

# Single Variable Calculus I

## Math 101, Spring 2020

This exam has 10 problems worth 117 points distributed over 9 pages, including this one.

**Instructions:** . This is a two hour exam. You may not consult any notes or books during the exam, and no calculators are allowed. Show all of your work on each problem and carefully justify all answers. Points will be deducted for irrelevant, incoherent or incorrect statements, and no points will be awarded for illegible work. If you run out of room, you may work answers on the back of pages or on attached scratch paper. Be sure to clearly indicate when work is continued on another page.

Name:

Honor Pledge: *On my honor, I have neither given nor received any unauthorized aid on this exam.*

Signature:

Question	Points	Score
1	10	
2	23	
3	15	
4	8	
5	6	
6	16	
7	6	
8	7	
9	17	
10	9	
Total:	117	

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1. (10 points) Answer true or false and circle it. No explanation is necessary.

(a) If  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x)$ , then  $f$  is continuous at  $x = 1$ .

*TRUE*

*FALSE*

(b) The function  $f(x) = |x|$  is not differentiable at  $x = 0$ .

*TRUE*

*FALSE*

(c) An object moves along a straight line. If its velocity is negative at  $t = 1$  and its acceleration is negative at  $t = 1$ , then the object is speeding up at  $t = 1$ .

*TRUE*

*FALSE*

(d) The derivative of  $f(x) = \sqrt{x}$  is not defined at  $x = 0$ .

*TRUE*

*FALSE*

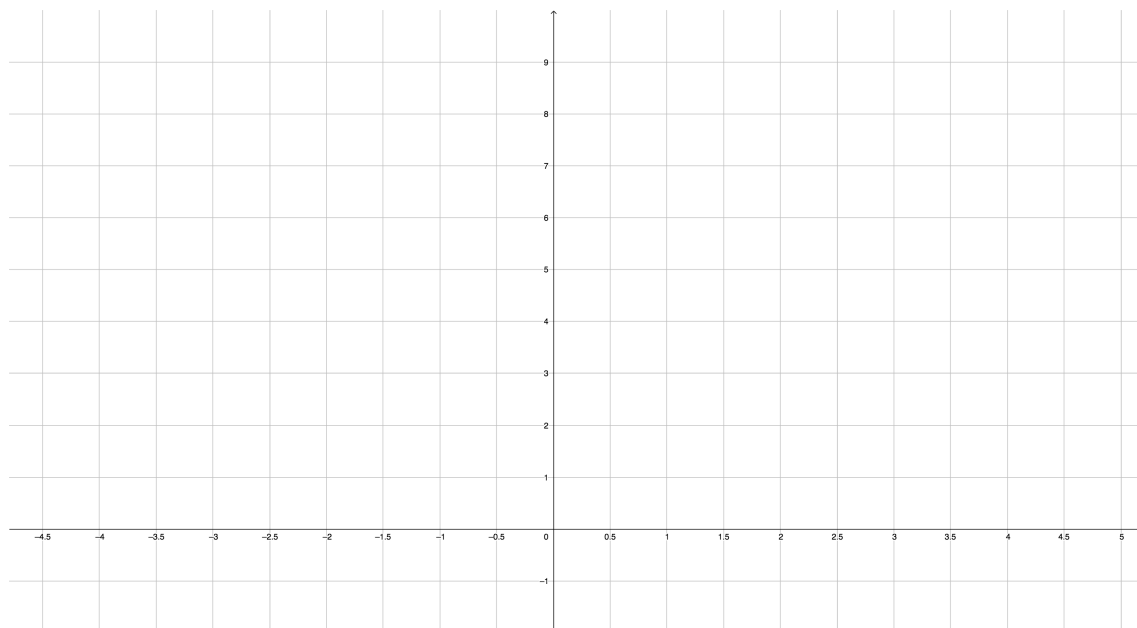
(e) If  $f(x)$  and  $g(x)$  are continuous, then  $\frac{f(x)}{g(x)^2+1}$  is continuous.

*TRUE*

*FALSE*

2. (a) (5 points) Draw the graph of the following function

$$f(x) = \begin{cases} -2x & x < -2 \\ 8 + 2x & -2 \leq x < 0 \\ [x] & 0 \leq x < 3 \\ x - 2 & x \geq 3 \end{cases}$$



- (b) (4 points) Indicate at which points the function  $f(x)$  is not continuous.
- (c) (2 points) Indicate at which points the function is continuous but not differentiable.
- (d) (12 points) Compute then the following limits (if they exist)

$$\lim_{x \rightarrow -2^+} f(x)$$

$$\lim_{x \rightarrow 3} (x - 3)f(x)$$

$$\lim_{x \rightarrow 0^-} f(x)^2$$

$$\lim_{x \rightarrow -2^+} \sqrt{f(x)}$$

$$\lim_{x \rightarrow 1^-} f(x)$$

$$\lim_{x \rightarrow 0^+} \frac{1}{f(x)}$$

3. Compute the following limits

(a) (5 points)

$$\lim_{x \rightarrow 1^-} \frac{x^2 + 3x + 2}{x^2 - 3x + 2}$$

(b) (5 points)

$$\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{x - 2}$$

(c) (5 points)

