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**REVIEW 2**

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1. Compute the derivative of the following functions :

i)  $f(x) = \ln(3x^2 + 1) + \arctan(x)$

ii)  $f(x) = \frac{\cos(x^2)}{\log_3(x)}$

2. Compute the following limits :

i)  $\lim_{x \rightarrow 0} \frac{e^{3x} - 1 - 3x}{x^2}$

ii)  $\lim_{x \rightarrow +\infty} x^2 \ln \left(1 + \frac{1}{2x^2}\right)$

3. A box with an open top has vertical sides, a square bottom and a volume of 8 cubic meters. Given that the box has the least possible surface area, find its dimension.

4. If  $f(1) = 1$  and  $f'(x) = e^x f(x)$ , find  $f''(1)$ .

5. Show that the function  $f(x) = 7 \ln(x) - 17x + 6x^2 - \frac{2}{3}x^3$  has a critical point at  $x = 1$ . Is it a local maximum, a local minimum or neither?

6. Find the absolute maximum and minimum of the function  $f(x) = 3 + 2 \sin(x) + \cos^2(x) - \sin^2(x)$  on the interval  $[0, \pi]$ .

7. Consider the function  $f(x) = \frac{x}{\ln(x)}$ .

- i) Determine for which values of  $x$  the function is well-defined.
- ii) Find and classify the critical points of  $f(x)$ .
- iii) Determine the intervals where the function is increasing.
- iv) Find the inflection points of  $f(x)$ .
- v) Determine the intervals where the function is concave-up.
- vi) Compute  $\lim_{x \rightarrow +\infty} f(x)$  and  $\lim_{x \rightarrow 0^+} f(x)$ .
- vii) Compute  $\lim_{x \rightarrow 1^+} f(x)$  and  $\lim_{x \rightarrow 1^-} f(x)$ .
- viii) Sketch the graph of  $f(x)$ .