BY GIMUSI 21/01/15

Derivate 1

Argomenti: calcolo di derivate **Difficoltà:** ⋆

Prerequisiti: regole algebriche per il calcolo di derivate

Calcolare la derivata prima e seconda delle funzioni indicate. Sarebbe opportuno anche precisare l'insieme dei punti in cui $f(x)$ è continua, derivabile una volta, derivabile due volte (cosa che comunque sarà l'argomento di un esercizio futuro).

Qualche volta la casella potrebbe essere un po' piccola per far stare la risposta …

	a) $f(x)$	$f'(x)$	$f''(x)$	b) $f(x)$	$f'(x)$	$f''(x)$
1)	$\cos(2x)$	$-2\cdot\sin(2x)$	$-4\cos(2x)$	$2\cos(3x)$	$-6\sin(3x)$	$-12\cos(3x)$
2)	$\sin(x^2)$	$2x\cos(x^2)$	$2\cos(x^2)-4x^2\sin(x^2)$	$\sin^2 x$	$\sin(2x)$	$2\cos(2x)$
3)	$\cos(e^x)$	$-e^x\sin(e^x)$	$-e^x\sin(e^x)-e^{2x}\cos(e^x)$	$e^{\cos x}$	$-\sin x\, e^{\cos x}$	$-\cos x\, e^{\cos x}+\sin^2 x\, e^{\cos x}$
4)	$x\sin x$	$\sin x+x\cos x$	$2\cos x-x\sin x$	$\cos(2x)\sin(3x)$	$-2\sin(2x)\sin(3x)+3\cos(2x)\cos(3x)$	$-13\cos(2x)\sin(3x)+-12\sin(2x)\cos(3x)$
5)	$xe^x\cos x$	$f(x)\left(\frac{1}{x}+1-\tan x\right)$	$f(x)\left[\left(\frac{1}{x}+1-\tan x\right)^2-\left(\frac{1}{x^2}+\frac{1}{\cos^2 x}\right)\right]$	$x^2e^{3x}\cos x$	$f(x)\left(\frac{2}{x}+3-\tan x\right)$	$f(x)\left[\left(\frac{2}{x}+3-\tan x\right)^2-\left(\frac{2}{x^2}+\frac{1}{\cos^2 x}\right)\right]$
6)	$\log^7 x$	$7\frac{\log^6 x}{x}$	$\frac{42x\log^5 x-\log^6 x}{x^3}$	$\log(7x)$	$1/x$	$-1/x^2$
7)	$\frac{1}{2x+3}$	$\frac{-2}{(2x+3)^2}$	$\frac{8}{(2x+3)^3}$	$\frac{1}{2x^4+3}$	$\frac{-8x^3}{(2x^4+3)^2}$	$\frac{80x^6-72x^2}{(2x^4+3)^3}$
8)	$\frac{x}{3-x}$	$\frac{3}{(3-x)^2}$	$\frac{6}{(3-x)^3}$	$\frac{x}{3-x^2}$	$\frac{3+x^2}{(3-x^2)^2}$	$\frac{18x+2x^3}{(3-x^2)^3}$
9)	$\sqrt{x}$	$1/2\sqrt{x}$	$-1/4\sqrt{x^3}$	$\sqrt[3]{x}+\sqrt{2}$	$1/3\sqrt[3]{x^2}$	$-2/9\cdot 1/\sqrt[3]{x^5}$
10)	$x^2\sqrt{x}$	$5\sqrt{x^3}/2$	$15\sqrt{x}/4$	$x^7\sqrt[3]{x^5}$	$\frac{26}{3}x^7\sqrt[3]{x^2}$	$\frac{598}{3}x^6\sqrt[3]{x^2}$
11)	$\sqrt{x+3}$	$1/2\sqrt{x+3}$	$-1/4\sqrt{(x+3)^3}$	$\sqrt[5]{7-2x}$	$-2/5\sqrt[5]{(7-2x)^4}$	$-16/25\sqrt[5]{(7-2x)^4}$
12)	$\sqrt{x^2+3x}$	$\frac{2x+3}{2\sqrt{x^2+3x}}$	$-9/4\sqrt{(x^2+3x)^3}$	$\sqrt[4]{5-x^7}$	$\frac{-7x^6}{4\sqrt[4]{(5-x^7)^3}}$	$\frac{850x^5+21x^{12}}{16\sqrt[4]{(5-x^7)^7}}$
13)	$\sin(\sqrt{x})$	$\frac{\cos\sqrt{x}}{2\sqrt{x}}$	$\frac{-\sqrt{x}\sin\sqrt{x}-\cos\sqrt{x}}{4x\sqrt{x}}$	$\sqrt{\sin x}$	$\frac{\cos x}{2\sqrt{\sin x}}$	$\frac{-\sin^2 x-1}{4\sin x\sqrt{\sin x}}$
14)	$\arccos(e^{-x})$	$\frac{e^{-x}}{\sqrt{1-e^{-2x}}}$	$\frac{-e^{-x}}{(1-e^{-2x})^{3/2}}$	$\log(\sin(e^{x^2}))$	$\frac{2xe^{x^2}}{\tan(e^{x^2})}$	$\frac{(e^{x^2}+2x^2e^{x^2})\cdot\sin(2e^{x^2})+4xe^{2x^2}}{\sin^2(e^{x^2})}$
15)	$7^x+\log 3$	$7^x\log 7$	$7^x(\log 7)^2$	$\log_7 x$	$\log_7 e/x$	$-\log_7 e/x^2$
16)	$\frac{\sin x}{\cos(2x)+3}$	$\frac{-2\cos^3 x+6\cos x}{(\cos 2x+3)^2}$	$\frac{(6\cos^2 x\sin x-6\sin x)(\cos 2x+3)+4\sin 2x(-2\cos^3 x+6\cos x)}{(\cos 2x+3)^3}$	$\frac{e^{2x}}{\sqrt{x^2-1}}$	$\frac{e^{2x}(2x^2-2-x)}{\sqrt{(x^2-1)^3}}$	$\frac{(4x^4-4x^3-6x^2+4x+5)e^{2x}}{\sqrt{(x^2-1)^5}}$
17)	x^x	$x^x(1+\log x)$	$x^x\left[(1+\log x)^2+\frac{1}{x}\right]$	$x^{\arctan x}$	$f(x)\left(\frac{\log x}{1+x^2}+\frac{\arctan x}{x}\right)$	
18)	$\sqrt{3+\sqrt[5]{3-x^2}}$	$\frac{-x}{5\sqrt{3+\sqrt[5]{3-x^2}}\,(3-x^2)^{4/5}}$		$(\sin x)^{(\cos x)^x}$	*	

