BY GIMUSI 18/10/15

$$\lim_{x\to 0}\frac{\cos(x^{2k})+x^{4k}/2-1}{x+x^k}$$

per quali valori del parametro k il limite suddetto dà zero come risultato?

$\boxed{k \in \mathbb{Z}}$

$\boxed{k > 0}$

$$\lim_{x\to 0}\frac{\cos(x^{2k})+x^{4k}/2-1}{x+x^k}=0$$

$$\cos(x^{2k}) = 1-\frac{x^{4k}}{2}+\frac{x^{8k}}{24}+o(x^{12k}) \qquad x^{2k}\to 0$$

$$\frac{\cos(x^{2k})+x^{4k}/2-1}{x+x^k}=\frac{\frac{x^{8k}}{24}+o(x^{12k})}{x+x^k}=\frac{1}{24}\,\frac{x^{8k}+o(x^{12k})}{x+x^k}$$

$$=\frac{1}{24}\,\frac{x^{8k}}{x^k}\,\frac{1+o(x^{4k})}{x^{1-k}+1}=\frac{x^{7k}}{24}\,\frac{1+o(x^{4k})}{x^{1-k}+1}$$

$$0<k<1 \qquad \lim_{x\to 0}\frac{\overset{\to 0}{x^{7k}}}{24}\,\frac{\overset{\to 1}{1+o(x^{4k})}}{\underset{\to 1}{x^{1-k}+1}}=0$$

$$k>1 \qquad \lim_{x\to 0}\frac{\overset{\to 0}{x^{7k}}}{24}\,\frac{\overset{\to 1}{1+o(x^{4k})}}{\underset{\to\pm\infty}{x^{1-k}+1}}=0$$

$$k=1 \qquad \lim_{x\to 0}\frac{\overset{\to 0}{x^{7}}}{24}\,\overset{\to 1/2}{\frac{1+o(x^{4})}{1+1}}=0$$

$\boxed{n=0}$

$$\lim_{x\to 0} \frac{\cos(x^{2n}) + x^{5n}/2 - 1}{x + x^n} = \cos(1) - 1/2$$

$$\frac{\cos(x^{2n}) + x^{5n}/2 - 1}{x + x^n} = \frac{\cos(1) - 1/2}{x+1}$$

$\boxed{n<0}$

$$\lim_{x\to 0} \frac{\cos(x^{2n}) + x^{5n}/2 - 1}{x + x^n} = \begin{cases} +\infty & n \text{ PARI} \\ \text{N.E.} & n \text{ DISPARI} \begin{cases} +\infty & x\to 0^+ \\ -\infty & x \to 0^- \end{cases} \end{cases}$$

$\alpha = -n$

$$\frac{\cos(x^{2n}) + x^{5n}/2 - 1}{x + x^n} = \frac{\cos(1/x^{2\alpha}) + 2/x^{5\alpha} - 1}{x + 1/x^{\alpha}} =$$

$$= \frac{x^{\alpha}}{x^{5\alpha}} \; \frac{x^{5\alpha}\cos(1/x^{2\alpha}) + 2 - x^{5\alpha}}{x^{\alpha+1} + 1} =$$

$$= \overset{\to \pm\infty}{\frac{1}{x^{3\alpha}}} \; \frac{\overset{\to 0}{x^{5\alpha}\cos(1/x^{2\alpha})} + 2 - \overset{\to 0}{x^{5\alpha}}}{\underset{\to 0}{x^{\alpha+1}} + 1} \longrightarrow \begin{cases} +\infty & \alpha \text{ PARI} \\ \pm\infty & \alpha \text{ DISPARI} \end{cases}$$