Esercitazione 20-12-2021

Esercizio 1 Ipotesi equivale a
guarito dalla defecatio isterica $\Rightarrow$ curato dal Prof. Sassaroli

Laura	F
Alessia	F
Francesco	V
Andrea	F

Esercizio 2 $A = \bigcup_{i=0}^{100} \left(i - \frac{1}{2}, i + \frac{1}{2}\right]$

$$\left(-\frac{1}{2}, \frac{1}{2}\right] \cup \left(\frac{1}{2}, \frac{3}{2}\right] \cup \left(\frac{3}{2}, \frac{5}{2}\right] \cup \dots = \left(-\frac{1}{2}, \frac{201}{2}\right] = A$$

$\sup(A) = \max(A) = \frac{201}{2}$ $\quad \inf(A) = -\frac{1}{2}$ $\quad \min(A)$ N.E.

Esercizio 3 $\quad A = \bigcup_{i \in \mathbb{N}} \left[i - \frac{1}{2}, i + \frac{1}{2} \right)$

$$\left[-\frac{1}{2}, \frac{1}{2}\right) \cup \left[\frac{1}{2}, \frac{3}{2}\right) \cup \left[\frac{3}{2}, \frac{5}{2}\right) \dots = \left[-\frac{1}{2}, +\infty\right)$$

$\sup(A) = +\infty \quad \max(A)$ N.E. $\quad \inf(A) = \min(A) = -\frac{1}{2}$

Esercizio 4 $\quad A = \bigcup_{i \in \mathbb{Z}} \left(i - \frac{1}{2}, i + \frac{1}{2} \right]$

$$\dots\left(-\frac{5}{2}, -\frac{3}{2}\right] \cup \left(-\frac{3}{2}, -\frac{1}{2}\right] \cup \left(-\frac{1}{2}, \frac{1}{2}\right] \cup \left(\frac{1}{2}, \frac{3}{2}\right] \cup \dots = (-\infty, +\infty) = \mathbb{R}$$

Se fosse $\quad B = \bigcup_{i \in \mathbb{Z}} \left(i - \frac{1}{2}, i + \frac{1}{2} \right)$

$$B \neq \mathbb{R} \qquad B = \mathbb{R} - \left\{ \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \dots \right\}$$

$\sup(A) = \sup(B) = +\infty \quad \inf(A) = \inf(B) = -\infty \quad \min(A), \max(A)$ N.E.

Esercizio 5 $\quad A = \left\{ (-1)^n \, e^{\frac{n+2}{n-1}} : n \in \mathbb{N} - \{1\} \right\}$

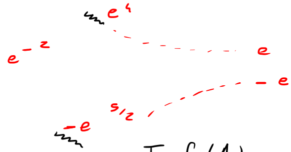
$\frac{n+2}{n-1} \qquad n_1 > n_2 > 1 \qquad$ andamento dell'esponente

$$\frac{n_1+2}{n_1-1} - \frac{n_2+2}{n_2-1} = \frac{\cancel{n_1 n_2} - n_1 + 2n_2 - \cancel{2} - \cancel{n_1 n_2} + n_2 - 2n_1 + \cancel{2}}{(n_1-1)(n_2-1)}$$

$$= \frac{3(n_2 - n_1)}{(n_1-1)(n_2-1)} < 0 \quad \text{mon decrescente per } n_1, n_2 > 1$$

