Abstracts

Riccardo Adami, *Towards nonlinear hybrids: the Nonlinear Schrödinger Equation in dimension two with a point interaction*

We study the Nonlinear Schrödinger equation in dimension two, with an external interaction concentrated at a point. The presence of such interaction enlarges the energy space, allowing a logarithmic singularity, and this results in a more complicated analysis. We prove existence of ground states in the subcritical case for every mass. This is a joint project with Filippo Boni, Raffaele Carbone, and Lorenzo Tentarelli.

Antonio Azzolini, *The 2-D Schrödinger-Maxwell system*

In this talk we will introduce a model to describe the interaction between a charged particle moving on a plane and the electromagnetic field generated by itself. Such a system, which reads

$$\begin{cases} -\Delta u + V(x)u - q\phi u + W'(u) = 0 \text{ in } \mathbb{R}^2, \\ \Delta\phi = u^2 \text{ in } \mathbb{R}^2, \end{cases} \tag{$\mathcal{P}_0$}$$

represents the two dimensional version of the well known Schrödinger-Maxwell system in the electrostatic case.

In spite of the wide literature dealing with it in $\mathbb{R}^3$, (($\mathcal{P}_0$)) is new and definitely unexplored in $\mathbb{R}^2$. Analogies and, mainly, unexpected different features will be highlighted. We refer to [1,2].

[1] Azzollini A., *The planar Schrödinger - Poisson system with a positive potential*, Nonlinearity, **34**, pp 5799–5821, (2021).

[2] Azzollini A. and Pimenta M.T.O. *The 2-dimensional nonlinear Schrödinger - Maxwell system*, (in preparation).

Luca Battaglia, *Blow-up phenomena for a curvature problem in a disk*

We consider the problem of prescribing Gaussian and geodesic curvatures for a conformal metric on the unit disk, which is equivalent to a Liouville-type PDE with nonlinear Neumann boundary conditions.
We build a family of solutions which blow up on the boundary at a critical point of a functional which is a combination of the curvatures we are prescribing.
The talk is based on joint works with M. Medina and A. Pistoia.

Jacopo Bellazzini, *Ground state energy threshold and blow-up for NLS with competing nonlinearities*

Aim of the talk is to discuss qualitative properties of the nonlinear Schrödinger equation with combined nonlinearities, where the leading term is an intracritical focusing power-type nonlinearity, and the perturbation is given by a power-type defocusing one. Fixed the mass of the problem, we completely answer the question whether the ground state energy , which is a threshold between global existence and formation of singularities, is achieved. As a byproduct of the variational characterization of the ground state energy, we show the existence of blowing-up solutions in finite time, for any initial data with energy below the ground state energy threshold in case of cylindrical symmetry.

Pietro d'Avenia, *Ground states for an Hartree-Fock type system*

We introduce an Hartree-Fock type system made by two Schrödinger equations in presence of a Coulomb interacting term and a *cooperative* pure power and subcritical nonlinearity depending on a parameter $\beta \geq 0$.
We present some results about the existence of radial ground states solutions and their

semitriviality or *vectoriality* covering the whole range $\beta \geq 0$.
Joint work in collaboration with Liliane Maia and Gaetano Siciliano.

Simone Dovetta, *Action versus energy ground states in nonlinear Schrödinger equations*

The talk investigates the relations between normalized critical points of the nonlinear Schrödinger energy functional and critical points of the corresponding action functional on the associated Nehari manifold. First, we show that the ground state levels are strongly related by the following duality result: the (negative) energy ground state level is the Legendre–Fenchel transform of the action ground state level. Furthermore, whenever an energy ground state exists at a certain frequency, then all action ground states with that frequency have the same mass and are energy ground states too. We see that the converse is in general false and that the action ground state level may fail to be convex. Next we analyze the differentiability of the ground state action level and we provide an explicit expression involving the mass of action ground states. Finally we show that similar results hold also for local minimizers, and we exhibit examples of domains where our results apply.

The matter of the talk refers to joint works with Enrico Serra and Paolo Tilli.

Francesca Gladiali, *Symmetry and monotonicity results for solutions of semilinear PDEs in sector like domains*

I will consider semilinear PDEs, with a convex nonlinearity, in a sector-like domain under Dirichlet-Neumann boundary conditions. Using cylindrical coordinates, I will prove that any nonradial solution with Morse index less than two must be strictly monotone with respect to the angular variable. The result holds in a circular sector as well as in an annular and it can also be extended to a rectangular domain. The proof is based on a moving plane-type argument. The result is part of a collaboration with Antonio Greco.

María Medina de la Torre, *Blow-up analysis of a curvature prescription problem in the disk*

We will establish necessary conditions on the blow-up points of conformal metrics of the disk with prescribed Gaussian and geodesic curvatures, where a non local restriction will appear. Conversely, given a point satisfying these conditions, we will construct an explicit family of approximating solutions that explode at such a point. These results are contained in several works in collaboration with A. Jevnikar, R. López-Soriano and D. Ruiz, and with L. Battaglia and A. Pistoia.

L. Battaglia, M. Medina, A. Pistoia, *A blow-up phenomenon for a non-local Liouville-type equation.* Available at arxiv.org/pdf/2011.01883.pdf.

A. Jevnikar, R. López-Soriano, M. Medina and D. Ruiz, *Blow-up analysis of conformal metrics of the disk with prescribed Gaussian and geodesic curvatures.* To appear in Analysis and PDEs.

Anna Maria Micheletti, *Compactness and non compactness of Yamabe problem on manifold with boundary*

Let (M, g) be a compact Riemannian manifold of dimension n with umbilic boundary. It is well known that in the conformal class of the metric g there are scalar flat metrics with a constant mean curvature on the boundary. We prove that these metrics are a compact set provided $n > 5$ and the Weyl tensor is always different from zero on the boundary. We also give some results of stability or instability of compactness under perturbation of the problem. These results are in collaboration with Marco Ghimenti and Angela Pistoia.

Dimitri Mugnai, *The Fractional Laplacian with nonlocal Neumann boundary conditions*

We will report on some recent regularity and existence results for the Fractional Laplacian with nonlocal Neumann boundary conditions

Angela Pistoia, *Critical Lane-Emden systems*

I will present some recent results concerning non-degeneracy, existence and multiplicity of solutions to a Lane-Emden critical system obtained in collaboration with R.Frank and S.Kim.

Frédéric Robert, *Blowing-up solutions for second-order critical elliptic equations: the impact of the scalar curvature*

Given a closed manifold (M^n, g), $n \geq 3$, Olivier Druet proved that a necessary condition for the existence of energy-bounded blowing-up solutions to perturbations of the equation

$$\Delta_g u + h_0 u = u^{\frac{n+2}{n-2}},\ u > 0 \text{ in } M$$

is that $h_0 \in C^1(M)$ touches the Scalar curvature somewhere when $n \geq 4$ (the condition is different for $n = 6$). In this paper, we prove that Druet's condition is also sufficient provided we add its natural differentiable version. For $n \geq 6$, our arguments are local. For the low dimensions $n \in \{4, 5\}$, our proof requires the introduction of a suitable mass that is defined only where Druet's condition holds. This mass carries global information both on h_0 and (M, g).

Joint work with Jérôme Vétois (McGill University, Canada)