

EXERCISE SHEET: FINAL REVIEW

1. Compute the following integrals

i) $\int_0^\pi \sin(x)e^{\cos(x)} dx$

ii) $\int_1^3 2^x dx$

2. Compute the following limits

i) $\lim_{x \rightarrow +\infty} \frac{\sqrt{2x^4-1}}{17x^2-2}$

ii) $\lim_{x \rightarrow 0} \frac{\ln(1+x)-x}{x^2}$

3. True or false? Decide whether the following statements are true or false.

i) There exists a function such that $f(0) = \ln(2)$ and $\int_0^1 f(x) dx = 0$.

ii) If $h''(x) = 0$, then $h(x)$ is a polynomial of degree 2.

iii) If F and G are two antiderivatives of the same function f , then $F(b) - F(a) = G(b) - G(a)$ for every a and b .

iv) For every positive function $f(x)$, we have $\int_a^b \sqrt{f(x)} dx = \sqrt{\int_a^b f(x) dx}$.

v) If $f(x)$ is continuous on $[0, 2]$ and $\int_0^2 f(x) dx = 0$, then $f(x) = 0$ for all $x \in [0, 2]$.

vi) If f is a continuous function, then $f'(4) = \lim_{x \rightarrow 4} \frac{\int_4^x f(t) dt}{x-4}$.

4. Find the points (x, y) on the curve $x^2 - xy + y^2 = 3$ where the tangent lines are horizontal.

5. Find the volume of the solid obtained by rotating the region enclosed by $y = x^3$ and $y = x^2$ around the x -axis.

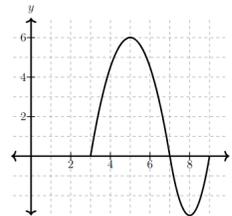
6. Find the area of the region enclosed by $x = y^2 - 2 + \cos(y)$ and $x = y + \cos(y)$.

7. Suppose that $f(5) = 13$ and $\int_5^{10} f'(x) dx = 30$. Find $f(10)$.

8. Beside is the graph of a function $f(t)$. Let $g(x) = \int_3^x f(t) dt$.

i) For what value of $x \in [3, 9]$ does $g(x)$ have an absolute maximum?

ii) In what intervals is $g(x)$ concave-up?



9. Find the critical points of the function

$$f(x) = \int_1^{x^2-2x} (\sin^2(t^2) + 1)(t+1) dt .$$