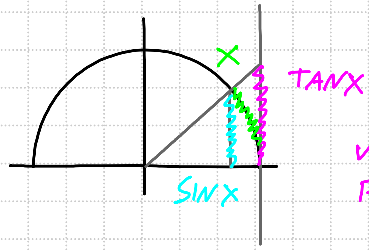
BY GIMUSI 10/12/16

dimostrare lim x che tende a zero di (1/tan x - 1/x) risposta 0

$\underline{x \to 0^+}$ $\quad 0 \le \sin x \le x \le \tan x$



VD. DIM. FATTA PER $\frac{\sin x}{x} \to 1$

$$0 \le \frac{1}{\tan x} \le \frac{1}{x} \le \frac{1}{\sin x}$$

$$0 \le \underbrace{\frac{1}{x} - \frac{1}{\tan x}}_{\to 0} \le \underbrace{\frac{1}{\sin x} - \frac{1}{\tan x}}_{\to 0}$$

$$\frac{1}{\sin x} - \frac{1}{\tan x} = \frac{1 - \cos x}{\sin x} = \underbrace{\frac{1 - \cos x}{x^2}}_{\to 1/2} \cdot \underbrace{\frac{x}{\sin x}}_{\to 1} \cdot \underbrace{x}_{\to 0} \to 0$$

OPPURE

$$= \frac{1 - \cos x}{\sin x} \, \frac{1 + \cos x}{1 + \cos x} = \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}(1 + \cos x)} \to 0$$

$$\leadsto \frac{1}{\tan x} - \frac{1}{x} \to 0 \qquad x \to 0^+$$

$$\underline{x \to 0^-} \quad \frac{1}{\tan x} - \frac{1}{x} = -\frac{1}{\tan(-x)} + \frac{1}{-x} \overset{y=-x}{=} -\left(\frac{1}{\tan y} - \frac{1}{y}\right) \to 0$$

$$\leadsto \frac{1}{\tan x} - \frac{1}{x} \to 0 \qquad x \to 0$$