$*\ (\sin x)^{(\cos x)^x}\left[(\cos x)^x\left(\log\cos x+\frac{x}{\cos x}\right)+\frac{(\cos x)^{1+x}}{\sin x}\right]$

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1.a) $\cos(2x)$

$f'(x) = -2 \cdot \sin(2x)$ $\qquad$ $f''(x) = -4\cos(2x)$

1.b) $2\cos(3x)$

$f'(x) = -6\sin(3x)$ $\qquad$ $f''(x) = -12\cos(3x)$

2.a) $\sin(x^2)$

$f'(x) = 2x\cos(x^2)$ $\qquad$ $f''(x) = 2\cos(x^2) - 4x^2\sin(x^2)$

2.b) $\sin^2 x$

$f'(x) = 2\sin x\cos x = \sin(2x)$ $\qquad$ $f''(x) = 2\cos(2x)$

3.a) $\cos(e^x)$

$f'(x) = -e^x\sin(e^x)$ $\qquad$ $f''(x) = -e^x\sin(e^x) - e^{2x}\cos(e^x)$

3.b) $e^{\cos x}$

$f'(x) = -\sin x\, e^{\cos x}$ $\qquad$ $f''(x) = -\cos x\, e^{\cos x} + \sin^2 x\, e^{\cos x}$

