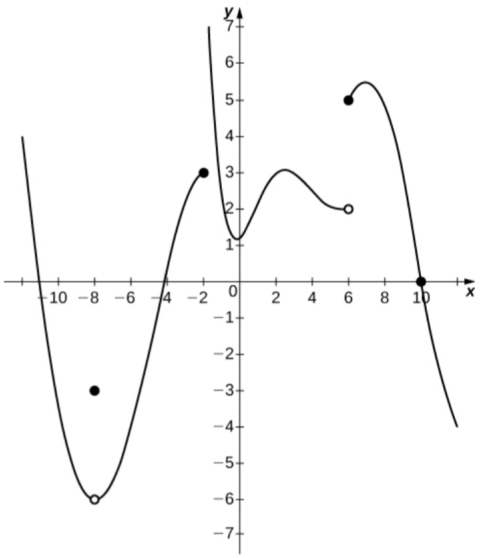
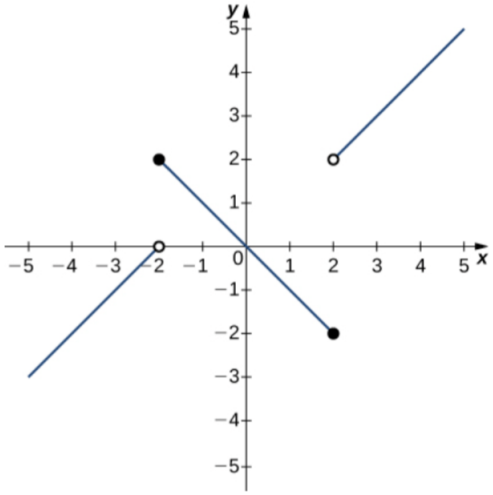
For questions: at52@rice.edu January 22

IN-CLASS ACTIVITY : LIMIT LAWS AND CONTINUITY

1. Consider the functions $f(x)$ and $g(x)$ graphed below.





Using the limit laws, compute the following limits (if they exist) :

i) $\lim_{x\to -8^-} f(x)$

ii) $\lim_{x\to -8} f(x)^2$

iii) $\lim_{x\to -2^+} f(x)g(x)$

iv) $\lim_{x\to 2^-}(f(x)+g(x))$

v) $\lim_{x\to 2^+}(f(x)-g(x))$

vi) $\lim_{x\to 2}(f(x)g(x))$

vii) $\lim_{x\to -2^-} \frac{f(x)}{g(x)}$

viii) $\lim_{x\to 6^+} \frac{1}{f(x)}$

ix) $\lim_{x\to 6^-} \sqrt{f(x)}$

Can you find an explicit formula for the function $g(x)$?
(*Hint : it is a function defined by cases. Do you remember how to write the equation of a line ?*)

2. Using a calculator, compute the values of the following quotients :

i) $\frac{0.013}{0.000048}$

ii) $\frac{0.000053}{0.018}$

iii) $\frac{0.00011}{0.000093}$

What does it suggest about the limit of a quotient when both numerator and denominator tend to 0 ?

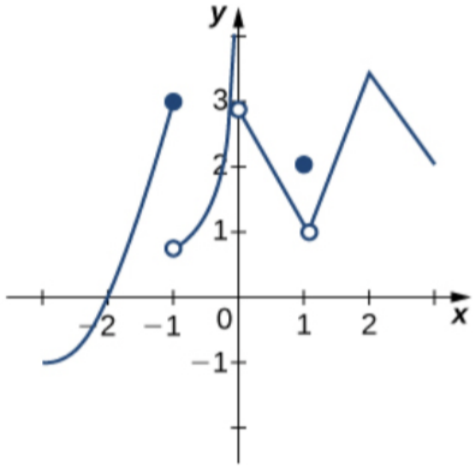
3. Assume that $\lim_{x\to 6} f(x) = 4$ and $\lim_{x\to 6} g(x) = 9$. Use the limit laws to evaluate each of the following limits :

i) $\lim_{x\to 6} 2f(x)g(x)$
ii) $\lim_{x\to 6} \frac{g(x)-1}{f(x)}$
iii) $\lim_{x\to 6} \left(f(x) + \frac{1}{3}g(x)\right)$
iv) $\lim_{x\to 6} \frac{f(x)^3}{2}$
v) $\lim_{x\to 6}(xf(x))$
vi) $\lim_{x\to 6}(f(x)^2 - xg(x))$

4. Compute the following limits using the limit laws :

i) $\lim_{x\to 0}(4x^2 - 2x + 3)$
ii) $\lim_{x\to 1} \frac{x^3+3x^2+5}{4-7x}$
iii) $\lim_{x\to -2} \sqrt{x^2 - 6x + 3}$
iv) $\lim_{x\to -1}(9x + 1)^2$
v) $\lim_{x\to 7} x^2$
vi) $\lim_{x\to -2}(4x^2 - 1)$
vii) $\lim_{x\to 0} \frac{1}{1+\sin(x)}$
viii) $\lim_{x\to 1} \frac{2-7x}{x+6}$
ix) $\lim_{x\to 3} e^{3x}$

5. Consider the function $f(x)$ graphed below :



Find all values x_0 for which the function is not continuous at $x = x_0$. For all such values, compute, if they exist, the left and right limits.

6. Find the value of k that makes the follwing function continuous :

$$f(x) = \begin{cases} \sqrt{kx} & \text{if } 0 \leq x \leq 3 \\ x + 1 & \text{if } 3 < x \leq 10 \end{cases}.$$