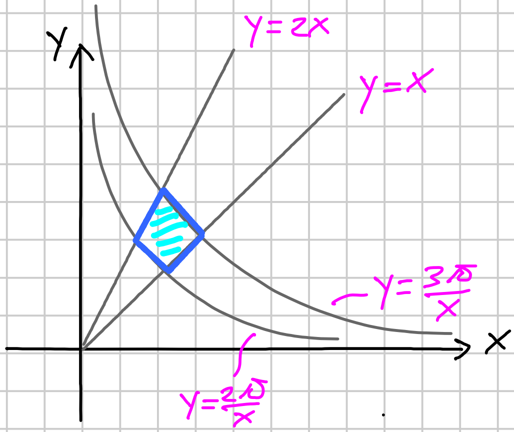
BY GIMUSI 03/06/15

Si calcoli:

$$\iint_T x^2(y-x^3)e^{(y+x^3)}dxdy$$

dove $T=\{(x,y)\in\mathbb{R}^2:\ x<y<2x,\ \frac{2}{x}\pi<y<\frac{3}{x}\pi,\ x>0\}$

(suggerimento: si trasformi T in un rettangolo)



CAMBIO DI VARIABILI 1

$$yx=u \qquad \frac{y}{x}=v \quad \rightsquigarrow \quad 1\le v\le 2 \qquad 2\pi\le u\le 3\pi$$

$$y=\frac{u}{x}=vx \quad \rightsquigarrow x^2=\frac{u}{v} \qquad x=\sqrt{\frac{u}{v}} \qquad y=\sqrt{uv}$$

$$\begin{cases} du = y\,dx + x\,dy \\ dv = -\frac{y}{x^2}dx+\frac{1}{x}dy \end{cases} \qquad du\,dv=\begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x}\end{vmatrix}dx\,dy$$

$$du\,dv=\left(\frac{y}{x}+\frac{y}{x}\right)dx\,dy \rightsquigarrow dx\,dy=\frac{1}{2v}\,du\,dv$$

$$x^2(y-x^3)e^{(y+x^3)} = \frac{u}{v}\left(\sqrt{uv}-\frac{u}{v}\sqrt{\frac{u}{v}}\right)e^{\sqrt{uv}+\frac{u}{v}\sqrt{\frac{u}{v}}} =$$

$$=\left(\frac{u}{v}\sqrt{uv}-\frac{u^2}{v^3}\sqrt{uv}\right)e^{\sqrt{uv}+\frac{u}{v^2}\sqrt{uv}}$$

NON MOLTO INTEGRABILE...!!!

CAMBIO DI VARIABILI 2

$$xy=u^2 \qquad \frac{y}{x}=v^2 \quad \rightsquigarrow \quad 1\le v\le\sqrt{2} \qquad \sqrt{2\pi}\le u\le\sqrt{3\pi}$$

$$y=\frac{u^2}{x}=v^2\cdot x \rightsquigarrow x=\frac{u}{v} \qquad y=uv$$

$$x^2(y-x^3)e^{(y+x^3)} = \frac{u^2}{v^2}\left(uv-\frac{u^3}{v^3}\right)e^{(uv+u^3/v^3)}$$

$$=\left(\frac{u^3}{v}-\frac{u^5}{v^5}\right)e^{(uv+u^3/v^3)}$$

$$\begin{cases} dx = \frac{1}{v} du - \frac{u}{v^2} dv \\ dy = v\,du + u\,dv \end{cases} \rightsquigarrow dx\,dy = \begin{vmatrix} 1/v & -u/v^2 \\ v & u \end{vmatrix} du\,dv$$

$$dx\,dy = (u/v + u/v)\,du\,dv = 2\frac{u}{v}\,du\,dv$$

$$\int_T x^2(y-x^3)\,e^{(y+x^3)}\,dx\,dy = \int_1^{\sqrt{2}}\int_{\sqrt{2\pi}}^{\sqrt{3\pi}} 2\left(\frac{u^5}{v^2} - \frac{u^6}{v^6}\right) e^{(uv + u^3/v^3)}\,du\,dv$$

NON MOLTO MEGLIO DI PRIMA!!!

CAMBIO DI VARIABILI 3

$y - x^3 = u \qquad y + x^3 = v$

$$\begin{cases} du = -3x^2\,dx + dy \\ dv = 3x^2\,dx + dy \end{cases}$$

$$du\,dv = -6x^2\,dx\,dy \qquad dx\,dy = -\frac{1}{6x^2}\,du\,dv$$

BUONO!!!

$$\int_T x^2(y-x^3)\,e^{(y+x^3)}\,dx\,dy = -\frac{1}{6}\int_{T'} u\,e^{v}\,du\,dv$$

INTEGRALE SEMPLICE

DOMINIO T' COMPLICATO!!!