4.a) $x\sin x$

$f'(x) = \sin x + x\cos x$ $\qquad$ $f''(x) = \cos x + \cos x - x\sin x = 2\cos x - x\sin x$

4.b) $\cos(2x)\sin(3x)$

$f'(x) = (-2\sin(2x))\sin(3x) + \cos(2x)(3\cos(3x)) =$

$= -2\sin(2x)\sin(3x) + 3\cos(2x)\cos(3x)$

$$f''(x) = -4\cos(2x)\sin(3x) - 6\sin(2x)\cos(3x) + -6\sin(2x)\cos(3x) - 9\cos(2x)\sin(3x) =$$

$$= -13\cos(2x)\sin(3x) - 12\sin(2x)\cos(3x)$$

5.a) $x e^x \cos x$

$$f'(x) = e^x\cos x + x e^x \cos x - x e^x \sin x =$$

$$= f(x)\left(\frac{1}{x} + 1 - \tan x\right)$$

$$f''(x) = f'(x)\left(\frac{1}{x} + 1 - \tan x\right) + f(x)\left(-\frac{1}{x^2} - \frac{1}{\cos^2 x}\right) =$$

$$= f(x)\left[\left(\frac{1}{x} + 1 - \tan x\right)^2 - \left(\frac{1}{x^2} + \frac{1}{\cos^2 x}\right)\right]$$

5.b) $x^2 e^{3x} \cos x$

$$f'(x) = 2x e^{3x}\cos x + x^2 \cdot 3e^{3x}\cos x - x^2 e^{3x}\sin x$$

$$= f(x)\left(\frac{2}{x} + 3 - \tan x\right)$$

$$f''(x) = f'(x)\left(\frac{2}{x} + 3 - \tan x\right) + f(x)\left(\frac{-2}{x^2} - \frac{1}{\cos^2 x}\right) =$$

$$= f(x)\left[\left(\frac{2}{x} + 3 - \tan x\right)^2 - \left(\frac{2}{x^2} + \frac{1}{\cos^2 x}\right)\right]$$

6.a) $\log^7 x$

$$f'(x) = 7\frac{\log^6 x}{x}$$

$$f''(x) = 7\,\frac{6\dfrac{\log^5 x}{x}\cdot x - \dfrac{\log^6 x}{x}}{x^2} = \frac{42x\log^5 x - \log^6 x}{x^3}$$

6.b) $\log(7x)$

$$f'(x) = \frac{1}{x} \qquad f''(x) = \frac{-1}{x^2} \qquad \log(7x) = \log x + \log 7$$

7.a) $\dfrac{1}{2x+3}$

$$f'(x) = \frac{-2}{(2x+3)^2} \qquad f''(x) = -2 \cdot (-2) \cdot 2\,(2x+3)^{-3} = \frac{8}{(2x+3)^3}$$

7.b) $\dfrac{1}{2x^4+3}$

$$f'(x) = \frac{-8x^3}{(2x^4+3)^2}$$

$$f''(x) = \frac{-24x^2(2x^4+3)^{\cancel{2}} - 2\cancel{(2x^4+3)} \cdot 8x^3\,(-8x^3)}{(2x^4+3)^{\cancel{4}\,3}} = \frac{-48x^6 - 72x^2 + 128x^6}{(2x^4+3)^3} = \frac{80x^6 - 72x^2}{(2x^4+3)^3}$$

8.a) $\dfrac{x}{3-x}$

$$f'(x) = \frac{(3-x)+x}{(3-x)^2} = \frac{3}{(3-x)^2} \qquad f''(x) = 3 \cdot (-2) \cdot (3-x)^{-3}(-1) = \frac{6}{(3-x)^3}$$

