALS FROM RIEMANN SUMS

1. Compute $\int_0^1 x^2\ dx$ using a Riemann sum :

i) Divide the interval $[0,1]$ in n subintervals. How long is each interval ?

ii) Evaluate the function $f(x) = x^2$ in the right endpoint of each subinterval.

iii) Compute the right Riemann sum and take the limit as $n \to +\infty$.

iv) Deduce the value of the integral.

Hint : the following summation formula may be useful : $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$.

2. Compute the following integrals

i) $\int_{-3}^3 (3 - |x|)\ dx$ ii) $\int_{-2}^2 \sqrt{4 - x^2}\ dx$

3. Consider the function

$$f(x) = \begin{cases} 3 & x < 3 \\ x & x \geq 3 \end{cases}$$

Compute $\int_0^5 f(x)\ dx$.

4. Use the properties of integrals to evaluate :

i) $\int_{-\pi}^{\pi} \frac{\sin(x)}{1+x^2}\ dx$ ii) $\int_{-1}^1 x^3\ dx$

5. Let $A = \int_0^{2\pi} \sin^2(x)\ dx$ and $B = \int_0^{2\pi} \cos^2(x)\ dx$. Show that $A + B = 2\pi$ and $A = B$.