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Spectral mass gauging of unsettled liquid with acoustic waves

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Abstract. Propellant mass gauging is one of the key technologies required to enable the next step in NASA's space exploration program. At present, there is no reliable method to accurately measure the amount of unsettled liquid propellant in a large-scale propellant tank in micro- or zero gravity. Recently we proposed a new approach to use sound waves to probe the resonance frequencies of the two-phase liquid-gas mixture and take advantage of the mathematical properties of the high frequency spectral asymptotics to determine the volume fraction of the tank filled with liquid. We report the current progress in exploring the feasibility of this approach in the case of large propellant tanks, both experimental and theoretical. Excitation and detection procedures using solenoids for excitation and both hydrophones and accelerometers for detection have been developed. A 3% uncertainty for mass-gauging was demonstrated for a 200-liter tank partially filled with liquid for various unsettled configurations, such as tilts and artificial ullages.

1. Introduction

The ability to quickly and accurately gauge the amount of the available propellant in a large-scale cryogenic propellant tank is one of the basic requirements to a successful tank design [1]. Under settled conditions, a wide range of mass gauging techniques are available [2–4]. These techniques usually work by determining the location of the liquid/gas interface in the tank with the help of, e.g., wet/dry sensors, and using it to infer the liquid propellant volume from the knowledge of the tank geometry. However, propellant mass gauging becomes a significant challenge under microgravity conditions, since in this case both the location and the shape of the propellant are a priori unknown.

At present, several advanced techniques for propellant mass gauging under reduced gravity conditions have been developed, including the radio frequency mass gauging

(RFMG) technology [2, 5, 6] and Modal Propellant Gauging (MPG) technique [7, 8]. Both techniques use proprietary pattern matching algorithm to compare the measured radio waves (RFMG) or acoustic (MPG) resonance frequencies with a database of the eigenfrequencies measured or computed under various assumptions about the liquid configurations. The best match thus obtained is used to predict the fill level. The pattern-matching techniques were demonstrated to perform well under fully settled conditions and are currently considered among the best to operate under microgravity conditions in large-scale propellant tanks. At the same time, the results of low-g testing exhibit strong temporal variability, indicating a significant amount of sloshing and/or fluid motion driven by capillary forces. As a consequence, the predicted fill level oscillated wildly, essentially precluding a possibility of an accurate fill level measurement.

The differences between the settled vs. unsettled RF mass-gauging performance of pattern-matching methods are not surprising, since there exists a fundamental mathematical difficulty, common also to many other inverse problems, in using spectral information alone to infer the shape of heterogeneous wave-carrying media [9, 10]. Such inverse problems are notoriously ill-posed (for a discussion of various mathematical issues in closely related contexts, see, e.g., [11, 12]). In particular, any kind of regularization of the continuum model required to infer the shape of the ullage from the spectral data would also suffer from strong sensitivity to noise. As a consequence, for small but fixed amount of noise the pattern-matching performance would be expected to degrade above a certain noise-dependent level of spatial resolution. This imposes intrinsic limits on the accuracy of pattern-matching approaches in general, even when measurements can be done very accurately.

An alternative to the pattern-matching techniques, which does not suffer from the aforementioned limitations, is a mass-gauging approach based on spectral invariants with respect to the propellant configurations in the tanks. In what follows we outline the corresponding approach to use sound waves to probe the resonance frequencies of the two-phase liquid-gas mixture and report initial experimental results on inferring the volume fraction of the tank filled with liquid, taking advantage of the mathematical properties of the high frequency spectral asymptotics, invariant with respect to the liquid configuration.

2. Spectral asymptotics for an acoustic cavity

In his famous mathematical work from 1911, H. Weyl proved that the high frequency asymptotics of the spectrum of the Laplacian in a three-dimensional spatial domain with Dirichlet boundary conditions depends on the domain only through its volume [13–15]. Ever since, Weyl's analysis has been greatly expanded and now provides rigorously justified asymptotic expansion formulas for the large eigenvalue asymptotics in the case of various differential operators and boundary conditions (for a recent review, see [16]). We note that sharp asymptotic expansion formulas for the eigenvalue problems involving heterogeneous media have been established only fairly recently and constitute a major advance in the mathematical analysis of partial differential equations [17].

