EXERCISE SHEET : RIEMANN SUMS AND DEFINITE INTEGRALS

1. State whether the given sums are equal or not.

i) $\sum_{i=1}^{10} i$ and $\sum_{k=1}^{10} k$

iii) $\sum_{i=1}^{10} i(i-1)$ and $\sum_{j=0}^{9}(j+1)j$

ii) $\sum_{i=1}^{10} i$ and $\sum_{i=6}^{15}(i-5)$

iv) $\sum_{i=1}^{10} i(i-1)$ and $\sum_{k=1}^{10}(k^2-k)$

2. Estimate the definite integral $\int_{2}^{3} \frac{1}{x-1} \, dx$ using Riemann sum :
— Divide the interval $[2,3]$ in ten subintervals ;
— In each subinterval, evaluate $f(x) = \frac{1}{x-1}$ at the left endpoint of the subinterval.
— Write and compute the Riemann sum.
Will the result be bigger or smaller than the actual value of the integral ?

3. Estimate the definite integral $\int_{0}^{2}(x^2 - 2x + 1) \, dx$ using Riemann sum :
— Divide the interval $[0,2]$ in ten subintervals ;
— In each subinterval, evaluate $f(x) = x^2 - 2x + 1$ at the right endpoint of the subinterval.
— Write and compute the Riemann sum.

4. Compute $\int_{0}^{1} \sin(2\pi x) \, dx$.

5. Compute $\int_{-2}^{2} |x - 1| \, dx$.