BY GIMUSI 25/06/16

$$\sum \frac{n^a}{1+a^2 e^{an}}$$

$\underline{a > 0}$ $\qquad \alpha_n = \dfrac{n^a}{1+a^2 e^{an}}$

$$\frac{\alpha_{n+1}}{\alpha_n} = \frac{(n+1)^a}{1+a^2 e^{a(n+1)}} \, \frac{1+a^2 e^{an}}{n^a} = \Big(\overbrace{\frac{n+1}{n}}^{\to 1}\Big)^a \, \overbrace{\frac{1+a^2 e^{an}}{1+a^2 e^a e^{an}}}^{\to 1/e^a} \to \frac{1}{e^a}$$

$\rightsquigarrow$ LA SERIE CONVERGE $\forall a > 0$

$\underline{a \leq 0}$ $\qquad a = -b \quad b \geq 0 \quad \alpha_n = \dfrac{n^{-b}}{1+b^2 e^{-bn}} = \dfrac{e^{bn}}{n^b e^{bn} + b^2 n^b} \sim \dfrac{1}{n^b}$

$$\beta_n = \frac{1}{n^b} \qquad \frac{\alpha_n}{\beta_n} = \frac{n^b e^{bn}}{n^b e^{bn} + b^2 n^b} \to 1$$

$\rightsquigarrow$ LA SERIE $\begin{cases} \text{CONVERGE PER } b > 1 \\ \text{DIVERGE PER } 0 \leq b \leq 1 \end{cases}$

$\rightsquigarrow$ LA SERIE $\begin{cases} \text{CONVERGE} & a \in (-\infty, -1) \cup (0, +\infty) \\ \text{DIVERGE} & a \in [-1, 0] \end{cases}$