Below we illustrate the Weyl's asymptotic formula arising from the studies of a propellant tank with thin walls filled with a heavy liquid propellant, such as liquid oxygen (LOx) or kerosine. For simplicity, we assume that the liquid has the shape of a simply connected bounded domain $\Omega \subset \mathbb{R}^3$, partially in contact with the tank walls. The portion of the wall which is in contact with the liquid will be denoted by $\partial\Omega_-$, and the liquid/gas interface is denoted by $\partial\Omega_+$. The boundary of the liquid is the union of the liquid/wall and gas/liquid surfaces: $\partial\Omega = \partial\Omega_- \cup \partial\Omega_+$.

The spectral asymptotics deals with the eigenfrequency counting function $N(f) = \sum_{l=0}^{\infty} \theta(f - f_l)$, where $\theta(x)$ is the Heaviside step function. Note that we have $f_l = c\sqrt{\lambda_l}/(2\pi)$, where λ_l are the eigenvalues of the Laplacian defined on the spatial domain occupied by the liquid with the corresponding boundary conditions. For this problem, Weyl conjectured [15], and several authors later proved [18,19] (for reviews, see [9,16]) that

$$N(f) = \frac{4\pi|\Omega|f^3}{3c^3} - \frac{\pi|\partial\Omega_+|f^2}{4c^2} - \frac{\pi|\partial\Omega_-|f^2}{4c^2} + o(f^2), \qquad f \to \infty, \tag{1}$$

where $|\Omega|$ is the volume of Ω and $|\partial\Omega_\pm|$ is the area of $\partial\Omega_\pm$. Observe that the leading order term in the above formula is proportional to the liquid volume and is *independent* of the liquid shape. Thus, measuring the eigenfrequency counting function $N(f)$ and fitting it to the functional form in Eq. (1) can in principle yield the liquid volume to arbitrary precision, provided the counting function is known accurately up to sufficiently high frequencies.

The asymptotic formula in Eq. (1) forms the basis of our mass gauging approach. Its utility is evaluated for a number of typical propellant tank geometries for which the spectra can be derived in closed form analytically. We note that for general propellant geometries and mixed boundary conditions, solving the acoustic eigenvalue problem, also known as the Zaremba eigenvalue problem, is a significant challenge, both analytically and numerically (for some recent progress, see [20]). We postpone the studies of more general unsettled propellant geometries to future work.

3. Analysis of the spectral asymptotics for the cylindrical geometry

3.1. Acoustic eigenfrequencies

We consider a cylindrical propellant shape of radius R and height H, with the gas in contact with the liquid at the settled bottom horizontal surface. Modes f_l in the liquid can be calculated using the pressure release (vanishing acoustic pressure) condition over the liquid boundary $\partial\Omega$. The expression for Weyl's asymptotic formula from Eq. (1) becomes:

$$N(f) = \frac{4\pi^2 R^2 H f^3}{3c^3} - \frac{\pi^2 R^2}{2c^2}\left(1 + \frac{H}{R}\right) f^2 + o(f^2), \qquad f \to \infty. \tag{2}$$

Choosing $R = 2.5$ m, $H = 4$ m and velocity of sound $c = 181$ m/s corresponding to LOx in a large tank, we obtain the theoretical eigenfrequency counting function shown in Fig. 1 (a). One can see an excellent agreement between $N(f)$ from counting the exact eigenmodes f_l (dashed line) and the asymptotic prediction in Eq. (2) (solid line), for the frequency range yielding the first 400 eigenfrequencies.

