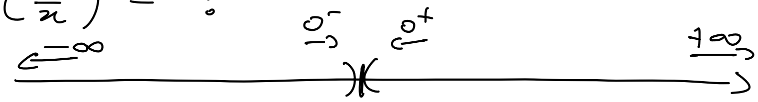
ES $\lim_{x \to +\infty} x \cdot \sin\left(\frac{1}{x}\right) = ?$



$f(x) = x \cdot \sin\left(\frac{1}{x}\right)$, $D = \{x \in \mathbb{R} / x \neq 0\} = \mathbb{R} \setminus \{0\}$, $Acc(\mathbb{R} \setminus \{0\}) = \mathbb{R}^*$

$$\lim_{x \to +\infty} \underbrace{x \cdot \sin\left(\frac{1}{x}\right)}_{+\infty \cdot 0} = \lim_{t \to 0^+} \frac{1}{t} \sin(t) = 1 \quad \text{(limite notevole)}$$

$$t = \frac{1}{x} \xrightarrow[x \to +\infty]{} 0^+$$

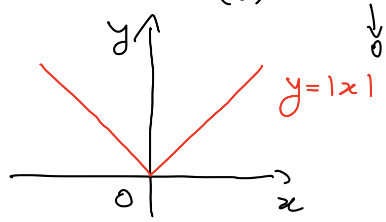
$$\lim_{x \to -\infty} \underbrace{x \cdot \sin\left(\frac{1}{x}\right)}_{-\infty \cdot 0} = 1 ,$$

$$f(-x) = (-x) \cdot \sin\left(\frac{1}{-x}\right) = (-x) \cdot \left(-\sin\left(\frac{1}{x}\right)\right) =$$
$$= x \cdot \sin\left(\frac{1}{x}\right) = f(x) , \quad f \text{ è PARI}$$

$$\lim_{x \to 0^+} x \cdot \sin\left(\frac{1}{x}\right) = 0$$

$$0 \cdot \boxed{\lim_{t \to +\infty} \sin(t)} \quad \nexists$$

$$0 \le \left|x \cdot \sin\left(\frac{1}{x}\right)\right| = |x| \cdot \left|\sin\left(\frac{1}{x}\right)\right| \le |x| \xrightarrow[x \to 0]{} 0$$

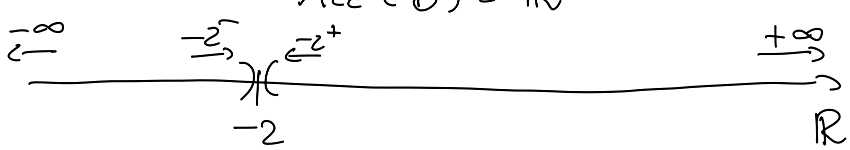


ES Studiare gli asintoti a $+\infty$ di:

$$f(x) = \frac{x^2 - 1}{x + 2} , \quad f(x) = \sqrt{x^2 + 4x - 1}$$

- $f(x) = \dfrac{x^2 - 1}{x + 2}$, $D = \{x \in \mathbb{R} / x + 2 \neq 0\} = \mathbb{R} \setminus \{-2\}$

$$Acc(D) = \mathbb{R}^*$$



$$\lim_{x \to +\infty} \frac{x^2 - 1}{x + 2} = \lim_{x \to +\infty} \frac{x^2\left(1 - \frac{1}{x^2}\right)}{x\left(1 + \frac{2}{x}\right)} = \lim_{x \to +\infty} x \frac{\left(1 - \frac{1}{x^2}\right)}{\left(1 + \frac{2}{x}\right)} = +\infty$$

$$+\infty \cdot \frac{1}{1}$$

$$a = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\frac{x^2 - 1}{x + 2}}{x} = \lim_{x \to +\infty} \frac{x^2 - 1}{(x + 2)x} = \lim_{x \to +\infty} \frac{x^2 - 1}{x^2 + 2x} =$$

$$= \lim_{x \to +\infty} \frac{x^2\left(1-\frac{1}{x^2}\right)}{x^2\left(1+\frac{2}{x}\right)} = \lim_{x \to +\infty} \frac{1-\frac{1}{x^2}}{1+\frac{2}{x}} = 1 \in \mathbb{R}$$

$$b = \lim_{x \to +\infty} \left[f(x) - ax\right] = \lim_{x \to +\infty} \left[\frac{x^2-1}{x+2} - x\right] = \lim_{x \to +\infty} \frac{x^2-1-x(x+2)}{x+2} =$$

$$+\infty \quad -\infty$$

$$= \lim_{x \to +\infty} \frac{x^2-1-x^2-2x}{x+2} = \lim_{x \to +\infty} \frac{-2x-1}{x+2} = \lim_{x \to +\infty} \frac{x\left(-2-\frac{1}{x}\right)}{x\left(1+\frac{2}{x}\right)} =$$

$$= \lim_{x \to +\infty} \frac{-2-\frac{1}{x}}{1+\frac{2}{x}} = -2 \in \mathbb{R}$$

