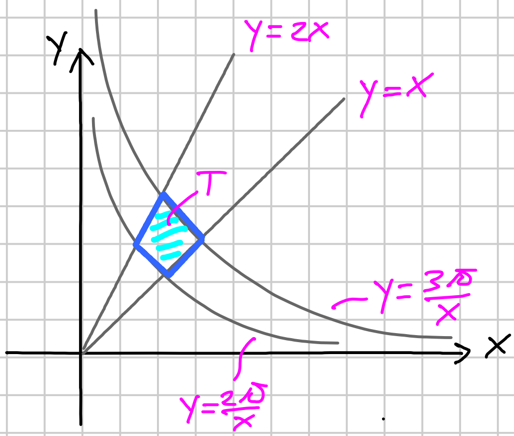
BY GIMUSI 05/06/15

Si calcoli:

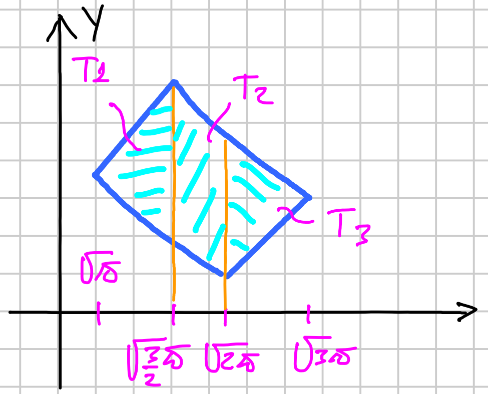
$$\iint_T x^2(y-x^3)e^{(y+x^3)}dxdy$$

dove $T=\{(x,y)\in\mathbb{R}^2:\ x<y<2x,\ \frac{2}{x}\pi<y<\frac{3}{x}\pi,\ x>0\}$

(suggerimento: si trasformi T in un rettangolo)



DOMINIO DI INTEGRAZIONE:



$$T=T_1\cup T_2\cup T_3$$

$$\begin{cases} T_1=\{(x,y):\ \sqrt{\pi}\le x\le\sqrt{\tfrac{3}{2}\pi},\ \tfrac{2}{x}\pi\le y\le 2x\}\\ T_2=\{(x,y):\ \sqrt{\tfrac{3}{2}\pi}\le x\le\sqrt{2\pi},\ \tfrac{2}{x}\pi\le y\le\tfrac{3}{x}\pi\}\\ T_3=\{(x,y):\ \sqrt{2\pi}\le x\le\sqrt{3\pi},\ x\le y\le\tfrac{3}{x}\pi\}\end{cases}$$

CAMBIO DI VARIABILI: $x^3=u \qquad y=v$

$$\begin{cases} du=3x^2dx\\ dv=dy\end{cases} \qquad du\,dv=\begin{vmatrix}3x^2 & 0\\ 0 & 1\end{vmatrix}dx\,dy=3x^2dx\,dy$$

$$\rightsquigarrow dx\,dy=\frac{1}{3x^2}du\,dv$$

$$\int_T x^2(y-x^3)e^{y+x^3}dx\,dy=\int_T \cancel{x^2}(v-u)e^{v+u}\frac{1}{3\cancel{x^2}}du\,dv=$$

$$=\frac{1}{3}\int_T(v-u)e^{v+u}du\,dv=\frac{1}{3}\int_{T_1\cup T_2\cup T_3}(v-u)e^{v+u}du\,dv$$

CALCOLO DELL'INTEGRALE SU T_1

$$\int_{T_1}(v-u)e^{v+u}du\,dv=\int_{\pi\sqrt{\pi}}^{\frac{3}{2}\pi\sqrt{\frac{3}{2}\pi}}\int_{2\pi/\sqrt[3]{u}}^{2\sqrt[3]{u}}(v-u)e^{v+u}dv\,du$$

$$\int (v-\mu)\, e^{v+\mu}\, dv = \int \overset{f}{v}\, \overset{g'}{e^{v+\mu}}\, dv - \int \mu\, e^{v+\mu}\, dv =$$

$$= v\, e^{v+\mu} - \int 1 \cdot e^{v+\mu}\, dv - \mu\, e^{v+\mu} =$$

$$= v e^{v+\mu} - e^{v+\mu} - \mu\, e^{v+\mu} = (v-\mu-1)\, e^{v+\mu}$$

$$\int_{2\pi/\sqrt[3]{\mu}}^{2\sqrt[3]{\mu}} (v-\mu)\, e^{v+\mu}\, dv = \left(2\sqrt[3]{\mu}-\mu-1\right) e^{2\sqrt[3]{\mu}+\mu} - \left(\frac{2\pi}{\sqrt[3]{\mu}}-\mu-1\right) e^{\frac{2\pi}{\sqrt[3]{\mu}}+\mu}$$

$$\int_{T_2} (v-\mu)\, e^{v+\mu}\, d\mu\, dv = \int_{\pi\sqrt{\pi}}^{\frac{3}{2}\pi\sqrt{\frac{3}{2}\pi}} \left[\left(2\sqrt[3]{\mu}-\mu-1\right) e^{2\sqrt[3]{\mu}+\mu} - \left(\frac{2\pi}{\sqrt[3]{\mu}}-\mu-1\right) e^{\frac{2\pi}{\sqrt[3]{\mu}}+\mu} \right] d\mu$$

... É INTEGRABILE???