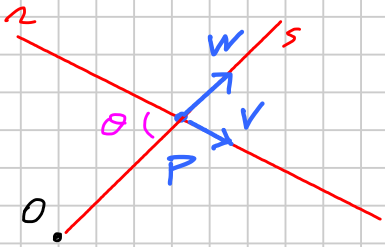
BY GIMUSI 20/06/16

Determinare l'equazione delle rette passanti per l'origine, incidenti la retta r: (x=2z+3; y=z) e formanti con essa un angolo di π/6

RETTA r : $\begin{cases} x = 2z+3 \\ y = z \end{cases} \rightsquigarrow (2\delta+3, \delta, \delta) \qquad v = (2, 1, 1)$



$$w = P - O = (2\delta+3, \delta, \delta)$$

$$|v| = \sqrt{6} \qquad |w| = \sqrt{6\delta^2 + 12\delta + 9}$$

$$\cos\theta = \frac{v \cdot w}{|v||w|} \qquad v \cdot w = |v||w| \cos\theta$$

$$4\delta + 6 + \delta + \delta = \sqrt{6}\sqrt{6\delta^2+12\delta+9} \cos\left(\frac{\pi}{6}\right)$$

$$(6\delta+6)^2 = \left(\sqrt{36\delta^2+72\delta+54}\ \frac{\sqrt{3}}{2}\right)^2$$

$$36\delta^2 + 72\delta + 36 = (36\delta^2+72\delta+54)\frac{3}{4}$$

$$(36-27)\delta^2 + (72-54)\delta + 36 - \frac{81}{2} = 0 \qquad 18\delta^2 + 36\delta - 9 = 0$$

$$2\delta^2 + 4\delta - 1 = 0 \qquad \delta = \frac{-4 \pm \sqrt{16+8}}{4} = -1 \pm \frac{\sqrt{6}}{2}$$

$$\rightsquigarrow \begin{cases} P_1 = (2(-1+\sqrt{6}/2) + 3, -1+\sqrt{6}/2, -1+\sqrt{6}/2) = \\ \quad = (1+\sqrt{6}, -1+\sqrt{6}/2, -1+\sqrt{6}/2) \\ P_2 = (1-\sqrt{6}, -1-\sqrt{6}/2, -1-\sqrt{6}/2) \end{cases}$$

$$\rightsquigarrow \begin{cases} s_1 : \ \delta(1+\sqrt{6}, -1+\sqrt{6}/2, -1+\sqrt{6}/2) \\ s_2 : \ \delta(1-\sqrt{6}, -1-\sqrt{6}/2, -1-\sqrt{6}/2) \end{cases}$$