Single Variable Calculus I
Math 101, Spring 2020

This exam has 9 problems worth 115 points distributed over 10 pages, including this one.

Instructions: You may work on this exam for at most two hours. Time is supposed to start when you turn this page. You may not consult any notes or books during the exam, and no calculators are allowed. Show all of your work on each problem and carefully justify all answers. Points will be deducted for irrelevant, incoherent or incorrect statements, and no points will be awarded for illegible work. If you have a tablet or a printer, please write your answers in the spaces provided. Otherwise, make sure to clearly indicate which question you are answering in each page and include the signed honor pledge at the top of the first page.

Deadline for submission: Please upload on canvas a pdf file with your answers by April 1, at 11am.

Name:

Honor Pledge: *On my honor, I have neither given nor received any unauthorized aid on this exam.*

Signature:

Question	Points	Score
1	10	
2	14	
3	20	
4	15	
5	8	
6	5	
7	20	
8	8	
9	15	
Total:	115	

1. (10 points) Answer true or false and circle it. No explanation is necessary.

   (a) Let $f(x)$ and $g(x)$ be differentiable functions. If $f(x)$ and $g(x)$ are increasing in the interval (a, b), then $f(x)g(x)$ is increasing in (a, b).

   TRUE *FALSE*

   (b) Let $f(x)$ be a differentiable function defined on the interval $(-\infty, +\infty)$ such that $\lim_{x\to\pm\infty} f(x) = -\infty$. Then $f(x)$ has a local maximum.

   TRUE *FALSE*

   (c) If $\lim_{x\to 1} f(x) = 0$ and $\lim_{x\to 1} g(x) = 0$, then $\lim_{x\to 1} f(x)^{g(x)} = 0$.

   TRUE *FALSE*

   (d) The rectangle with perimeter 16cm and biggest area is a square of side 4cm.

   TRUE *FALSE*

   (e) Every differentiable function has at least one critical point.

   TRUE *FALSE*

2. Compute the derivative of the following functions:

   (a) (7 points) $f(x) = \frac{e^x \cos(2x)}{x \tan(x)}$

   (b) (7 points) $g(x) = 2^x(\log_3(4x) - \arcsin(x^2))$

3. Consider the function $f(x) = e^{x^3-2x^2-7x}$.

   (a) (5 points) Find the critical points of $f(x)$.

   (b) (6 points) Classify the critical points of $f(x)$.

   (c) (4 points) Does $f(x)$ have a global maximum or minimum?

4. (15 points) Compute the following limits:

(a)

$$\lim_{x\to 0} \frac{\sin(x)}{\arctan(x)}$$

(b)

$$\lim_{x\to +\infty} x^2 \sin\left(\frac{1}{x}\right)$$

(c)

$$\lim_{x\to 0^+} \left(\frac{1}{x}\right)^x$$

5. (8 points) Find the equation of the line tangent to the curve of equation $y^3 - xy + e^{xy} = 2$ at the point $x = 0$.

6. (5 points) It is given that $f(x)$ is a differentiable function such that $f(0) = 1$ and $f'(0) = -2$. Let $g(y) = f^{-1}(y)$ denote the inverse of $f(x)$. Compute the derivative of $h(y) = g(y)e^{g(y)}$ at $y = 1$.

7. Consider the function $f(x) = e^{2x} - 3e^x + 2$

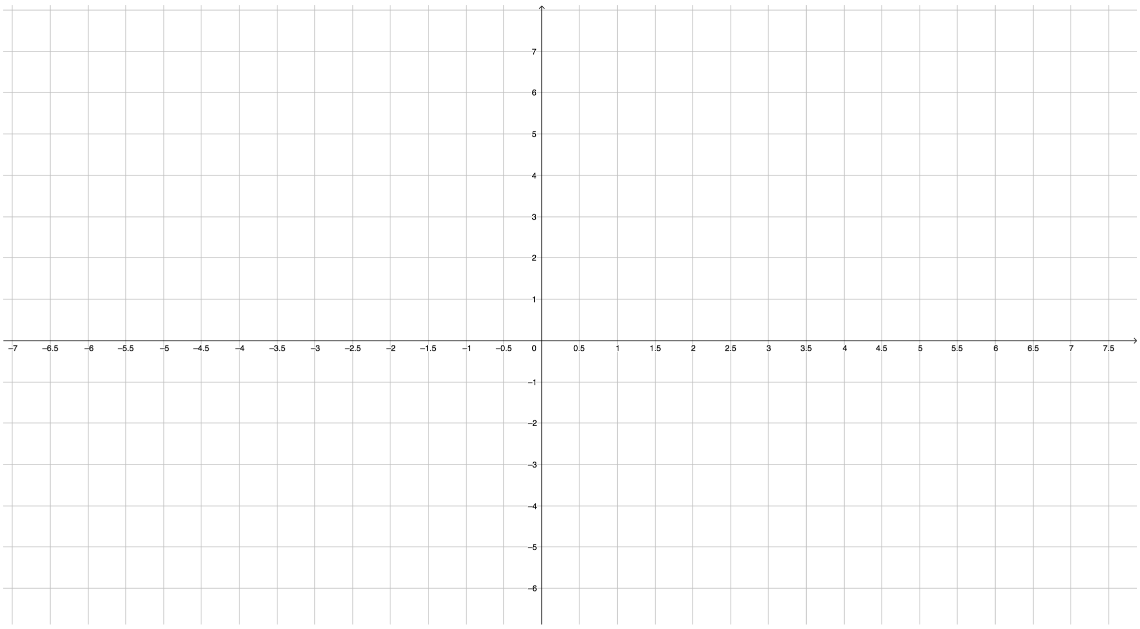
   (a) (4 points) Find and classify the critical points of $f(x)$.

