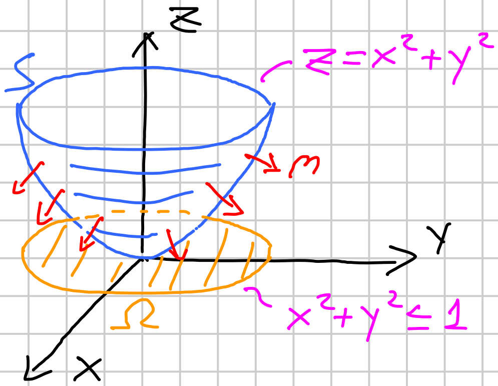
$$\vec{E} = (y^3, z-x, x^2) \qquad S: \; z = x^2+y^2 \;, \; x^2+y^2 \leq 1$$



MODO 1

PARAMETRIZZAZIONE DI S

$$\begin{cases} \phi(u,v) = (u, v, u^2+v^2) \\ \Omega = \{(u,v) \in \mathbb{R}^2 : u^2+v^2 \leq 1\} \end{cases}$$

VERSORE USCENTE $\quad n = \dfrac{\vec{N}}{|\vec{N}|}$

$$\begin{pmatrix} \phi_u \\ \phi_v \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2u \\ 0 & 1 & 2v \end{pmatrix} \rightsquigarrow \vec{N}' = (M_1, M_2, M_3) = (-2u, -2v, 1)$$

$$(u,v) = (0,0) \rightsquigarrow \vec{N}' = (0,0,1) \rightsquigarrow \vec{N} = -\vec{N}' = (2u, 2v, -1)$$

CALCOLO DEL FLUSSO

$$\int_S \vec{E}\cdot\vec{n}\, d\sigma = \int_\Omega (2uv^3 + (u^2+v^2-u)\cdot 2v - u^2)\, du\, dv =$$

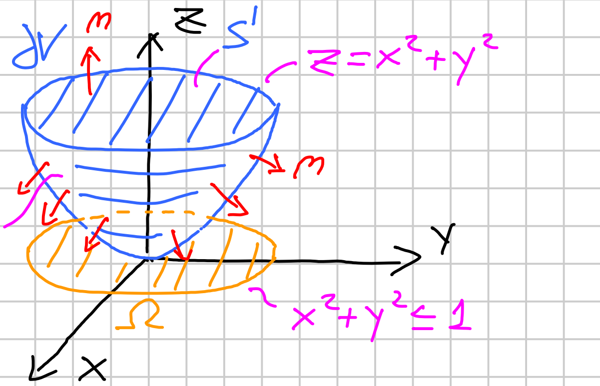
$$= \int_\Omega (\cancel{2uv^3} + \cancel{2u^2v} + \cancel{2v^3} - \cancel{2uv} - u^2)\, du\, dv = \int_\Omega -u^2 \, du\, dv =$$

$$= \int_0^{2\pi}\int_0^1 -\rho^2\cos^2\theta \cdot \rho\, d\rho\, d\theta = \int_0^{2\pi}\int_0^1 -\rho^3 \cos^2\theta\, d\rho\, d\theta =$$

$$= -\int_0^{2\pi} \cos^2\theta \left[\rho^4/4\right]_0^1 d\theta = -\frac{1}{4}\int_0^{2\pi}\cos^2\theta\, d\theta = -\frac{\pi}{4}$$

MODO 2 GAUSS-GREEN $\int_V \operatorname{div} \vec{E}\, dV = \int_{\partial V} \vec{E} \cdot \vec{n}\, d\sigma$

$\partial V = S \cup S'$



PARAMETRIZZAZIONE S'

$(u, v, 1) \quad \Omega = \{(u,v) \in \mathbb{R}^2 : u^2+v^2 \le 1\}$

APPLICO GAUSS-GREEN

$$\operatorname{div} \vec{E} = \frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial C}{\partial z} = \frac{\partial y^3}{\partial x} + \frac{\partial (z-x)}{\partial y} + \frac{\partial x^2}{\partial z} = 0$$

$$\rightsquigarrow \int_{\partial V} \vec{E} \cdot \vec{n}\, d\sigma = \int_S \vec{E} \cdot \vec{n}\, d\sigma + \int_{S'} \vec{E} \cdot \vec{n}\, d\sigma = 0$$

$$\rightsquigarrow \int_S \vec{E} \cdot \vec{n}\, d\sigma = -\int_{S'} \vec{E} \cdot \vec{n}\, d\sigma$$

$$\int_{S'} \underbrace{\vec{E} \cdot \vec{n}}_{(0,0,1)}\, d\sigma = \int_\Omega u^2\, du\, dv \underbrace{=}_{\text{FATTO PRIMA}} \frac{\pi}{4}$$