$f(x)$ ha asintoto obliquo a $+\infty$ dato da $y = x-2$

$$\lim_{x \to -\infty} \frac{x^2-1}{x+2} = \lim_{x \to -\infty} \frac{x^2\left(1-\frac{1}{x^2}\right)}{x\left(1+\frac{2}{x}\right)} = \lim_{x \to -\infty} x \, \frac{1-\frac{1}{x^2}}{1+\frac{2}{x}} = -\infty$$

$$a = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{x^2-1}{x(x+2)} = \lim_{x \to -\infty} \frac{x^2\left(1-\frac{1}{x^2}\right)}{x^2\left(1+\frac{2}{x}\right)} = 1 \in \mathbb{R}$$

$$b = \lim_{x \to -\infty} \left[f(x) - ax\right] = \lim_{x \to -\infty} \left[\frac{x^2-1}{x+2} - x\right] = \lim_{x \to -\infty} \frac{-2x-1}{x+2} =$$

$$= \lim_{x \to -\infty} \frac{x\left(-2-\frac{1}{x}\right)}{x\left(1+\frac{2}{x}\right)} = -2 \in \mathbb{R}$$

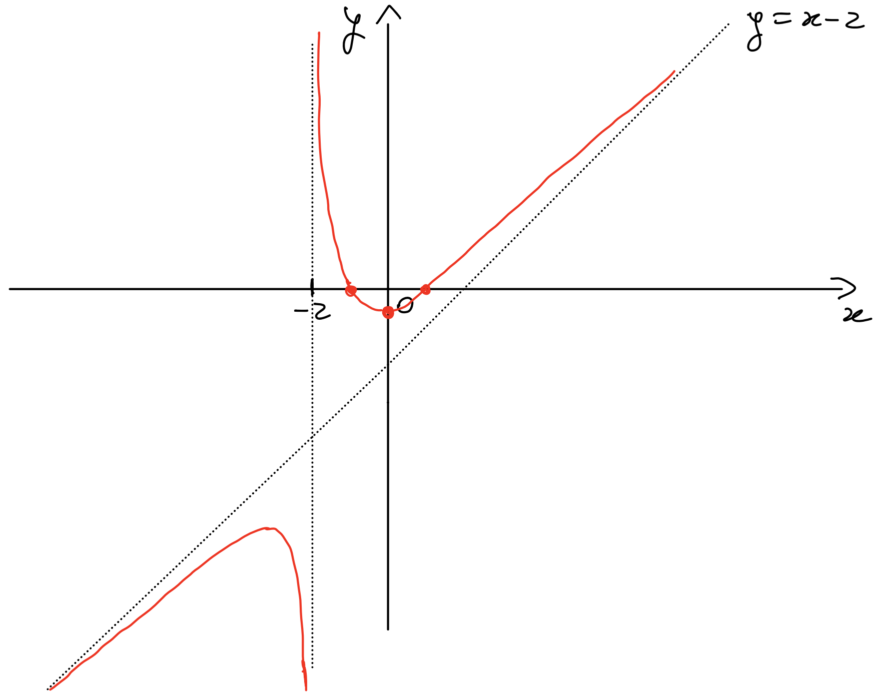
$f(x)$ ha asintoto obliquo a $-\infty$ dato da $y = x-2$

$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \frac{x^2-1}{x+2} = +\infty$, f ha asintoto verticale in -2^+

$$\frac{3}{0^+} = 3 \cdot +\infty$$

$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \frac{x^2-1}{x+2} = -\infty$, f ha asintoto verticale in -2^-.