$$\lim_{x \rightarrow -1} \frac{2x^2 + 3x + 1}{x^2 + 3x + 2}$$

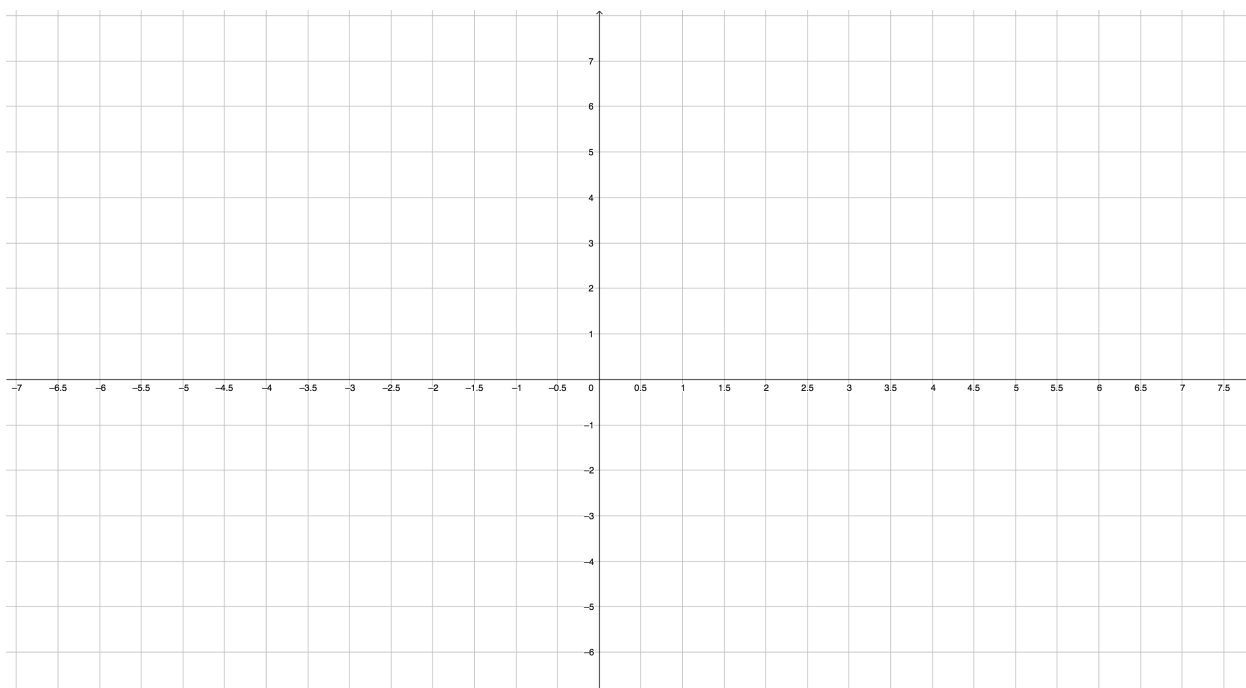
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4. (8 points) Using the definition of derivative, compute  $f'(1)$  where  $f(x) = \frac{1}{\sqrt{x+1}}$ .

5. (6 points) Find the equation of the tangent line to the graph of  $f(x) = e^x + x + \cos(x)$  at the point  $x = 0$ .

6. (16 points) Sketch the graph of a function  $f(x)$  that satisfies all of the following properties:

- $f(x)$  is continuous for  $x \neq 0$ .
- $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = 1$ .
- $f'(x) > 0$  for  $x > 1$ .
- $f''(x) < 0$  for  $x > 1$ .
- $f(x)$  is not differentiable at  $x = 1$ .
- $f'(x) = 0$  for  $0 < x < 1$ .
- $f(-1) = -1$ .
- $f'(x) > 0$  for  $x < 0$ .



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7. (6 points) Show that the equation  $x \sin(x) = 1$  has a solution in the interval  $[0, \frac{\pi}{2}]$ .

8. (7 points) Find the intervals in which the function  $f(x) = \frac{\sqrt{2}}{2}x + \cos(x)$  defined for  $x \in [0, 2\pi]$  is increasing.

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9. The function  $s(t) = -t^2 + 2t + 3$  represents the height in inches of a ball thrown vertically, where time  $t \geq 0$  is measured in seconds.

- (a) (2 points) From which height was the ball thrown? (Use the correct units)
  
  
  
  
  
  
  
  
  
  
- (b) (2 points) What was the initial velocity of the ball? (Use the correct units)
  
  
  
  
  
  
  
  
  
  
- (c) (3 points) At what time does the ball reach its maximum height? (Use the correct units)
  
  
  
  
  
  
  
  
  
  
- (d) (2 points) Is the acceleration 0, when the ball reaches its maximum height?
  
  
  
  
  
  
  
  
  
  
- (e) (2 points) When does the ball hit the ground? (Use the correct units)
  
  
  
  
  
  
  
  
  
  
- (f) (3 points) What is the velocity of the ball when it hits the ground? (Use the correct units)
  
  
  
  
  
  
  
  
  
  
- (g) (3 points) What is the average velocity of the ball from  $t = 0$  to the time it hits the ground? (Use the correct units)

10. Below are the graphs of a function  $f(x)$  and its first and second derivative. Indicate which is which.