8.b) $\dfrac{x}{3-x^2}$

$$f'(x) = \frac{3-x^2+2x(x)}{(3-x^2)^2} = \frac{3+x^2}{(3-x^2)^2}$$

$$f''(x) = \frac{2x(3-x^2)^{\cancel{2}} - 2\cancel{(3-x^2)}(-2x)\,(3+x^2)}{(3-x^2)^{\cancel{4}\,3}} = \frac{6x-2x^3+12x+4x^3}{(3-x^2)^3} = \frac{18x+2x^3}{(3-x^2)^3}$$

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9.a) $\sqrt{x}$

$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$ $\qquad$ $f''(x) = -\frac{1}{4} x^{-3/2} = -\frac{1}{4\sqrt{x^3}}$

9.b) $\sqrt[3]{x} + \sqrt{2}$

$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$ $\qquad$ $f''(x) = -\frac{2}{9} x^{-5/3} = -\frac{2}{9} \frac{1}{\sqrt[3]{x^5}}$

10.a) $x^2\sqrt{x} \quad = x^{5/2}$

$f'(x) = \frac{5}{2} x^{3/2} = \frac{5}{2}\sqrt{x^3}$ $\qquad$ $f''(x) = \frac{15}{4} x^{1/2} = \frac{15}{4}\sqrt{x}$

10.b) $x^7\sqrt[3]{x^5} \quad = x^{26/3}$

$f'(x) = \frac{26}{3} x^{23/3} = \frac{26}{3} x^7 \sqrt[3]{x^2}$ $\qquad$ $f''(x) = \frac{598}{9} x^{20/3} = \frac{598}{3} x^6 \sqrt[3]{x^2}$

11.a) $\sqrt{x+3}$

$f'(x) = \frac{1}{2}(x+3)^{-1/2} = \frac{1}{2\sqrt{x+3}}$ $\qquad$ $f''(x) = -\frac{1}{4}(x+3)^{-3/2} = \frac{-1}{4\sqrt{(x+3)^3}}$

11.b) $\sqrt[5]{7-2x} \quad = (7-2x)^{1/5}$

$f'(x) = \frac{1}{5}(7-2x)^{-4/5}(-2) = -\frac{2}{5}\frac{1}{\sqrt[5]{(7-2x)^4}}$

$f''(x) = \frac{8}{25}(7-2x)^{-9/5}(-2) = -\frac{16}{25}\frac{1}{\sqrt[5]{(7-2x)^9}}$

12.a) $\sqrt{x^2+3x} = (x^2+3x)^{1/2}$

$$f'(x) = \frac{1}{2}(x^2+3x)^{-1/2}(2x+3) = \frac{2x+3}{2\sqrt{x^2+3x}}$$

$$f''(x) = -\frac{1}{4}(x^2+3x)^{-3/2}(2x+3)^2 + (x^2+3x)^{-1/2} = \frac{-(2x+3)^2}{4\sqrt{(x^2+3x)^3}} + \frac{1}{\sqrt{x^2+3x}} =$$

$$= \frac{-\cancel{4x^2} - \cancel{12x} - 9 + \cancel{4x^2} + \cancel{12x}}{4\sqrt{(x^2+3x)^3}} = \frac{-9}{4\sqrt{(x^2+3x)^3}}$$