$$\frac{3}{0^-} = 3 \cdot -\infty$$



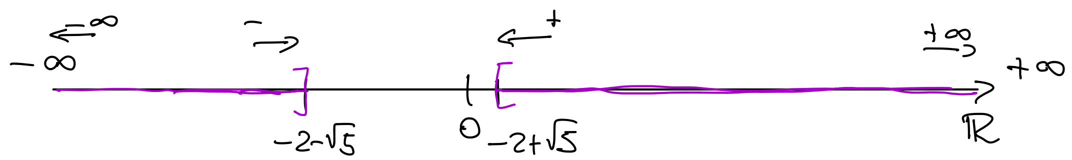
$f(0) = \frac{0-1}{0+2} = -\frac{1}{2}$

$f(1) = 0 = f(-1)$

• $f(x) = \sqrt{x^2+4x-1}$, $D = \{ x \in \mathbb{R} \,/\, x^2+4x-1 \geq 0 \}$

$x^2+4x-1 \geq 0$, $x^2+4x-1 = 0 \iff x = -2 \pm \sqrt{4+1} = \begin{cases} -2+\sqrt{5} \\ -2-\sqrt{5} \end{cases}$

$D = (-\infty, -2-\sqrt{5}] \cup [-2+\sqrt{5}, +\infty)$, $Acc(D) = D \cup \{\pm\infty\}$



$\lim\limits_{x \to (-2+\sqrt{5})^+} \underbrace{\sqrt{x^2+4x-1}}_{\sqrt{0}} = 0$, $\lim\limits_{x \to (-2-\sqrt{5})^-} \underbrace{\sqrt{x^2+4x-1}}_{\sqrt{0}} = 0$

$\lim\limits_{x \to \pm\infty} \sqrt{x^2+4x-1} = +\infty$

$a = \lim\limits_{x \to +\infty} \frac{\sqrt{x^2+4x-1}}{x} = \lim\limits_{x \to +\infty} \frac{\sqrt{x^2\left(1+\frac{4}{x}-\frac{1}{x^2}\right)}}{x} = \lim\limits_{x \to +\infty} \frac{\overset{\sqrt{x^2}}{|x|} \cdot \sqrt{1+\frac{4}{x}-\frac{1}{x^2}}}{x} =$

$= \lim\limits_{x \to +\infty} \frac{x \sqrt{1+\frac{4}{x}-\frac{1}{x^2}}}{x} = 1 \in \mathbb{R}$

$b = \lim\limits_{x \to +\infty} [f(x) - a x] = \lim\limits_{x \to +\infty} \left[\sqrt{x^2+4x-1} - x\right] =$

$$= \lim_{x \to +\infty} \left[\sqrt{x^2+4x-1} - x \right] \cdot \frac{\sqrt{x^2+4x-1} + x}{\sqrt{x^2+4x-1} + x} = \lim_{x \to +\infty} \frac{\left(\sqrt{x^2+4x-1}\right)^2 - x^2}{\sqrt{x^2+4x-1} + x} =$$

$$= \lim_{x \to +\infty} \frac{x^2+4x-1-x^2}{\sqrt{x^2+4x-1} + x} = \lim_{x \to +\infty} \frac{4x-1}{\sqrt{x^2+4x-1} + x} =$$

$$= \lim_{x \to +\infty} \frac{x\left(4-\frac{1}{x}\right)}{x\sqrt{1+\frac{4}{x}-\frac{1}{x^2}} + x} = \lim_{x \to +\infty} \frac{4-\frac{1}{x}}{\sqrt{1+\frac{4}{x}-\frac{1}{x^2}} + 1} = 2 \in \mathbb{R}$$

$$\frac{4}{\sqrt{1}+1} = \frac{4}{2}$$

$f(x)$ ha asintoto obliquo a $+\infty$ dato da $y = x+2$

$$a = \lim_{x \to -\infty} \frac{\sqrt{x^2+4x-1}}{x} = \lim_{x \to -\infty} \frac{|x|\sqrt{1+\frac{4}{x}-\frac{1}{x^2}}}{x} = \lim_{x \to -\infty} \frac{(-x)\sqrt{1+\frac{4}{x}-\frac{1}{x^2}}}{x}$$

$$= \lim_{x \to -\infty} -\frac{\sqrt{1+\frac{4}{x}-\frac{1}{x^2}}}{1} = -1 \in \mathbb{R}$$

$$b = \lim_{x \to -\infty} \left[f(x) - ax \right] = \lim_{x \to -\infty} \left[\sqrt{x^2+4x-1} + x \right] =$$

