
HOMEWORK 9

1. Write the Riemann sum for the function $f(x) = \ln(x)$ defined on $[1, 2]$ using 10 sub-intervals of the same length and evaluating the function at the left end-point of each sub-interval.
2. Consider the function $f(x)$ defined by

$$f(x) = \begin{cases} 1 - |x - 1| & 0 \leq x < 2 \\ 2 - |x - 4| & 2 \leq x < 6 \\ 3 - |x - 9| & 6 \leq x \leq 12 \end{cases}.$$

Compute $\int_0^{12} f(x) \, dx$.

3. Consider the function $f(x)$ defined by

$$f(x) = \begin{cases} \sqrt{1 - (x - 1)^2} & 0 \leq x < 2 \\ |x - 4| - 2 & 2 \leq x < 6 \end{cases}.$$

Compute $\int_0^6 f(x) \, dx$.

Hint : The graph of the first function is a semi-circle centered at $(1, 0)$ of radius 1.

4. It is given that $\int_0^4 f(x) \, dx = 5$, $\int_0^2 f(x) \, dx = -3$, $\int_0^4 g(x) \, dx = -1$ and $\int_0^2 g(x) \, dx = 2$. Compute the following integrals

i) $\int_0^4 (f(x) + g(x)) \, dx$

iii) $\int_0^2 (f(x) - g(x)) \, dx$

ii) $\int_2^4 (f(x) + g(x)) \, dx$

iv) $\int_2^4 (4f(x) - 3g(x)) \, dx$