12.b) $\sqrt[4]{5-x^7} = (5-x^7)^{1/4}$

$$f'(x) = \frac{1}{4}(5-x^7)^{-3/4}(-7x^6) = \frac{-7x^6}{4\sqrt[4]{(5-x^7)^3}}$$

$$f''(x) = -\frac{3}{16}(5-x^7)^{-7/4}(-7x^6)^2 + \frac{1}{4}(5-x^7)^{-3/4}(-42x^5) =$$

$$= \frac{-147x^{12} + 4(5-x^7)(-42x^5)}{16\sqrt[4]{(5-x^7)^7}} = \frac{-147x^{12} + 840x^5 + 168x^{12}}{16\sqrt[4]{(5-x^7)^7}} =$$

$$= \frac{840x^5 + 21x^{12}}{16\sqrt[4]{(5-x^7)^7}}$$

13.a) $\sin\sqrt{x}$

$$f'(x) = \cos\sqrt{x} \cdot \frac{1}{2}x^{-1/2} = \frac{\cos\sqrt{x}}{2\sqrt{x}}$$

$$f''(x) = \frac{-\sin\sqrt{x} \cdot \frac{1}{\cancel{2}}\cancel{x^{-1/2}} \cdot \cancel{2}\cancel{\sqrt{x}} - x^{-1/2}\cos\sqrt{x}}{4x} =$$

$$= \frac{-\sqrt{x}\sin\sqrt{x} - \cos\sqrt{x}}{4x\sqrt{x}}$$

13.b) $\sqrt{\sin x}$

$$f'(x) = \frac{1}{2}(\sin x)^{-1/2}\cos x = \frac{\cos x}{2\sqrt{\sin x}}$$

$$f''(x) = \frac{-\sin x \cdot 2\sqrt{\sin x} - (\sin x)^{-1/2}\cos^2 x}{4\sin x} =$$

$$= \frac{-2\sin^2 x - \cos^2 x}{4\sin x\sqrt{\sin x}} = \frac{-\sin^2 x - 1}{4\sin x\sqrt{\sin x}}$$

15.a) $\arccos(e^{-x})$

$$f'(x) = \frac{-1}{\sqrt{1-e^{-2x}}}\cdot(-e^{-x}) = \frac{e^{-x}}{\sqrt{1-e^{-2x}}}$$

$$f''(x) = \frac{-e^{-x}\sqrt{1-e^{-2x}} - \frac{1}{\cancel{2}}(1-e^{-2x})^{-1/2}(\cancel{2}e^{-2x})\,e^{-x}}{1-e^{-2x}} =$$

$$= \frac{-e^{-x}(1-e^{-2x}) - e^{-3x}}{(1-e^{-2x})^{3/2}} = \frac{-e^{-x}}{(1-e^{-2x})^{3/2}}$$

15.b) $\log(\sin(e^{x^2}))$

$$f'(x) = \frac{1}{\sin(e^{x^2})}\cdot\cos(e^{x^2})\cdot e^{x^2}\cdot 2x = \frac{2x e^{x^2}}{\tan(e^{x^2})}$$

$$f''(x) = \frac{(2e^{x^2}+4x^2e^{x^2})\tan(e^{x^2}) - \cos^{-2}(e^{x^2})\cdot(2xe^{x^2})^2}{\tan^2(e^{x^2})} =$$

$$= \frac{(2e^{x^2}+4x^2e^{x^2})\sin(e^{x^2})\cos(e^{x^2}) + 4x^2e^{2x^2}}{\sin^2(e^{x^2})} =$$

$$= \frac{(e^{x^2}+2x^2e^{x^2})\cdot\sin(2e^{x^2}) + 4x\,e^{2x^2}}{\sin^2(e^{x^2})}$$

15.a) $7^x + \log 3$

$$f'(x) = 7^x\log 7 \qquad f''(x) = 7^x(\log 7)^2$$

15.б) $\log_7 x = \log_7 e \cdot \log x$

$$f'(x) = \log_7 e \cdot \frac{1}{x} \qquad f''(x) = \log_7 e \left(\frac{-1}{x^2}\right)$$

