BY GIMUSI 15/07/16

$$\sum_{n\geq 0}(\sqrt{(1+n^2)}-n)^a$$

$$\sum_{n=0}^{\infty}\left(\sqrt{1+n^2}-n\right)^a = \sum_{n=0}^{\infty} a_n$$

BRUTAL MODE $\left(\sqrt{1+n^2}-n\right)^a = \left(n\sqrt{1+1/n^2}-n\right)^a =$

$$= \left[n\left(1+\frac{1}{2n^2}+o\left(\frac{1}{n^2}\right)\right)-n\right]^a = \left[\frac{1}{2n}+o\left(\frac{1}{n}\right)\right]^a \sim \frac{1}{(2n)^a}$$

CONFRONTO ASINTOTICO CON $b_n = \dfrac{1}{n^a}$

$$\frac{a_n}{b_n} = \left(n\sqrt{1+n^2}-n^2\right)^a = \left(n^2\sqrt{1+1/n^2}-n^2\right)^a =$$

$$= \left[n^2\left(1+\frac{1}{2n^2}+o\left(\frac{1}{n^2}\right)\right)-n^2\right]^a = \left[\frac{1}{2}+o(1)\right]^a \to \frac{1}{2^a}$$

$\leadsto$ $\sum_{n=0}^{\infty} a_n$ HA LO STESSO COMPORTAMENTO DI $\sum_{n=0}^{\infty} b_n$

E QUINDI $\sum_{n=0}^{\infty} a_n$ PER $\begin{cases} a>1 & \text{CONVERGE} \\ a\leq 1 & \text{DIVERGE} \end{cases}$