

Integrali doppi 7 dell'eserciziario.

Insieme di integrazione in ellisse + retta:

$$4x^2 + 9y^2 \leq 36, 3y \geq 2x$$

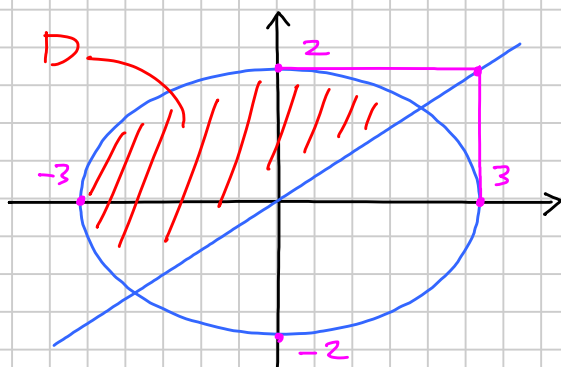
Funzione

X

Per risolvere l'esercizio su suggerimento del testo ho adottato un cambio di coordinate:

$$X = 3 \rho \cos \theta$$

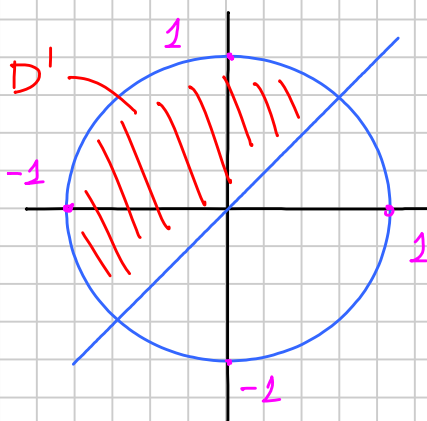
$$Y = 2 \rho \sin \theta$$



$$D: \begin{cases} 4x^2 + 9y^2 \leq 36 \\ 3y \geq 2x \end{cases}$$

$$\begin{cases} X = 3\rho \cos \theta \\ Y = 2\rho \sin \theta \end{cases} \leadsto D': \begin{cases} 36\rho^2 \cos^2 \theta + 36\rho^2 \sin^2 \theta \leq 36 \\ 6\rho \sin \theta \geq 6\rho \cos \theta \end{cases}$$

$$\begin{cases} \rho^2 \leq 1 \\ \sin \theta \geq \cos \theta \end{cases} \quad \begin{cases} 0 \leq \rho \leq 1 \\ \cos \theta (\tan \theta - 1) \geq 0 \end{cases} \quad \begin{cases} \cos \theta \geq 0 & \tan \theta \geq 1 \\ \cos \theta < 0 & \tan \theta < -1 \end{cases}$$

oss si può passare prima a

$$\begin{cases} X = 3\tilde{X} \\ Y = 2\tilde{Y} \end{cases} \leadsto \begin{cases} \tilde{X}^2 + \tilde{Y}^2 \leq 1 \\ \tilde{Y} \geq \tilde{X} \end{cases}$$

E poi in polari

$$\int_D x \, dA \leadsto \int_{D'} 3\rho \cos \theta \, dA' \quad dA' = 6\rho \, d\rho \, d\theta$$

$$\int_0^1 \int_{\pi/4}^{5\pi/4} 18\rho^2 \cos \theta \, d\theta \, d\rho$$