

CURVE ELLITTICHE – ELLIPTIC CURVES 2024/25

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An elliptic curve over a field K is a smooth projective plane curve E that is defined by a cubic equation with coefficients in K and has a point O with coordinates in K . What distinguishes these curves from others is that there is a geometrically definable addition operation on their set of points that gives it the structure of an abelian group with zero element O . (Changing O gives an isomorphic group.) Over the complex numbers one may identify E together with its group structure with a complex torus (i.e. the quotient of the complex plane by a lattice Λ). But over smaller fields the group of points may be much smaller. For instance, by a theorem of Mordell over $K = \mathbf{Q}$ it is a finitely generated abelian group.

Our main focus in this class will be on fields of arithmetic interest such as finite fields, the field \mathbf{Q} or fields of p -adic numbers. The main theorems we shall prove are valid in much greater generality, so what we shall see are in fact baby cases of some of the fundamental results and conjectures in arithmetic geometry. This is true of Mordell's theorem mentioned above but also of the Hasse–Weil theorem which was historically the first nontrivial proven case of Weil's Riemann Hypothesis for varieties over finite fields. We shall also encounter the Birch–Swinnerton-Dyer conjecture which is a million dollar open problem in itself but at the same time only a tiny piece in a fascinating set of very general conjectures proposed by Deligne and Beilinson. If the course is successful, you will be eager to learn more about arithmetic geometry once it is over.

Prerequisites. Background from basic algebraic geometry, algebraic number theory and homological algebra will be introduced along the way. It helps if you have already followed courses on some of these topics (or are following them in this semester) but it is not strictly necessary.

Endorsements

Rocco S: Fantastico! Mi piaciono le curve, in generale.

Donald T: Same as last time. COMPLETE BULLSHIT. @Rocco: Drill, baby, drill!

IMPORTANT: The class starts on Monday 7 October 2024. There will be no lectures in the weeks of 23 and 30 September.