ISOMETRIE NELLO SPAZIO 2 – ESERCIZIO 2.(e)

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12/05/2016

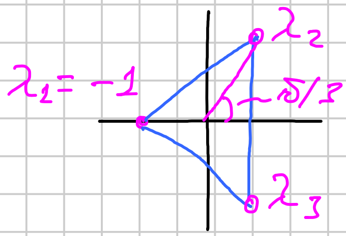
(e) $(y,-z,x) \rightsquigarrow f(x)=Ax+b \quad A=\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} \quad b=0 \quad AA^T = I \rightsquigarrow$ ISOMETRIA

PUNTI FISSI: $Ax=x \rightsquigarrow (A-I)x = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} x = 0 \rightsquigarrow \quad x=0$

$\rightsquigarrow$ 1 P.TO FISSO $\equiv$ ROT. RISPETTO A RETTA r^* DI ANGOLO θ^*
$\oplus$ SIMMETRIA RISPETTO A PIANO $\pi^* \perp r^*$

$$|A-\lambda I| = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & -1 \\ 1 & 0 & -\lambda \end{vmatrix} = -\lambda(\lambda^2) - 1(1) = -\lambda^3 - 1 = 0$$

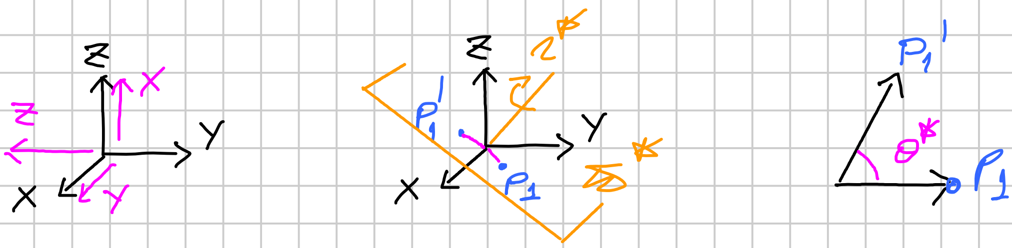
$\lambda^3 = -1 \quad \rightsquigarrow \quad \lambda_1 = -1 \quad \lambda_{2,3} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$



EQUAZIONE RETTA $r^* \equiv$ AUTOSPAZIO DI $\lambda_1 = -1$

$$(A+I)x = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix} x = 0 \qquad x = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$\rightsquigarrow r^*: \ t(-1,1,1) \qquad \theta^* = \pm \pi/3 \qquad \pi^*: \ x-y-z=0$



(ex) $P_1 \in \pi^* \quad P_1 = (1,1,0) \qquad P_1' = AP_1 = (1,0,1)$

$\cos \widehat{P_1 P_1'} = \frac{\langle P_1, P_1' \rangle}{\|P_1\| \|P_1'\|} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2} \rightsquigarrow \theta^* = \pm \frac{\pi}{3}$

SCOMPOSIZIONE DI A : $A = S \cdot R$

R MATRICE DI ROTAZIONE INTORNO A r^* DI $\pi/3$ IN SENSO ORARIO RISPETTO A OMINO ORIENTATO SECONDO ASSE z^+

S MATRICE DI SIMMETRIA RISPETTO A PIANO $\pi^* \perp r^*$

CALCOLO DI R

BASE: $v_1 \in r^*$ $\quad v_2, v_3 \perp r^*$ $\quad (v_2, v_3 \in \pi^*)$

$$v_1 = (-1, 1, 1) \qquad v_2 = (1, 1, 0) \qquad v_3 = (1, 0, 1)$$

BASE ORTOGONALE: $\langle v_1, v_2 \rangle = 0 \quad \langle v_1, v_3 \rangle = 0$

$$\hat{v}_1 = v_1 \qquad \hat{v}_2 = v_2 \qquad \hat{v}_3 = v_3 - \frac{\langle v_2, v_3 \rangle}{\|v_2\|^2} v_2 = v_3 - \frac{1}{2} v_2$$

$$\rightsquigarrow \hat{v}_3 = (1, 0, 1) - \left(\frac{1}{2}, \frac{1}{2}, 0\right) = (1/2, -1/2, 1)$$

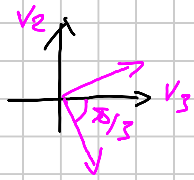
$$\|\hat{v}_1\| = \sqrt{3} \qquad \|\hat{v}_2\| = \sqrt{2} \qquad \|\hat{v}_3\| = \sqrt{\frac{3}{2}}$$

BASE ORTONORMALE

$$v_1 = \frac{\hat{v}_1}{\|\hat{v}_1\|} = \frac{1}{\sqrt{3}}(-1, 1, 1) \qquad v_2 = \frac{\hat{v}_2}{\|\hat{v}_2\|} = \frac{1}{\sqrt{2}}(1, 1, 0) \qquad v_3 = \frac{\hat{v}_3}{\|\hat{v}_3\|} = \sqrt{\frac{2}{3}}\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$$

OSS $\quad v_2 \wedge v_3 = \dfrac{1}{\sqrt{3}} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 0 \\ 1/2 & -1/2 & 1 \end{vmatrix} = \dfrac{1}{\sqrt{3}}(1, -1, -1) = -v_1$

