III° Compitino Analisi 2 2017/2018

Es. 1

ⓐ

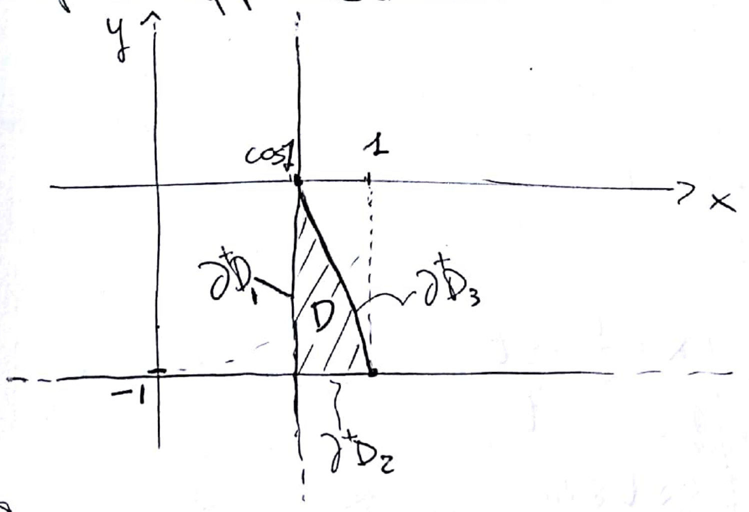
$\gamma(0) = (1, -1)$

$\gamma(1) = (\cos 1, 0)$

Verifico semplicità di γ.

Entrambe le componenti sono monotone per $0 \leq t \leq 1$.
$\Rightarrow \gamma$ è semplice.

Grafico approssimativo:



ⓑ Per il calcolo dell'area, utilizzo il teorema di Gauss-Green.

$$\int_D 1 \, dx\, dy = \int_{\partial^+ D_1} F_1 dy - F_2 dx + \int_{\partial^+ D_2} F_1 dy - F_2 dx + \int_{\partial^+ D_3} F_1 dy - F_2 dx =$$

$$\partial^+ D_1 = \begin{cases} x = \cos 1 \\ y = -t \end{cases} 0 \leq t \leq +1 \qquad \partial^+ D_2 = \begin{cases} x = t \\ y = -1 \end{cases} \cos 1 \leq t \leq 1 \qquad \partial^+ D_3 = \begin{cases} x = \cos(t) \\ y = t^2 - 1 \end{cases} 0 \leq t \leq 1$$

Prendo $F = (x, 0)$

$$\text{Area}(D) = \int_0^1 \cos 1 (-1)\, dt + \int_{\cos 1}^1 \cancel{x(0)}^{\,0}\, dt + \int_0^1 \underset{y}{\cos(t)}\underset{F}{(2t)}\, dt$$

$$= -\cos 1 + 2\left[t \,\text{sen}\, t - \int \text{sen}\, t \, dt\right]_0^1$$

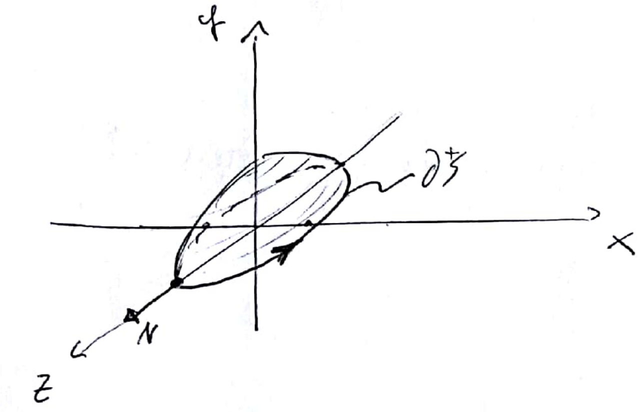
$$= -\cos 1 + 2\left[t \,\text{sen}\, t + \cos t\right]_0^1 = -\cos 1 + 2(\text{sen}\, 1 + \cos 1 - 1)$$

$$= 2 \,\text{sen}\, 1 + \cos 1 - 2$$

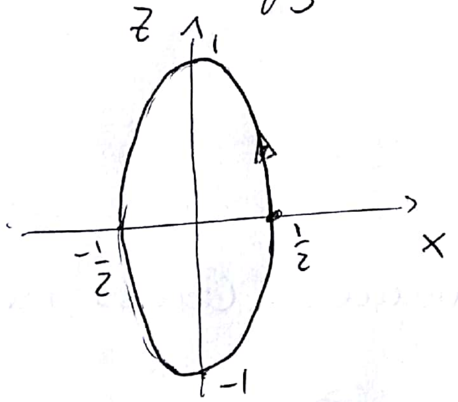
Es. 2

$S = \{ 4x^2 + y^2 e^z + z^2 = 1 \ ; \ y \geq 0 \}$ con $N(0,0,1) = (0,0,1)$

$F = (\text{sen}\, y, \ x e^z, \ x z^2)$. Calcola Flux RotF S



$$\int_S \langle \text{Rot}F, \vec{n} \rangle \, d\sigma = \int_{\partial^+ S} F_1\, dx + F_2\, dy + F_3\, dz$$



$$\partial^+ S = \begin{cases} x = \frac{1}{2}\cos t \\ z = \text{sen}\, t \\ 0 \leq t \leq 2\pi \end{cases}$$

$$= \int_0^{2\pi} \cancel{\text{sen}(0)\, dx} + \frac{1}{2}\cos t\, \cancel{e^{\text{sen}\, t}\, dy} + \frac{1}{2}\cos^2 t\, \text{sen}^2 t \, dt$$