Per $n = 0$ si ottiene -2

$e^{\frac{n+2}{n-1}}$ è mon dec per $n > 1$



Per n pari > 1 $\quad e^{\frac{n+2}{n-1}}$ è dec

Per n dispari > 1 $\quad -e^{\frac{n+2}{n-1}}$ è cresc

$n = 0 \quad e^{-2} \qquad n = 2 \quad e^4 \qquad$ pari dec fino a e

$n = 3 \quad -e^{5/2} \qquad$ dispari cres fino a $-e$

$\text{Inf}(A) = \min(A) = -e^{5/2}$

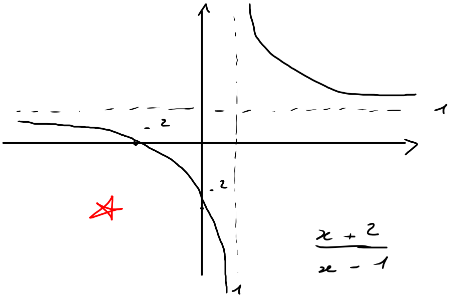
$\text{Sup}(A) = \max(A) = e^4$

Esercizio 6

$$f(x) = \ln\left|\frac{x+2}{x-1}\right| \qquad x \neq -2 \quad x \neq 1$$

$$\mathrm{Dom}(f) = \mathbb{R} - \{1, -2\}$$

$$f(x) = \begin{cases} \ln\left(\frac{x+2}{x-1}\right) & x \in (-\infty, -2) \cup (1, +\infty) \\ \ln\left(\frac{x+2}{1-x}\right) & x \in (-2, 1) \end{cases}$$



$\frac{x+2}{x-1}$

Con calcoli simili a quelli dell'Es 5 si osserva che $\frac{x+2}{x-1}$ è mon dec in $(-\infty, 1)$ e in $(1, +\infty)$

$$\lim_{x \to \pm\infty} \frac{x+2}{x-1} = 1$$

$$\lim_{x \to 1^-} \frac{x+2}{x-1} = -\infty \qquad \lim_{x \to 1^+} \frac{x+2}{x-1} = +\infty$$

✱ Grafico di $\left|\dfrac{x+2}{x-1}\right|$

$\xrightarrow{\ln}$ Rispetta la monotonia perché ln cresc

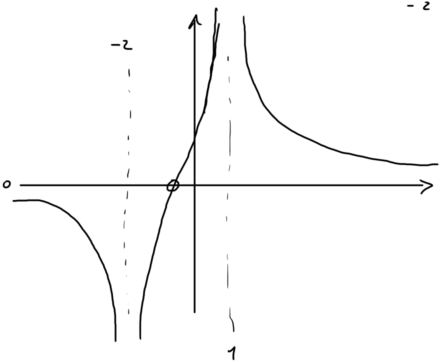


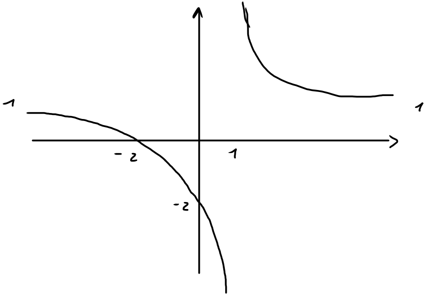
Grafico di $\ln\left|\dfrac{x+2}{x-1}\right|$

$Im(f) = \mathbb{R}$

f non è iniettiva $\Rightarrow$ non è invertibile

Esercizio 7

$f(x) = \operatorname{arctg}\left(\frac{x+2}{x-1}\right)$ $\qquad$ $\mathrm{Dom}(f) = \mathbb{R} - \{1\}$



$\frac{x+2}{x-1} \xrightarrow{\operatorname{arctg}}$ risp la monotonia

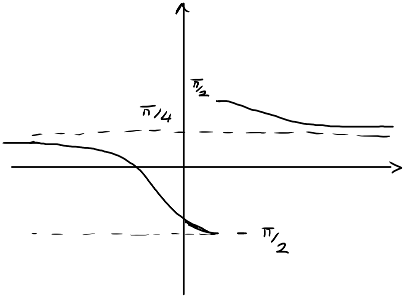


Grafico di $\operatorname{arctg}\left(\frac{x+2}{x-1}\right)$

$\mathrm{Im}(f) = \left(-\frac{\pi}{2}, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$f: \mathbb{R} - \{1\} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ è biunivoca