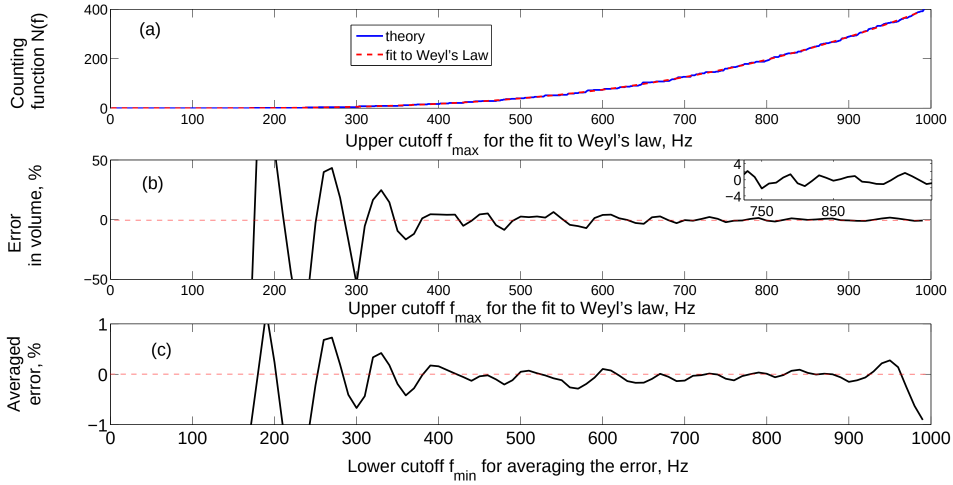

Figure 1. Model calculations: (a) Counting function for a cylindrical tank, $R = 2.5m$, $H = 4m$ filled with LOx; (b)Error in inferred volume as a function of the cutoff frequency; (c) Averaged error in the range between a lower cutoff and the upper cutoff $f_{max} = 1kHz$ as a function of the lower cutoff. Error is seen to be $< 2\%$ for $N > 300$. Averaged error is $< 0.3\%$. .

3.2. Comparison with Weyl's asymptotics

Here we discuss the accuracy of the approximation in more details. Coming back to Figure 1, in panel (b) we see the relative error in the volume inference as a function of the upper cut-off frequency f_{max} oscillates around zero as f_{max} increases. The error is calculated as $\varepsilon = (V_w - V)/V \cdot 100\%$, where V is the actual volume of the propellant $V = \pi R^2 H$ and V_w is the value inferred using the best fit to Weyl's formula. The error is seen to fall below 3% for $N(f_{max}) > 400$. Oscillations of the error with the cut-off frequency is a generic feature of the convergence and is observed in other geometries (boxes, spheres) and boundary conditions (Neumann, i.e., a rigid wall). This gives an idea that averaging of the inferred volume over the oscillations can accelerate its convergence to the actual value. Figure 1 (c) displays the result of such an averaging. The error is averaged over the interval between the upper cut-off f_{max} and a lower cut-off f_{min} as a function of f_{min}. Indeed, the averaged error fall below 0.3% for $N > 200$.

3.3. Peaks resolution

If the number of resolvable peaks were unlimited, one could arrive at arbitrarily small error in volume with the mode counting technique. In reality the resolution is limited by the dissipation and other factors such as temperature inhomogeneity of the fluid. The contribution of the latter factor depends on the specific situation, is negligible close to thermal equilibrium and is not considered here. The tests reported below were performed under conditions very close to thermal equilibrium, where the effect of inhomogeneity was small. The dissipation of the acoustic waves occurs both in the bulk of the liquid or gas and in the boundary layer near the wall. The bulk contribution is generally negligible and the boundary layer dissipation will determine the available resolution in thermal equilibrium. We obtain the following condition for the resolution, expressed in terms of the maximal number of peaks N_{max} resolved for the given tank of characteristic linear dimension $L \sim V^{1/3}$ and fluid properties:

$$N \ll N_{max} \equiv \left[\sqrt{\frac{\nu}{cL}} + \sqrt{\frac{\chi}{cL}} \left(\frac{c_p}{c_v} - 1 \right) \right]^{-6/5} . \tag{3}$$

where μ is the kinematic viscosity, $\chi = \kappa/(\rho c_p)$ is the thermal diffusivity, κ is the thermal conductivity, c_p is the isobaric heat capacity and c_v is isochoric heat capacity of the gas or liquid. Similar calculations show that the relative width contribution from bulk dissipation scales as L^{-1} and that generally $N_{max}|_{bulk} \sim (N_{max}|_{boundary})^{3/2}$, justifying neglecting the bulk dissipation. Eq.(3) implies that the resolved N will increase and the error will decrease with the characteristic size of the tank L for similar shapes, fluid properties and boundary conditions. In addition, we expect that due to significantly lower dimensionless numbers $\nu/(cL)$ and $\chi/(cL)$ in liquids compared to gases higher resolution N can generally be achieved in gauging liquids. For oxygen at saturation temperature at $p = 1.6atm$, and for a tank of $L = 1m$ we find $N_{max} = 5.2 \cdot 10^4 (4.3 \cdot 10^5)$ for $GOx(LOx)$. These observations imply that liquid modes counting will generally lead to a more accurate volume inference than the ullage mode counting.