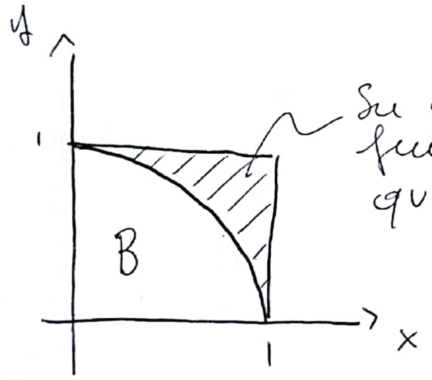
$$= \frac{1}{2}\int_0^{2\pi} \cos^2 t - \cos^4 t \, dt = \frac{1}{2}\pi - \frac{1}{2}\int_0^{2\pi} \cos^4 t\, dt = \frac{1}{2}\pi - \frac{1}{2\,4}\int_0^{2\pi} (1 + \cos(2t))^2 dt$$

$$= \frac{1}{2}\pi - \frac{1}{8}\int_0^{2\pi} 1 + 2\cos(2t) + \cos^2(2t) = \frac{1}{2}\pi - \frac{1}{8}2\pi - \frac{1}{8}\left[\text{sen}(2t) + \frac{1}{2}\int 1 + \cos(4t)\, dt\right]_0^{2\pi}$$

$$= \frac{1}{2}\pi - \frac{1}{4}\pi - \frac{1}{8}\left[\frac{1}{2}t + \frac{1}{8}\cancel{\text{sen}(4t)}\right]_0^{2\pi} = \frac{1}{2}\pi - \frac{1}{4}\pi - \frac{1}{8}\pi = \frac{1}{8}\pi$$

Es. 3

$$\int_{[0,1]\times[0,1]} \frac{\operatorname{sen}(x^2 y)}{x^\alpha + y^\alpha}\,dx\,dy$$



Su questa parte di Dominio la funzione non da problemi, quindi $\int_D f \sim \int_B f$.

In polari:

STIMA DALL'ALTO

$$\int_B f = \int_0^1 d\rho \int_0^{\frac{\pi}{2}} \frac{\operatorname{sen}(\rho^3\cos^2\vartheta \operatorname{sen}\vartheta)}{\rho^\alpha(\cos^\alpha\vartheta+\operatorname{sen}^\alpha\vartheta)}\cdot\rho\cdot d\vartheta$$

Poiché $\operatorname{sen} x \le x$

$$\le \int_0^1 d\rho\int_0^{\frac{\pi}{2}}\frac{\rho^3\cos^2\vartheta\operatorname{sen}\vartheta}{\rho^\alpha(\cos^\alpha\vartheta+\operatorname{sen}^\alpha\vartheta)}\,\rho\, d\vartheta \quad \cancel{\int_0^1 d\rho\int_0^{\frac{\pi}{2}}\frac{1}{\rho^{\alpha-4}}} = \int_0^1 \frac{1}{\rho^{\alpha-4}}\,d\rho\int_0^{\frac{\pi}{2}}\frac{\cos^2\vartheta\operatorname{sen}\vartheta}{\cos^\alpha\vartheta+\operatorname{sen}^\alpha\vartheta}\,d\vartheta$$

noto che:

$$\int_0^{\frac{\pi}{2}}\frac{\cos^2\vartheta\operatorname{sen}\vartheta}{\cos^\alpha\vartheta+\operatorname{sen}^\alpha\vartheta}\,d\vartheta = K \neq 0$$ Poiché integrale su un insieme limitato di funzione continua

$$= K\int_0^1 \frac{1}{\rho^{\alpha-4}}\,d\rho$$ converge per $\alpha-4<1 \Rightarrow \alpha<5$

STIMA DAL BASSO

$$\int_B f \ge \cancel{\int_0^1 \frac{1}{\rho^{\alpha-4}}\,d\rho\int_0^{\frac{\pi}{2}}\frac{1}{2}} \quad \int_0^1 d\rho\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\rho^{\alpha-4}}\cdot\frac{\cos^2\vartheta\operatorname{sen}\vartheta}{\cos^\alpha\vartheta+\operatorname{sen}^\alpha\vartheta}\,d\vartheta$$

$$\ge \int_0^1 d\rho\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\rho^{\alpha-4}}\,\frac{m}{2}\,d\vartheta$$ con m = minimo della funzione $\cos^2\vartheta\operatorname{sen}\vartheta$ per $\frac{\pi}{6}\le\vartheta\le\frac{\pi}{3}$

$$= m\pi\int_0^1 \frac{1}{\rho^{\alpha-4}}\,d\rho$$ che conv. per $\alpha<5$

$$\Rightarrow \int_D f$$ converge per $\alpha<5$