$\rightsquigarrow \{v_3, v_2, v_1\}$ LEVOGIRA



MATRICE R^* IN BASE $\{v_3, v_2, v_1\}$

$$R^* = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

MATRICE R^* IN BASE CANONICA: $R = M R^* M^{-1}$

$$M = \begin{array}{c} \begin{matrix} \quad v_3 & \quad v_2 & \quad v_1 \end{matrix} \\ \begin{pmatrix} \sqrt{2}/2\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{3} \\ -\sqrt{2}/2\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{3} \\ \sqrt{2}/\sqrt{3} & 0 & 1/\sqrt{3} \end{pmatrix} \end{array} \qquad M^{-1} = M^T = \begin{pmatrix} \sqrt{2}/2\sqrt{3} & -\sqrt{2}/2\sqrt{3} & \sqrt{2}/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$

BASE $\{v_3, v_2, v_1\} \rightsquigarrow$ CANONICA CANONICA $\rightsquigarrow$ BASE $\{v_3, v_2, v_1\}$

$$R = M R^* M^{-1} = M \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{6}/6 & -\sqrt{6}/6 & \sqrt{6}/3 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \end{pmatrix} =$$

$$= M \begin{pmatrix} \sqrt{6}/12 + \sqrt{6}/4 & -\sqrt{6}/12 + \sqrt{6}/4 & \sqrt{6}/6 \\ \cancel{-3\sqrt{2}/12} \cancel{+\sqrt{2}/4} & 3\sqrt{2}/12 + \sqrt{2}/4 & -3\sqrt{2}/6 \\ -\sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \end{pmatrix} =$$

$$= \begin{pmatrix} \sqrt{6}/6 & \sqrt{2}/2 & -\sqrt{3}/3 \\ -\sqrt{6}/6 & \sqrt{2}/2 & \sqrt{3}/3 \\ \sqrt{6}/3 & 0 & \sqrt{3}/3 \end{pmatrix} \begin{pmatrix} \sqrt{6}/3 & \sqrt{6}/6 & \sqrt{6}/6 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ -\sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \end{pmatrix}$$

$$= \begin{pmatrix} 1/3 + 1/3 & 1/6 + 1/2 - 1/3 & 1/6 - 1/2 - 1/3 \\ -1/3 - 1/3 & -1/6 + 1/2 + 1/3 & -1/6 - 1/2 + 1/3 \\ 2/3 - 1/3 & 1/3 + 1/3 & 1/3 + 1/3 \end{pmatrix} = \begin{pmatrix} 2/3 & 1/3 & -2/3 \\ -2/3 & 2/3 & -1/3 \\ 1/3 & 2/3 & 2/3 \end{pmatrix}$$

VER. $P_1 = (1,1,0)$ $R P_1 = P_1' = (1,0,1)$

CALCOLO DI S

MATRICE S^* IN BASE $\{v_3, v_2, v_1\}$

$$S^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

MATRICE S^* IN BASE CANONICA: $S = M S^* M^{-1}$

$$M = \begin{pmatrix} \sqrt{6}/6 & \sqrt{2}/2 & -\sqrt{3}/3 \\ -\sqrt{6}/6 & \sqrt{2}/2 & \sqrt{3}/3 \\ \sqrt{6}/3 & 0 & \sqrt{3}/3 \end{pmatrix} \qquad M^{-1} = M^T = \begin{pmatrix} \sqrt{6}/6 & -\sqrt{6}/6 & \sqrt{6}/3 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \end{pmatrix}$$

BASE $\{V_3, V_2, V_1\}$ ⇝ CANONICA $\qquad$ CANONICA ⇝ BASE $\{V_3, V_2, V_1\}$

$$S = M S^* M^{-1} = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \sqrt{6}/6 & -\sqrt{6}/6 & \sqrt{6}/3 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \end{pmatrix} =$$

$$= \begin{pmatrix} \sqrt{6}/6 & \sqrt{2}/2 & -\sqrt{3}/3 \\ -\sqrt{6}/6 & \sqrt{2}/2 & \sqrt{3}/3 \\ \sqrt{6}/3 & 0 & \sqrt{3}/3 \end{pmatrix} \begin{pmatrix} \sqrt{6}/6 & -\sqrt{6}/6 & \sqrt{6}/3 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ \sqrt{3}/3 & -\sqrt{3}/3 & -\sqrt{3}/3 \end{pmatrix} =$$

$$= \begin{pmatrix} 1/6+1/2-1/3 & -1/6+1/2+1/3 & 1/3+1/3 \\ -1/6+1/2+1/3 & 1/6+1/2-1/3 & -1/3-1/3 \\ 1/3+1/3 & -1/3-1/3 & 2/3-1/3 \end{pmatrix} = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix}$$

VER. $P_1=(1,1,0)$ $\quad SP_1 = P_1$ $\quad P_2=(-1,1,1)$ $\quad SP_2=(1,-1,-1)=-P_2$

$$\rightsquigarrow A = S \cdot R = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix} \begin{pmatrix} 2/3 & 1/3 & -2/3 \\ -2/3 & 2/3 & -1/3 \\ 1/3 & 2/3 & 2/3 \end{pmatrix} =$$

$$= \frac{1}{9} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & -2 \\ -2 & 2 & -1 \\ 1 & 2 & 2 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 0 & 9 & 0 \\ 0 & 0 & -9 \\ 9 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$