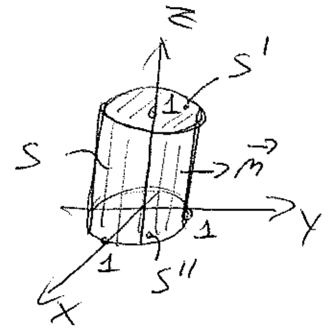
BY GIMUSI 03/06/15

$\vec{E} = (x^2+y, x-z, 0)$ $\qquad S = \{x^2+y^2=1, 0 \le z \le 1\}$

MODO 1 CALCOLO DIRETTO DEL FLUSSO



IN COORDINATE CILINDRICHE $\begin{cases} x = \cos\theta \\ y = \sin\theta \quad \rho = 1 \\ z = z \end{cases}$

$$\begin{cases} \vec{E} = (\cos^2\theta + \sin\theta, \cos\theta - z, 0) \\ \vec{n} = (\cos\theta, \sin\theta, 0) \end{cases}$$

$$\int_S \vec{E}\cdot\vec{n}\, d\sigma = \int_S (\cos^3\theta + \sin\theta\cos\theta + \sin\theta\cos\theta - z\sin\theta)\, d\sigma =$$

$$= \int_0^{2\pi}\int_0^1 (\cos^3\theta + \sin 2\theta - z\sin\theta)\, dz\, d\theta =$$

$$= \underbrace{\cancel{\int_0^{2\pi} \cos^3\theta\, d\theta}}_{=0} + \underbrace{\cancel{\int_0^{2\pi} \sin 2\theta\, d\theta}}_{=0} - \frac{1}{2}\underbrace{\cancel{\int_0^{2\pi} \sin\theta\, d\theta}}_{=0} = 0$$

MODO 2 TEOREMA DI GAUSS-GREEN

$$\int_\Omega \operatorname{div}\vec{E}\, dx\,dy\,dz = \int_{\partial\Omega} \langle \vec{E}, \vec{n}\rangle\, d\sigma \qquad \left\| \begin{array}{l} \Omega \equiv \text{CILINDRO} \\ x^2+y^2 \le 1 \quad 0 \le z \le 1 \end{array}\right.$$

$$\int_{\partial\Omega} \vec{E}\cdot\vec{n}\, d\sigma = \int_S \vec{E}\cdot\vec{n}\, d\sigma + \int_{S', S''} \vec{E}\cdot\vec{n}\, d\sigma \qquad \operatorname{div}\vec{E} = 2x + 0 + 0 = 2x$$

$$\int_{S', S''} \vec{E}\cdot\vec{n}\, d\sigma = 0 \qquad \vec{n} = (0, 0, \pm 1) \rightsquigarrow \vec{E}\cdot\vec{n} = 0$$

$$\rightsquigarrow \int_S \vec{E}\cdot\vec{n}\, d\sigma = \int_\Omega 2x\, dx\,dy\,dz = 0 \qquad (2x \text{ DISPARI SU } \Omega)$$