

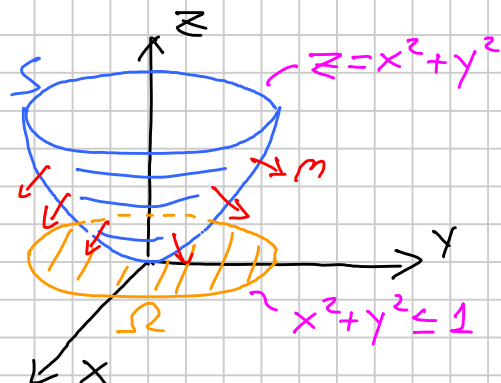
$$\vec{E} = (y^3, z-x, x^2)$$

$$S: z = x^2 + y^2, \quad x^2 + y^2 \leq 1$$

MODO 1

PARAMETRIZZAZIONE DI S

$$\begin{cases} \phi(\mu, \nu) = (\mu, \nu, \mu^2 + \nu^2) \\ \Omega = \{(\mu, \nu) \in \mathbb{R}^2 : \mu^2 + \nu^2 \leq 1\} \end{cases}$$



VERSO RE USCENTE $M = \frac{\vec{N}}{|\vec{N}|}$

$$\begin{pmatrix} \phi_\mu \\ \phi_\nu \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2\mu \\ 0 & 1 & 2\nu \end{pmatrix} \leadsto \vec{N}' = (M_1, M_2, M_3) = (-2\mu, -2\nu, 1)$$

$$(\mu, \nu) = (0, 0) \leadsto \vec{N}' = (0, 0, 1) \leadsto \vec{N} = -\vec{N}' = (2\mu, 2\nu, -1)$$

CALCOLO DEL FLUSSO

$$\int_S \vec{E} \cdot \vec{N} \, d\sigma = \int_\Omega (2\mu\nu^3 + (\mu^2 + \nu^2 - \mu) \cdot 2\nu - \mu^2) \, d\mu \, d\nu =$$

$$= \int_\Omega (\cancel{2\mu\nu^3} + \cancel{2\mu^2\nu} + \cancel{2\nu^3} - \cancel{2\mu\nu} - \mu^2) \, d\mu \, d\nu = \int_\Omega -\mu^2 \, d\mu \, d\nu =$$

$$= \int_0^{2\pi} \int_0^1 -\rho^2 \cos^2 \theta \cdot \rho \, d\rho \, d\theta = \int_0^{2\pi} \int_0^1 -\rho^3 \cos^2 \theta \, d\rho \, d\theta =$$

$$= - \int_0^{2\pi} \cos^2 \theta \left[\rho^{4/4} \right]_0^1 \, d\theta = - \frac{1}{4} \int_0^{2\pi} \cos^2 \theta \, d\theta = - \frac{\pi}{2}$$

MODO 2

GAUSS-GREEN

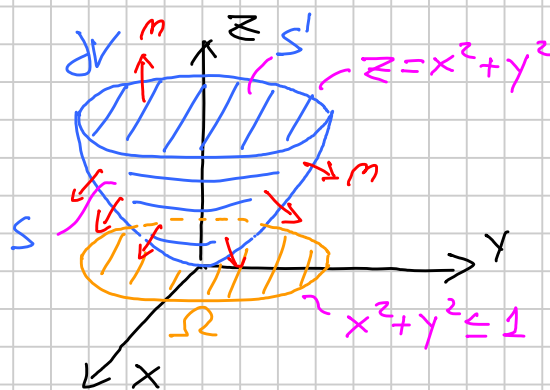
$$\int_V dV \vec{E} = \int_{\partial V} \vec{E} \cdot \vec{n} d\sigma$$

$$\partial V = S \cup S'$$

PARAMETRIZZAZIONE S'

$$(\mu, \nu, 1) \quad \Omega = \{(\mu, \nu) \in \mathbb{R}^2 : \mu^2 + \nu^2 \leq 1\}$$

APPLICO GAUSS-GREEN



$$\operatorname{div} \vec{E} = \frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial C}{\partial z} = \frac{\partial y^3}{\partial x} + \frac{\partial (z-x)}{\partial y} + \frac{\partial x^2}{\partial z} = 0$$

$$\leadsto \int_{\partial V} \vec{E} \cdot \vec{n} d\sigma = \int_S \vec{E} \cdot \vec{n} d\sigma + \int_{S'} \vec{E} \cdot \vec{n} d\sigma = 0$$

$$\leadsto \int_S \vec{E} \cdot \vec{n} d\sigma = - \int_{S'} \vec{E} \cdot \vec{n} d\sigma$$

$$\int_{S'} \vec{E} \cdot \vec{n} d\sigma = \int_{\Omega} \mu^2 d\mu d\nu = \frac{\pi}{5}$$

\downarrow $(0,0,1)$ \downarrow FATTO PALMA

$$\leadsto \int_S \vec{E} \cdot \vec{n} d\sigma = - \int_{S'} \vec{E} \cdot \vec{n} d\sigma = -\frac{\pi}{5}$$