For questions: at52@rice.edu April 13

EXERCISE SHEET : U SUBSTITUTION

1. Compute the antiderivative of the following functions :

i) $f(x) = x\sqrt{x+1}$
ii) $f(x) = \frac{x^2}{\sqrt{x-1}}$
iii) $f(x) = x\sqrt{4x^2+9}$
iv) $f(x) = \frac{x}{\sqrt{4x^2+9}}$
v) $f(x) = \frac{x}{(4x^2+9)^2}$
vi) $f(x) = (x+1)^4$
vii) $f(x) = (2x-3)^{-7}$
viii) $f(x) = x(1-x)^{99}$
ix) $f(x) = \cos(x)\sin(x)$
x) $f(x) = \sin^7(x)\cos(x)$
xi) $f(x) = x\sin(x^2)\cos(x^2)$
xii) $f(x) = \frac{x^3}{\sqrt{1-x^2}}$

2. Evaluate the following definite integrals

i) $\int_0^1 x\sqrt{1-x^2}\,dx$
ii) $\int_0^1 \frac{x^2}{\sqrt{x^3+1}}\,dx$
iii) $\int_0^{\frac{\pi}{4}} \sec^2(x)\,dx$
iv) $\int_0^{\frac{\pi}{4}} \frac{\sin(x)}{\cos^4(x)}\,dx$

3. If $h(a) = h(b)$, what is the value of the integral $\int_a^b g'(h(x))h'(x)\,dx$?

4. Consider the function $f(t) = \frac{t}{(1+t^2)^2}$.

i) Compute the area $A(x)$ under the graph of $f(t)$ from $t = 0$ to $t = x$.
ii) Evaluate $\lim_{x\to+\infty} A(x)$.