16.а) $\dfrac{\sin x}{\cos 2x + 3}$

$$f'(x) = \frac{\cos x(\overset{2\cos^2 x - 1}{\cos 2x} + 3) - (-2\overset{2\sin x \cos x}{\sin 2x})\sin x}{(\cos 2x + 3)^2} =$$

$$= \frac{2\cos^3 x + 2\cos x + 4\cos x - 4\cos^3 x}{(\cos 2x + 3)^2} =$$

$$= \frac{-2\cos^3 x + 6\cos x}{(\cos 2x + 3)^2}$$

$$f''(x) = \frac{(6\cos^2 x \sin x - 6\sin x)(\cos 2x + 3)^{\cancel{2}} - 2\cancel{(\cos 2x + 3)}(-2\sin 2x)(-2\cos^3 x + 6\cos x)}{(\cos 2x + 3)^{\cancel{4}\,3}} =$$

$$= \frac{(6\cos^2 x \sin x - 6\sin x)(\cos 2x + 3) + 4\sin 2x(-2\cos^3 x + 6\cos x)}{(\cos 2x + 3)^3}$$

16.б) $\dfrac{e^{2x}}{\sqrt{x^2 - 1}}$

$$f'(x) = \frac{2e^{2x}\sqrt{x^2-1} - \frac{1}{\cancel{2}}\frac{\cancel{2}x}{\sqrt{x^2-1}} \cdot e^{2x}}{x^2 - 1} =$$

$$= \frac{e^{2x}(2x^2 - 2 - x)}{\sqrt{(x^2-1)^3}} = f(x)\frac{2x^2 - x - 2}{x^2 - 1}$$

$$f''(x) = f'(x)\frac{2x^2 - x - 2}{x^2 - 1} + f(x)\frac{(4x-1)(x^2-1) - 2x(2x^2 - x - 2)}{(x^2-1)^2} =$$

$$= f(x)\frac{(2x^2 - x - 2)^2 + (4x-1)(x^2-1) - 2x(2x^2 - x - 2)}{(x^2-1)^2}$$

$$(2x^2-x-2)^2+(5x-1)(x^2-1)-2x(2x^2-x-2) =$$

$$= 5x^5+\cancel{x^2}+5-\cancel{5x^3}-8x^2+\cancel{5x}+\cancel{5x^3}-\cancel{5x}-\cancel{x^2}+1-5x^3+2x^2+5x =$$

$$= 5x^5-5x^3-6x^2+5x+5$$

$$f''(x) = f(x)\,\frac{5x^5-5x^3-6x^2+5x+5}{(x^2-1)^2} = \frac{(5x^5-5x^3-6x^2+5x+5)\,e^{2x}}{\sqrt{(x^2-1)^5}}$$

17.a) $\quad x^x = e^{x\log x}$

$$f'(x) = e^{x\log x}\cdot(x\log x)' = x^x\left(\log x + x\cdot\frac{1}{x}\right) =$$

$$= x^x(1+\log x) \quad = f(x)\,(1+\log x)$$

$$f''(x) = f'(x)\,(1+\log x) + f(x)\cdot\frac{1}{x} =$$

$$= f(x)\left[(1+\log x)^2+\frac{1}{x}\right] = x^x\left[(1+\log x)^2+\frac{1}{x}\right]$$

17.b) $\quad x^{\arctan x} = e^{\arctan x \,\log x}$

$$f'(x) = f(x)\cdot(\arctan x\,\log x)' =$$

$$= f(x)\left(\frac{\log x}{1+x^2}+\frac{\arctan x}{x}\right)$$

$$f''(x) = f'(x)\left(\frac{\log x}{1+x^2}+\frac{\arctan x}{x}\right)+f(x)\left(\frac{\log x}{1+x^2}+\frac{\arctan x}{x}\right)'$$

$$\left(\frac{\log x}{1+x^2}+\frac{\arctan x}{x}\right)' = \frac{\frac{1+x^2}{x}-2x\log x}{(1+x^2)^2} + \frac{\frac{x}{1+x^2}-\arctan x}{x^2} =$$

$$= \frac{1+x^2-2x^2\log x}{x(1+x^2)^2} + \frac{x-(1+x^2)\arctan x}{x^2(1+x^2)}$$

$$f''(x)=f(x)\left[\left(\frac{\lg x}{1+x^2}+\frac{\operatorname{ARCTAN} x}{x}\right)^2+\frac{1+x^2-2x^2\lg x}{x(1+x^2)^2}+\frac{x-(1+x^2)\operatorname{ARCTAN} x}{x^2(1+x^2)}\right]$$