   (b) (4 points) Find the inflection points of $f(x)$ and determine the intervals where $f(x)$ is concave-up.

   (c) (2 points) Compute $\lim_{x\to+\infty} e^{2x} - 3e^x + 2$

(d) (2 points) Compute $\lim_{x\to-\infty} e^{2x} - 3e^{x} + 2$

(e) (8 points) Sketch the graph of $f(x)$.



8. (8 points) You need to construct a fence around an area of 1600 square feet. What are the dimensions of the rectangular pen to minimize the amount of material needed?

9. (15 points) Match the following functions with their graph.

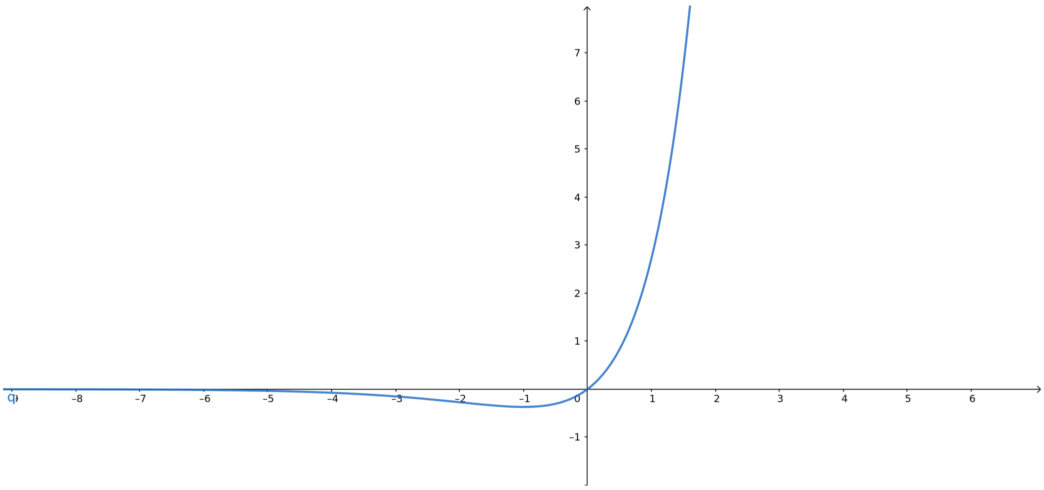
a) $f(x) = \frac{x+2}{x-3}$

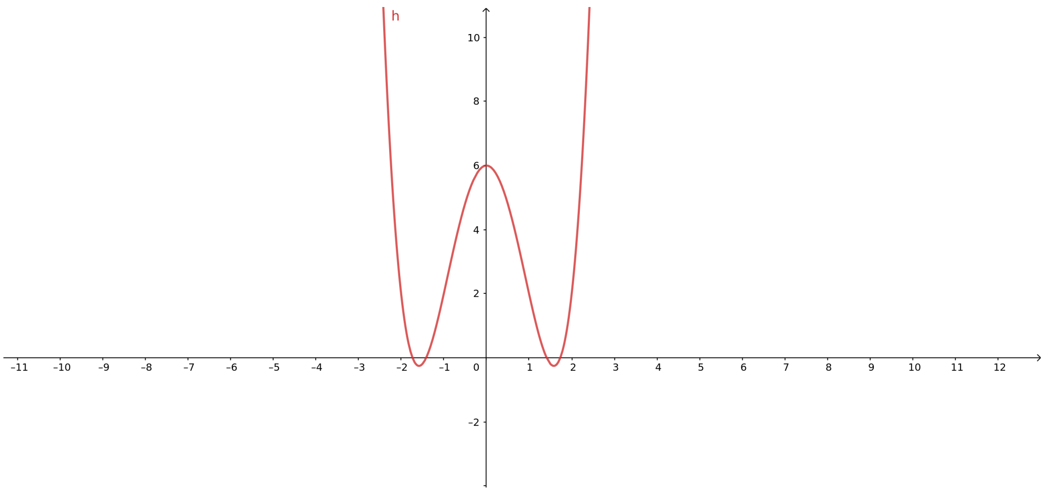
b) $f(x) = \sqrt{x^2 - 5x + 6}$

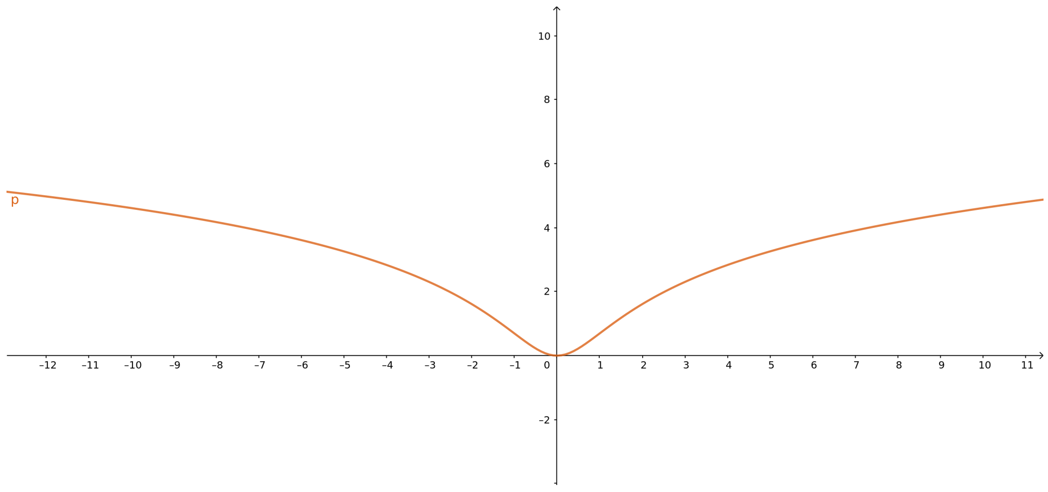
c) $f(x) = x^4 - 5x^2 + 6$

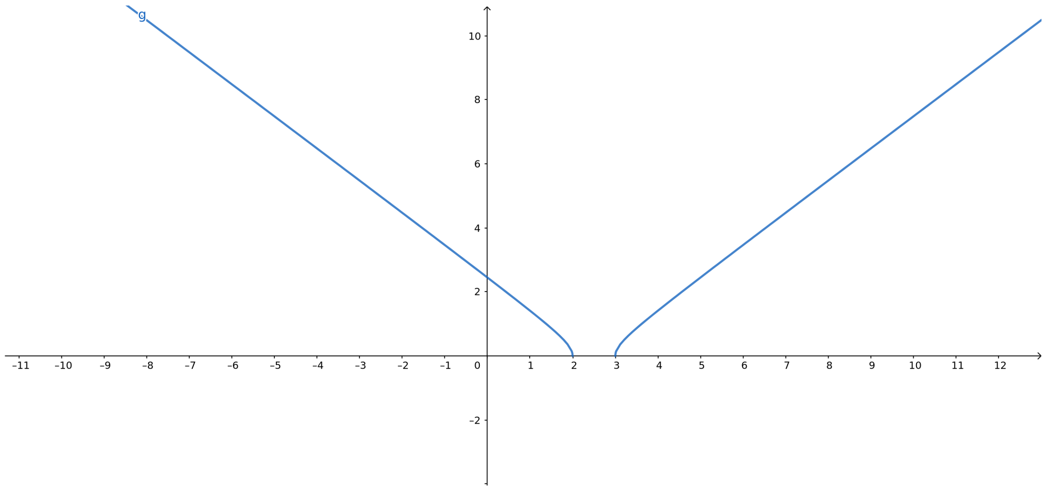
d) $f(x) = \ln(x^2 + 1)$

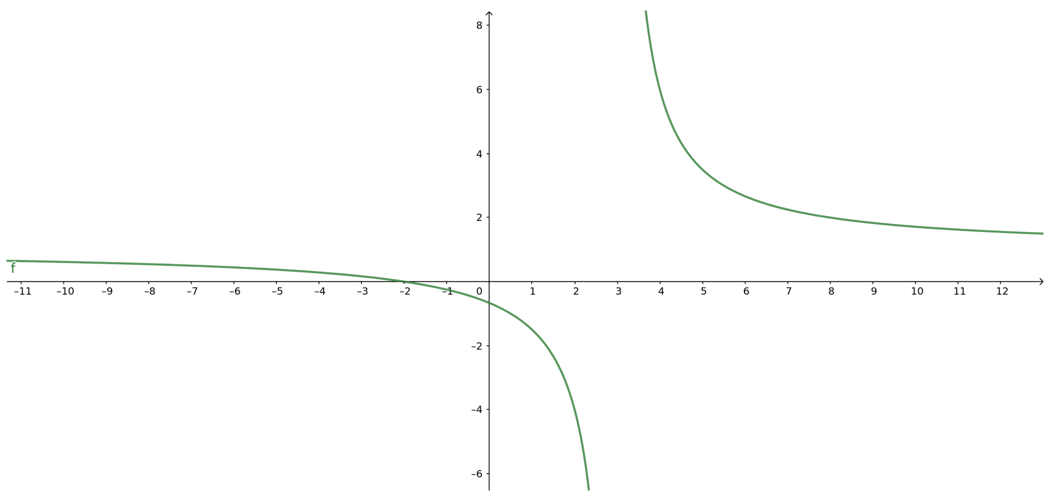
e) $f(x) = xe^x$






p


g


f