4. Comparison with the experimental results in cylindrical geometries

As our discussion in Section 3 and Ref. ([21]) implies, mode counting in the liquid compartment of a partially filled tank is expected to be advantageous (compared to the ullage mode counting) for the volume inference for the following reasons:

- Higher resolution available in terms of N_{max};
- Boundary conditions are closer to the pressure release condition, leading to faster convergence of the error with modes count number N.

In a series of experiments reported below water was used as the working fluid. The hardware is shown on Figure 2 (a-d). An actuation technique for the acoustic emission was developed using solenoid actuator, Figure 2 (b). The solenoid actuator was attached to the tank wall from the outside and applied a short ping to excite acoustic resonances in the liquid compartment. The detection was performed using accelerometers attached to the wall from the outside as well, Figure 2 (c). Initially, the detection was also performed with hydrophones immersed into the water. However, since accelerometers proved to give sufficiently good signal, they were adopted in latter tests. Using the solenoid actuators and accelerometers mounted on the wall from the outside makes the excitation/detection technique nonintrusive which is a very significant advantage for propellant volume gauging applications.

A 200 liter plastic tank was used, which adequately simulates the Dirichlet (pessure release) boundary condition expected for liquid propellants such as LOx in thin Aluminum alloy or steel tanks. Measurements were performed on both settled liquid and unsettled, by either tilting the tanks or immersing air-filled balloons in the liquid, Figure 2 (e-f). Various filling levels were used. About 10 excitations per configuration were

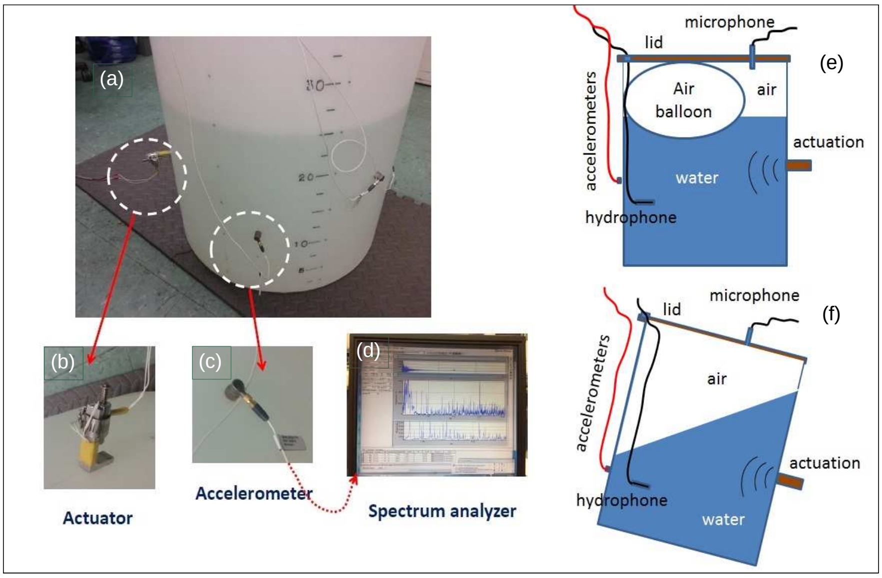

Figure 2. (a)-(d): Hardware used for liquid modes counting; (e)-(f): Representative geometries of unsettled liquid.

performed at various locations on the wall, giving 10 time series and the corresponding spectra. Each time series were recorded using 3 accelerometers and a hydrophone. Peaks were counted using the information in all the spectra as it was found that particular peaks are not excited or registered for a particular location of the actuator and detector, respectively, where the mode does not couple to either actuator or detector. The counting function N(f) was compiled and fitted to Weyls law.

