BY GIMUSI 28/02/16

$$\lim_{x\to+\infty}\frac{(x+1)\sqrt{x^2-2x}-x^2+x}{x-1} = 1$$

MODO 1

$$\frac{(x+1)\sqrt{x^2-2x}-x^2+x}{x-1} = \frac{x^{\cancel{2}}\left[\left(1+\frac{1}{x}\right)\sqrt{1-\frac{2}{x}}-1+\frac{1}{x}\right]}{\cancel{x}\left(1-\frac{1}{x}\right)} =$$

$$\sqrt{1-\frac{2}{x}} = 1-\frac{1}{x}+o\left(\frac{1}{x}\right)$$

$$= x\,\frac{\left(1+\frac{1}{x}\right)\left(1-\frac{1}{x}+o\left(\frac{1}{x}\right)\right)-1+\frac{1}{x}}{1-\frac{1}{x}} =$$

$$= x\,\frac{\cancel{1}-\cancel{\frac{1}{x}}+\cancel{\frac{1}{x}}-\cancel{1}+\frac{1}{x}+o\left(\frac{1}{x}\right)}{1-\frac{1}{x}} = \frac{1+o(1)}{1-1/x} \to 1$$

MODO 2

$$\frac{(x+1)\sqrt{x^2-2x}-(x^2-x)}{x-1}\cdot\frac{(x+1)\sqrt{x^2-2x}+(x^2-x)}{(x+1)\sqrt{x^2-2x}+(x^2-x)} =$$

$$= \frac{(x+1)^2(x^2-2x)-(x^2-x)^2}{(x^2-1)\sqrt{x^2-2x}+(x-1)^2(x+1)} = \frac{(x^2+2x+1)(x^2-2x)-x^4+2x^3-x^2}{(x^2-1)\sqrt{x^2-2x}+(x-1)^2(x+1)} =$$

$$= \frac{\cancel{x^4}-\cancel{2x^3}+\cancel{2x^3}-4x^2-2x-\cancel{x^4}+2x^3-x^2}{(x^2-1)\sqrt{x^2-2x}+(x-1)^2(x+1)} = \frac{2x^3-4x^2-2x}{(x^2-1)\sqrt{x^2-2x}+(x-1)^2(x+1)} \to 1$$