$y = \operatorname{arctg}\left(\frac{x+2}{x-1}\right) \xrightarrow{tg} tg(y) = \frac{x+2}{x-1}$ $\qquad$ $x(tg(y) - 1) = 2 + tg(y)$

$$x = \frac{2 + tg(y)}{tg(y) - 1} = f^{-1}(y)$$

Esercizio 8

A. $\lim_{x \to 1} \dfrac{e^{(x-1)^2} - 2x^2 + 4x - 3}{(x-1)\operatorname{sen}(x-1)} = -1$ $\qquad \dfrac{1-2+4-3}{0}$ F.I. $\dfrac{0}{0}$

$$\frac{e^{(x-1)^2} - 1 - 2x^2 + 4x - 2}{(x-1)^2} \cdot \frac{(x-1)}{\operatorname{sen}(x-1)}$$

$$= \Bigg[\underbrace{\frac{e^{(x-1)^2} - 1}{(x-1)^2}}_{\text{L.N. } 1} - \underbrace{\frac{2\cancel{(x-1)^2}}{\cancel{(x-1)^2}}}_{-2} \Bigg] \cdot \underbrace{\frac{(x-1)}{\operatorname{sen}(x-1)}}_{\text{L.N. } 1} \xrightarrow[x \to 1]{} -1$$

$x^{100} \ln(x+2)$

B. $\lim_{x \to +\infty} \dfrac{\ln (x+2)^{x^{100}} - 2^x + 5^x}{(2e)^x - \operatorname{sen}(e^{x^3}) + x^{210}} =$

$$\lim_{x \to +\infty} \underbrace{\frac{5^x}{(2e)^x}}_{\left(\frac{5}{2e}\right)^x} \; \frac{\left(\frac{\ln (x+2)^{x^{100}}}{5^x} - \left(\frac{2}{5}\right)^x + 1 \right)}{\left(1 - \frac{\operatorname{sen}(e^{x^3})}{(2e)^x} + \frac{x^{210}}{(2e)^x} \right)} = 0$$

$\dfrac{5}{2e} < 1$

numeratore $\nearrow 1$, denominatore $\searrow 1$

C. $\lim_{x \to +\infty} \left(\frac{3x^3 - 2x + 1}{3x^3 + 5} \right)^{\frac{1}{\operatorname{sen}^2(1/x)}} = e^{-2/3}$ F. I. 1^∞

$$\left(\frac{3x^3 - 2x + 1}{3x^3 + 5} \right)^{\frac{1}{\operatorname{sen}^2(1/x)}} = \left(1 + \frac{-2x - 4}{3x^3 + 5} \right)^{\frac{1}{\operatorname{sen}^2(1/x)}} =$$

$$\Bigg[\underbrace{\left(1 - \frac{2x + 4}{3x^3 + 5} \right)^{-\frac{3x^3+5}{2x+4}}}_{\to e} \Bigg]^{-\frac{2x+4}{3x^3+5} \cdot \underbrace{\frac{1/x^2}{\operatorname{sen}^2(1/x)}}_{\to 1} \cdot x^2} \longrightarrow e^{-2/3}$$

D. $\lim_{n \to +\infty} ((6n)!\,n!) - (7n)!$ F. I. $+\infty - \infty$

Per vedere chi domina calcolo $\lim_{n \to +\infty} \frac{(6n)!\,n!}{(7n)!}$ con il criterio del rapporto

$$\frac{(6n+6)!\,(n+1)!}{(7n+7)!} \cdot \frac{(7n)!}{(6n)!\,n!} \longrightarrow \frac{6^6}{7^7} < 1 \Rightarrow$$

$$\Rightarrow \lim_{n \to +\infty} \frac{(6n)!\,n!}{(7n)!} = 0$$

$$\Rightarrow \lim_{n \to +\infty} (7n)! \left[\underbrace{\frac{(6n)!\,n!}{(7n)!}}_{\to 0} - 1 \right] = -\infty$$