

HOMEWORK 5

1. Compute the first and second derivatives of the following functions

i) $f(x) = 2x^3 - 2\sqrt{x}$

vi) $f(x) = x^2 e^x$

ii) $f(x) = \sqrt[4]{x} + \frac{1}{x^2}$

vii) $f(x) = \sqrt[3]{x} e^x$

iii) $f(x) = \frac{e^x}{x^2}$

viii) $f(x) = \sin^2(x)(3x^2 - x + 2)$

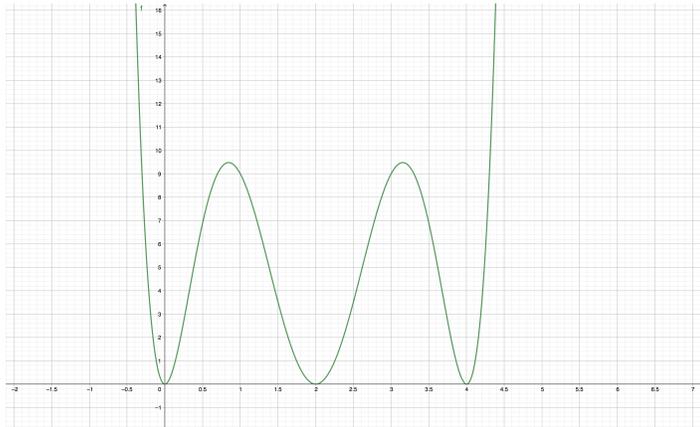
iv) $f(x) = \frac{x^2 - 1}{\sqrt{x}}$

ix) $f(x) = \cos^2(x) \sin(x)(x^2 + 1)$

v) $f(x) = \frac{\sin(x)}{\sqrt[3]{x}}$

x) $f(x) = x e^x \sin(x) \cos(x)$

2. Below is the graph of a function $f(x)$. Sketch the graph of its first and second derivative.



3. Sketch the graph of a function $f(x)$ satisfying all of the following properties :

- i) f is defined for all $x \geq 0$;
- ii) $\lim_{x \rightarrow 0^+} f(x) = f(0) = 0$;
- iii) f is not differentiable at $x = 2$;
- iv) f is not continuous at $x = 4$;
- v) $\lim_{x \rightarrow 2} f(x) = f(2)$;
- vi) $f(x) \geq 0$ for all $x \geq 0$;
- vii) $f'(x) < 0$ for $x > 4$;
- viii) $f''(x) < 0$ for $4 < x < 6$ and $f''(x) > 0$ for $x > 6$.

4. (Extra credit) Estimate the number $\sqrt{8.99}$ using the linear approximation of an appropriate function. Then justify why the estimate you find is smaller or greater than the actual value by using concavity.
5. (Extra credit) Find the linear approximation of the function $f(x) = \frac{x^2+2}{x}$ at the point $x = 1$.