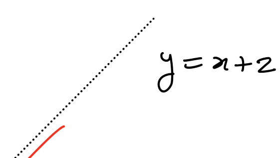
$$= \lim_{x \to -\infty} \left(\sqrt{x^2+4x-1} + x\right) \frac{\left(\sqrt{x^2+4x-1} - x\right)}{\left(\sqrt{x^2+4x-1} - x\right)} =$$

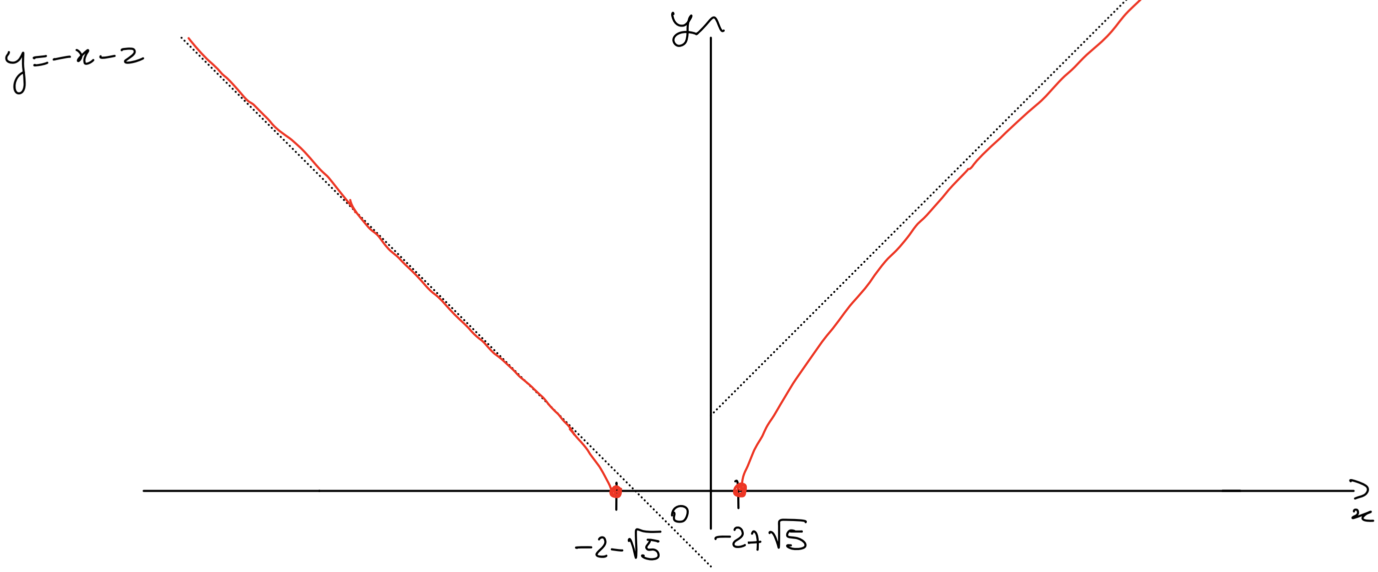
$$= \lim_{x \to -\infty} \frac{x^2+4x-1-x^2}{\sqrt{x^2+4x-1} - x} = \lim_{x \to -\infty} \frac{x\left(4-\frac{1}{x}\right)}{(-x)\sqrt{1+\frac{4}{x}-\frac{1}{x^2}} + (-x)} =$$

$$= \lim_{x \to -\infty} \frac{4-\frac{1}{x}}{-\sqrt{1+\frac{4}{x}-\frac{1}{x^2}} - 1} = -2 \in \mathbb{R}$$

$$\frac{4}{-1-1} = -\frac{4}{2}$$

$f(x)$ ha asintoto obliquo a $-\infty$ dato da $y = -x-2$

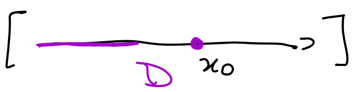




CONTINUITÀ

$f: \mathbb{R} \to \mathbb{R}$, D dom naturale

Def • Sia $x_0 \in D$ isolato ($\exists\, \mathcal{U}$ intorno di x_0 tale che $\mathcal{U} \cap D = \{x_0\}$)



allora si dice che f è continua in x_0.