18.a) $\sqrt{3+\sqrt[5]{3-x^2}}$

$$f'(x)=\frac{1}{\cancel{2}}\left(3+\sqrt[5]{3-x^2}\right)^{-1/2}\cdot\frac{1}{5}(3-x^2)^{-4/5}\,(-\cancel{2}x)=$$

$$=\frac{-x}{5\sqrt{3+\sqrt[5]{3-x^2}}\,(3-x^2)^{4/5}}$$

$$f''(x)=\frac{1}{5}\left[-\frac{1}{\cancel{2}}\left(3+\sqrt[5]{3-x^2}\right)^{-3/2}\frac{1}{5}(3-x^2)^{-4/5}(-\cancel{2}x)\,(3-x^2)^{-4/5}(-x)+\right.$$

$$\left.+\left(-\frac{4}{5}\right)(3-x^2)^{-9/5}(-2x)\left(3+\sqrt[5]{3-x^2}\right)^{-1/2}(-x)-\left(3+\sqrt[5]{3-x^2}\right)^{-1/2}(3-x^2)^{-4/5}\right]=$$

$$=\frac{-x^2}{25\left(3+\sqrt[5]{3-x^2}\right)^{3/2}(3-x^2)^{8/5}}+\frac{-8x^2}{25\left(3+\sqrt[5]{3-x^2}\right)^{1/2}(3-x^2)^{9/5}}+$$

$$-\frac{1}{5\sqrt{3+\sqrt[5]{3-x^2}}\,(3-x^2)^{4/5}}$$

18.b) $(\sin x)^{(\cos x)^x}=e^{(\cos x)^x\lg(\sin x)}$

$$f'(x)=f(x)\cdot\left[(\cos x)^x\lg(\sin x)\right]'=$$

$$=f(x)\left\{\left[(\cos x)^x\right]'\lg(\sin x)+\left[\lg(\sin x)\right]'(\cos x)^x\right\}=$$

$$\begin{cases}\left[(\cos x)^x\right]'=\left[e^{x\lg\cos x}\right]'=(\cos x)^x\left(\lg\cos x+\dfrac{x}{\cos x}\right)\\[2ex]\left[\lg(\sin x)\right]'=\dfrac{\cos x}{\sin x}\end{cases}$$

$$=(\sin x)^{(\cos x)^x}\left[(\cos x)^x\left(\lg\cos x+\frac{x}{\cos x}\right)+\frac{(\cos x)^{1+x}}{\sin x}\right]$$

$f''(x)=$

$$\frac{d^2}{dx^2}\left(\sin^{\cos^x(x)}(x)\right)=$$
$$\sin^{\cos^x(x)}(x)\left(\cos^{x+1}(x)\csc(x)+\cos^x(x)\log(\sin(x))(\log(\cos(x))-x\tan(x))\right)^2+$$
$$\sin^{\cos^x(x)}(x)\left(-\cos^{x+2}(x)\csc^2(x)+\cos^{x+1}(x)\csc(x)(\log(\cos(x))-x\tan(x))+\right.$$
$$\cos^{x+1}(x)\csc(x)(\log(\cos(x))-(x+1)\tan(x))+\cos^x(x)\log(\sin(x))$$
$$\left.(\log(\cos(x))-x\tan(x))^2+\cos^x(x)\log(\sin(x))\left(-2\tan(x)-x\sec^2(x)\right)\right)$$