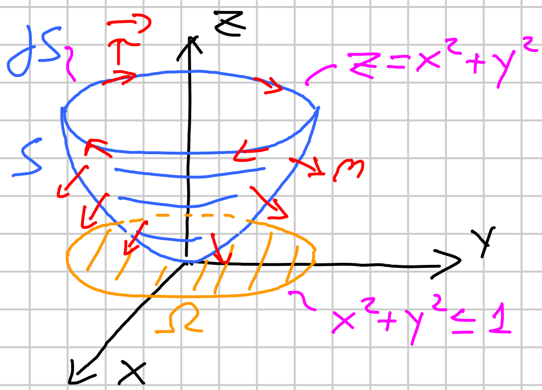
$$\rightsquigarrow \int_S \vec{E} \cdot \vec{n}\, d\sigma = -\int_{S'} \vec{E} \cdot \vec{n}\, d\sigma = -\frac{\pi}{4}$$

MODO 3 STOKES $\int_S \operatorname{rot} \vec{F} \cdot \vec{n}\, d\sigma = \int_{\partial S} \vec{F} \cdot \vec{\tau}\, ds$

$\operatorname{div} \vec{E} = 0$, Ω STELLATO

$\rightsquigarrow \vec{E} = \operatorname{rot} \vec{F}$

$$\vec{F} = (A, B, C) \quad \begin{cases} C_y - B_z = y^3 \\ A_z - C_x = z - x \\ B_x - A_y = x^2 \end{cases}$$



$$\leadsto C = \frac{y^4}{4} \qquad B = \frac{x^3}{3} \qquad A = \frac{z^2}{2} - xz$$

$$\int_S \vec{E}\cdot\vec{n}\, d\sigma = \int_S \operatorname{rot} \vec{F}\cdot\vec{n}\, d\sigma = \int_{\partial S} \vec{F}\cdot\vec{T}\, ds = \int_{\partial S} A\,dx + B\,dy + C\,dz$$

$$\begin{cases} x = \cos\theta \\ y = -\operatorname{sen}\theta \\ z = 1 \end{cases} \leadsto \begin{cases} dx = -\operatorname{sen}\theta\, d\theta \\ dy = -\cos\theta\, d\theta \\ dz = 0 \end{cases}$$

$$\int_{\partial S} A\,dx + B\,dy + C\,dz = \int_0^{2\pi} \left(\frac{1}{2} - \cos\theta\right)(-\operatorname{sen}\theta)\, d\theta + \int_0^{2\pi} \frac{\cos^3\theta}{3}(-\cos\theta)\, d\theta =$$

$$= \overset{=0}{\cancel{\int_0^{2\pi} -\frac{1}{2}\operatorname{sen}\theta\, d\theta}} + \overset{=0}{\cancel{\int_0^{2\pi} \frac{1}{2}\operatorname{sen}2\theta\, d\theta}} - \frac{1}{3}\int_0^{2\pi} \cos^4\theta\, d\theta$$

$$\int \cos^4\theta\, d\theta = \int \overset{f'}{\cos\theta}\cdot \overset{g}{\cos^3\theta}\, d\theta = \operatorname{sen}\theta\cos^3\theta - \int \operatorname{sen}\theta \cdot 3\cos^2\theta(-\operatorname{sen}\theta)\, d\theta =$$

$$= \operatorname{sen}\theta\cos^3\theta + 3\int \operatorname{sen}^2\theta\cos^2\theta\, d\theta = \operatorname{sen}\theta\cos^3\theta + 3\int \cos^2\theta\, d\theta - 3\int \cos^4\theta\, d\theta$$

$$\leadsto 4\int \cos^4\theta\, d\theta = \operatorname{sen}\theta\cos^3\theta + 3\int \cos^2\theta\, d\theta$$

$$\int \cos^4\theta\, d\theta = \frac{1}{4}\operatorname{sen}\theta\cos^3\theta + \frac{3}{4}\int \cos^2\theta\, d\theta$$

$$\leadsto \int_0^{2\pi} \cos^4\theta\, d\theta = \overset{=0}{\cancel{\left[\frac{1}{4}\operatorname{sen}\theta\cos^3\theta\right]_0^{2\pi}}} + \frac{3}{4}\overset{\pi}{\int_0^{2\pi} \cos^2\theta\, d\theta} = \frac{3}{4}\pi$$

$$\leadsto \int_{\partial S} A\,dx + B\,dy + C\,dz = -\frac{1}{3}\int_0^{2\pi} \cos^4\theta\, d\theta = -\frac{1}{3}\cdot\frac{3}{4}\pi = -\frac{\pi}{4}$$