• Sia $x_0 \in D \cap Acc(D)$, si dice che f è continua in x_0 se $\lim_{x \to x_0} f(x) = f(x_0)$

OSS $D = [a, +\infty)$, $\lim_{x \to a^+} f(x) = f(a)$

$D = (-\infty, a]$, $\lim_{x \to a^-} f(x) = f(a)$

Def Sia $x_0 \in D \cap Acc(D)$, si dice che

- f è continua in x_0 da destra se $\lim_{x \to x_0^+} f(x) = f(x_0)$
- f è continua in x_0 da sinistra se $\lim_{x \to x_0^-} f(x) = f(x_0)$

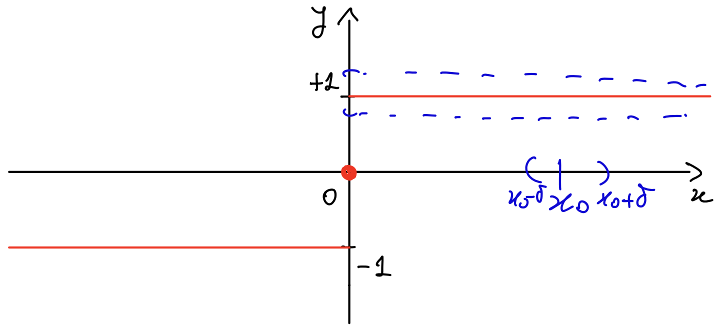
Def f si dice continua in D se è continua in ogni $x_0 \in D$.

Prop Se f, g sono continue nei loro domini naturali, allora $f \circ g$, $f+g$, $f \cdot g$, $|f|$, $\frac{1}{f}$, f/g, sono continue nei loro domini naturali.

Prop Sono continue nei loro domini naturali le funzioni:
- potenze, polinomi (x^α , $\alpha \in \mathbb{R}$; $x^n + \dots$)
- esponenziale, logaritmo
- trigonometriche

ES

- $f(x) = \operatorname{sgn}(x) = \begin{cases} +1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$, $D = \mathbb{R}$



$x_0 > 0$, f è continua in x_0? $\lim_{x \to x_0} f(x) = +1 \overset{?}{=} f(x_0)$ SÌ

($\forall$ intorno U di $\ell \in \mathbb{R}^*$, $\exists\, \delta > 0$ tale che $\forall x \in (x_0-\delta, x_0+\delta) \setminus \{x_0\}$ si ha $f(x) \in U$)

$x_0 < 0$, $\lim_{x \to x_0} f(x) = \lim_{x \to x_0} (-1) = -1 = f(x_0)$

f è continua in $(-\infty, 0) \cup (0, +\infty)$

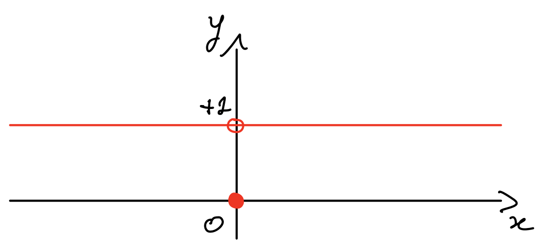
$x_0 = 0 \in D \cap Acc(D)$, $\lim_{x \to 0} f(x)$ esiste? NO

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (+1) = +1$, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-1) = -1$

$f(x)$ con $x \in (0, \varepsilon)$ $f(x)$ con $x \in (-\varepsilon, 0)$

f non è continua in 0, né da dx né da sx. ($f(0) = 0$)

- $f(x) = \begin{cases} +1, & x \in \mathbb{R} \setminus \{0\} \\ 0, & x = 0 \end{cases}$

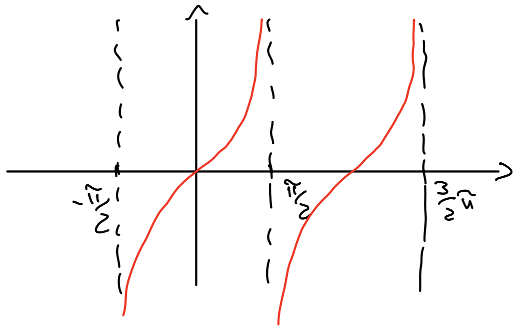


$x_0 = 0 \in D \cap Acc(D)$, $\lim\limits_{x \to 0} f(x) = +1 \neq f(0) = 0$

f non è continua in 0, né da dx né da sx.

f è continua in $\mathbb{R} \setminus \{0\}$.

- $f(x) = \operatorname{tg} x$, $D = \mathbb{R} \setminus \{ \frac{\pi}{2} + k\pi \,/\, k \in \mathbb{Z} \}$



$\frac{\pi}{2} \in \underset{\text{NO}}{D} \cap \underset{\text{sì}}{Acc(D)}$? $\underline{\underline{\text{NO}}}$