HOMEWORK 8

1. Compute the first derivative of the following functions

    i) $f(x) = \frac{\arcsin(x)}{1+e^{3x}}$ iii) $f(x) = x^e e^x$

    ii) $f(x) = x \ln(x^2 + 1)$ iv) $f(x) = \frac{x\sqrt[3]{x+2}}{e^{4x}\cos(x)}$

2. Compute the following limits

    i) $\lim_{x\to-\infty} \frac{x^5-3x^3+x+2}{x^5-2x^2+2x+1}$ iv) $\lim_{x\to\frac{\pi}{2}^+} \frac{\sqrt{1+\sin(x)}}{\cos(x)}$

    ii) $\lim_{x\to 0^+} (1+3x)^{\frac{1}{x}}$ v) $\lim_{x\to 0} x\sin\left(\frac{1}{x^3}\right)$

    iii) $\lim_{x\to+\infty} e^x - e^{2x} + e^{-x}$ vi) $\lim_{x\to+\infty} \frac{\sqrt{4x^4-1}}{16x^2+1}$

3. You have 800 feet of fencing to make a pen for hogs. If you have a river on one side of your property, what is the dimension of the rectangular pen that maximizes the area?

4. Find the tangent line to the curve of equation $x^3 - x\ln(y) + y^3 = 2x + 5$ at the point $(2,1)$.

5. Consider the function $f(x) = \frac{3x^2-3x+1}{3x-1}$.

    i) Determine for which values of x the function is continuous.

    ii) Find and classify the critical points of $f(x)$.

    iii) Determine the intervals where the function is increasing.

    iv) Determine the intervals where the function is concave-up.

    v) Compute $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

    vi) Compute $\lim_{x\to 1/3^+} f(x)$ and $\lim_{x\to 1/3^-} f(x)$.

    vii) Sketch the graph of $f(x)$.