For questions: at52@rice.edu January 27

IN-CLASS ACTIVITY : INTERMEDIATE VALUE THEOREM

1. Show that the polynomial $x^3 - x^2 - 3x + 1$ has a zero in the interval $[0, 1]$.

2. Consider the function

$$h(x) = \begin{cases} 3x^2 - 4 & x \leq 2 \\ 5 + 4x & x > 2 \end{cases} .$$

Sketch the graph of $h(x)$ and observe that there is no value of x in $[0, 4]$ such that $h(x) = 10$. On the other hand, $h(0) < 10$ and $h(4) > 10$. Explain why this does not contradict the intermediate value theorem.

3. Apply the intermediate value theorem to determine whether the equation $2^x = x^3$ has a solution in the interval $[1.25, 1.375]$ or in the interval $[1.375, 1.5]$.

4. Decide whether the following statements are true or false :

i) $\cos(x) - \sin(x) - x = 2$ has a solution in the interval $[-\pi, \pi]$.

ii) If $f(x)$ is continuous over the interval $[a, b]$ and $f(a)$ and $f(b)$ have opposite signs, then $f(x)$ has exactly one zero in the interval $[a, b]$.