

HOMEWORK 2

1. Decide whether the following statements are true or false.

- (i) If $\lim_{x \rightarrow a} g(x) = 0$, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ cannot exist.
- (ii) If $\lim_{x \rightarrow a} f(x) = 0$, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = 0$.
- (iii) If $\lim_{x \rightarrow a} f(x) = 0$ and $g(x)$ is continuous at $x = a$, then $\lim_{x \rightarrow a} f(x)g(x) = 0$.
- (iv) If $f(x)$ is continuous everywhere, then $\lim_{x \rightarrow a} f(x)$ exists for every a .
- (v) If $\lim_{x \rightarrow -1} f(x)g(x)$ exists, it must be $f(-1)g(-1)$.

2. For what value of the constant k , is the following function continuous for every x

$$f(x) = \begin{cases} \frac{x^2 - 5x + 4}{x^2 - 1} & \text{if } x > 1 \\ kx & \text{if } x \leq 1 \end{cases}$$

3. Compute the following limit

$$\lim_{x \rightarrow 2^-} \left[x \sin\left(\frac{\pi}{6}x\right) + [x] \right]$$

4. For each function, indicate whether it is continuous at every point or list its points of discontinuity.

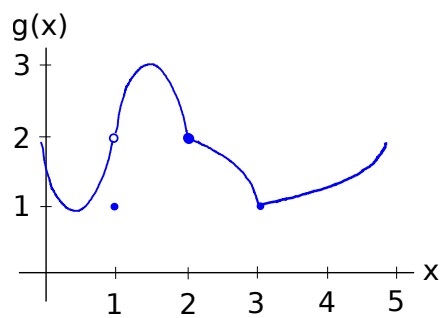
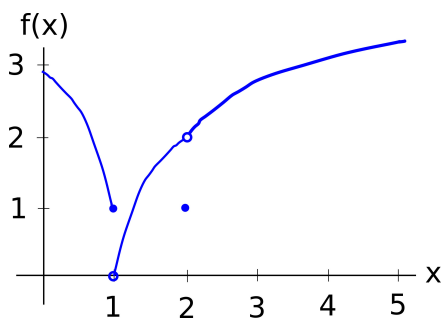
i) $\frac{x}{3x-1}$

iii) $\sqrt{x^4 + 1}$

ii) $\frac{5x^3 + 3x + 1}{15x^2 + 2}$

iv) $x^2 \sin(x) + x^{-2}$

5. Consider the functions $f(x)$ and $g(x)$ graphed below.



Compute the following limits or explain why they do not exist.

i) $\lim_{x \rightarrow 2} f(x)g(x)$

iii) $\lim_{x \rightarrow 1^-} \frac{g(x)}{f(x)}$

ii) $\lim_{x \rightarrow 1} [(x+1)^2 + g(x)]$

iv) $\lim_{x \rightarrow 1^+} \frac{g(x)}{f(x)}$