MOCK EXAM

1. True or False : indicate whether each of the following statement is true or false

   i) If $\lim_{x\to 1} f(x) = \lim_{x\to 1} g(x)$, then $\lim_{x\to 1} \frac{f(x)}{g(x)} = 1$.

   ii) $f'(a) = \lim_{b\to 0} \frac{f(b)-f(a)}{b-a}$.

   iii) The functions $\lfloor x - 7 \rfloor$ is continuous at $x = 0$.

   iv) The function $f(x) = \sqrt{x - 2}$ is differentiable at $x = 2$.

   v) An object s moving along a line. If its velocity at $t = 5$ is negative and it acceleration at $t = 5$ is positive, then the object is slowing down at $t = 5$.

2. Draw the graph of the following function

$$f(x) = \begin{cases} 2x + 1 & x < -2 \\ -3 & -2 \le x < 0 \\ \lceil x - 1 \rceil & 0 \le x < 2 \\ -x + 2 & x \ge 2 \end{cases}$$

   i) Indicate at which points the function $f(x)$ is not continuous.

   ii) Indicate at which points the function is continuous but not differentiable.

   iii) Compute then the following limits (if they exist)

   a) $\lim_{x\to -2^-} f(x)$

   b) $\lim_{x\to 2}(x - 2)f(x)$

   c) $\lim_{x\to 0^-} f(x)^2$

   d) $\lim_{x\to -2^+} \sqrt{xf(x)}$

   e) $\lim_{x\to 1^-} f(x)$

   f) $\lim_{x\to 1^-} \frac{1}{f(x)}$

3. Compute the following limits :

   a) $\lim_{x\to 2^-} \frac{|x-2|}{x-2}$

   b) $\lim_{x\to 1} \frac{x^2+2x-3}{x-1}$

   c) $\lim_{x\to \pi} \frac{\cos(x)+1}{x-\pi}$

   d) $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$

4. Let $f(x) = x^4 - \frac{1}{x} + e^2$. Compute $f'(x)$ and $f''(1)$.

5. Show that the equation $2^x = 3x$ has a solution in the interval $[0, 1]$.

6. Find the equation of the line tangent to the graph of $f(x) = \sin(x) + \frac{1}{x} + x^2$ at $x = 1$.

7. Sketch the graph of a function $f(x)$ that satisfies all of the following properties :

    i) f is defined in the interval $[-2, 2]$.

    ii) $\lim_{x \to -2^+} f(x) = 0$.

    iii) $f(-2) = -1$.

    iv) $f(0) = 0$ and $f(1) = -1$.

    v) $f'(x) < 0$ for $0 \le x < 1$.

    vi) $f'(1) = 0$.

    vii) f is not differentiable at $x = -1$.

    viii) $\lim_{x \to -1} f(x) = f(-1) = 2$

    ix) $f''(x) < 0$ for $x > \frac{3}{2}$.

8. Below are the graphs of a function $f(x)$ and its first and second derivative. Indicate which is which.