Figure 3 shows the mode counting results for the fill level of 100 liters and an 18 degrees tilt of the tank: the counting function $N(f)$, panel (a), the error in the inferred volume, panel (b) and the averaged error, as defined in Section 3.2, panel (c). It is found that accelerometers give similar accuracy as the hydrophone. Therefore, totally nonintrusive actuation and detection can be used. The error is seen to oscillate as observed in all the model calculations, panel (b). Averaging over the oscillations produces the average error of $< 3\%$ for the volume estimation. The data are compared to theoretical calculations for ideally cylindrical shape of water in the absence of the tilt.

Since 18 degrees is a small perturbation of the water shape the counting function is expected to be slightly perturbed compared to the ideal case. Panel (a)shows that this is indeed the case. It should be noted that while the counting function is only slightly perturbed by the tilt, the spectrum is perturbed substantially, in the sense that the perturbation lifts the degeneracy present in the ideally cylindrical case, allowing one to count the total number of modes. We note that the variation of the error with the upper cut-off, Figure 3 (b), is also significantly affected by the perturbation. Similar results were obtained with immersing air-filled balloons, although with a somewhat lower accuracy due to the technical challenges of immersing the baloons.

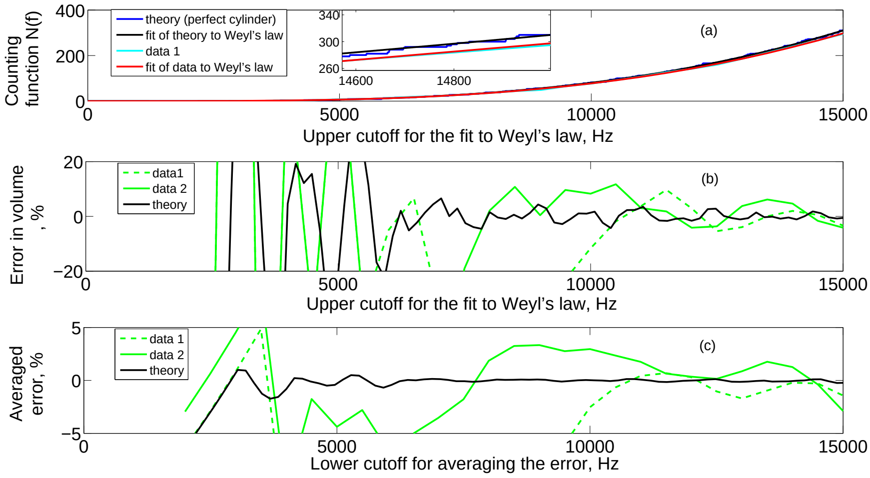

Figure 3. 100 liters of water in the tank, tilted by 18 degrees. 10 excitations locations have been used on the wall. Data 1 corresponds to detection by the hydrophone. Data 2 corresponds to detection by the accelerometers. Theory corresponds to ideally cylindrical shape of water in the absence of the tilt.

5. Summary

We have presented the results of our ongoing work on the spectral approach to propellant mass-gauging that uses mathematically rigorous results about the high-frequency asymptotics of the acoustic eigenfrequencies. To this end, we considered a cylindrical tank geometry commonly used in spacecraft. We assessed convergence properties of the error in volume inference with the counting number, and the contribution and scaling of the effect of dissipation on the resolution of the peaks. It is predicted that the peak resolution increases with the size of the tank for similar propellant configurations. Peak resolution is generically significantly better for liquids than for gases.

We performed mode counting of liquid (water) in partially filled tanks for a variety of unsettled configurations and filling levels. An actuation technique has been developed using solenoids mounted on the exterior of the tank wall. Accelerometers mounted on the exterior wall were used for detection of the acoustic resonances, compared to hydrophones in terms of the accuracy of the volume inference and found to perform equally well or better. As a consequence, a completely nonintrusive technique has been developed for liquid modes counting, which has significant advantages for the propellant volume gauging in-space applications. Using this technique for liquid (water) gauging in large 200 liter plastic tanks an uncertainty of $< 3\%$ has been achieved for "unsettled" configurations created with a tilt.

These results are encouraging, but more theoretical and experimental work is needed to understand better the convergence properties of the error with the counting number, to assess limitation on the resolution due to temperature inhomogeneity of propellant away from thermal equilibrium and sensitivity of the mass gauging to acoustic noise and to slosh and to optimize the actuation and detection protocol.

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