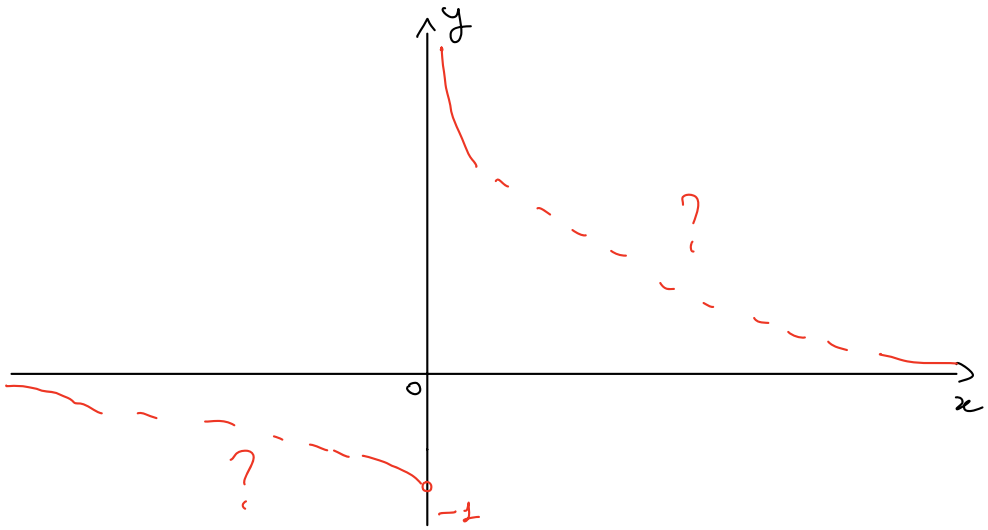
ES $f(x) = e^{\frac{x+x^3}{x^2+x^4}} - 1$ $\quad x \in \mathbb{R}$

$\left(= \exp\left(\frac{x+x^3}{x^2+x^4}\right) - 1 \right)$

$= (-\infty, 0) \cup (0, +\infty)$

Dominio naturale $D = \{ x \in \mathbb{R} : x^2 + x^4 \neq 0 \} = \mathbb{R} \setminus \{0\}$

$x^2 + x^4 = 0 \iff x^2 \underbrace{(1+x^2)}_{\neq 0} = 0 \iff x^2 = 0 \iff x = 0$



$\mathrm{Acc}(D) = \overline{\mathbb{R}} = \mathbb{R} \cup \{\pm\infty\}$

$\lim_{x \to +\infty} \left[e^{\frac{x+x^3}{x^2+x^4}} - 1 \right] = e^{0^+} - 1 = 1^+ - 1 = 0^+$

$\lim_{x \to +\infty} \frac{x+x^3}{x^2+x^4} = \lim_{x \to +\infty} \frac{x^3\left(\frac{1}{x^2}+1\right)}{x^4\left(\frac{1}{x^2}+1\right)} = \lim_{x \to +\infty} \frac{1}{x} \cdot \frac{\frac{1}{x^2}+1}{\frac{1}{x^2}+1} =$ ($\frac{1}{x^2}+1 \to 1$)

$\left(= \frac{+\infty}{+\infty} \text{ forme indet.} \right)$

$= \frac{1}{+\infty} \cdot \frac{\frac{1}{+\infty}+1}{\frac{1}{+\infty}+1} = 0^+ \cdot \frac{0+1}{0+1} = 0^+ \cdot 1$

$= 0^+$

$\lim_{x \to -\infty} \left[e^{\frac{x+x^3}{x^2+x^4}} - 1 \right] = e^{0^-} - 1 = 1^- - 1 = 0^-$

$\lim_{x \to -\infty} \frac{x+x^3}{x^2+x^4} = \lim_{x \to -\infty} \frac{x^3\left(\frac{1}{x^2}+1\right)}{x^4\left(\frac{1}{x^2}+1\right)} = \lim_{x \to -\infty} \frac{1}{x} \cdot \frac{\frac{1}{x^2}+1}{\frac{1}{x^2}+1} = \frac{1}{-\infty} \cdot 1 = 0^-$

$\left(= \frac{-\infty}{+\infty} \text{ forma indet.}\right)$

$$\left[\quad g(x) = \frac{x+x^3}{x^2+x^4}, \quad h(y) = e^y, \quad e^{\frac{x+x^3}{x^2+x^4}} = (h \circ g)(x) = e^{g(x)} \right.$$

$$\lim_{x \to x_0} (h \circ g)(x) = \lim_{y \to y_0} h(y)$$

$$y_0 = \lim_{x \to x_0} g(x)$$

$$\left. \lim_{x \to +\infty} e^{\frac{x+x^3}{x^2+x^4}} = \lim_{y \to 0} e^y, \quad 0 = \lim_{x \to +\infty} \frac{x+x^3}{x^2+x^4} \right]$$

$$\lim_{x \to 0^+} \left[e^{\frac{x+x^3}{x^2+x^4}} - 1 \right] = e^{+\infty} - 1 = \lim_{y \to +\infty} (e^y - 1) = +\infty - 1 = +\infty$$

$$\lim_{x \to 0^+} \frac{x+x^3}{x^2+x^4} = \lim_{x \to 0^+} \frac{x(1+x^2)}{x^2(1+x^2)} = \lim_{x \to 0^+} \frac{1}{x} \cdot \frac{1+x^2}{1+x^2}$$

$\left(= \frac{0^+}{0^+} \text{ forma indet.}\right)$

$$= \frac{1}{0^+} \cdot \underbrace{\frac{1+0^+}{1+0^+}}_{=1} = +\infty \cdot 1 = +\infty$$

ES $$\frac{x+x^3}{2x^2-x^5} = \frac{x^3}{x^5} \cdot \frac{1+\frac{1}{x^2}}{\frac{2}{x^3}-1} = \frac{1}{x^2} \cdot \frac{1+\infty}{+\infty - 1} = +\infty \cdot \underbrace{\frac{+\infty}{+\infty}}_{\text{f.i.}}$$

$$= \frac{x}{x^2} \cdot \frac{1+x^2}{2-x^3} = \frac{1}{x} \cdot \frac{1+0^+}{2-0^+} = \frac{1}{0^+} \cdot \frac{1}{2}$$

$$\lim_{x \to 0^-} \left[e^{\frac{x+x^3}{x^2+x^4}} - 1 \right] = \lim_{y \to -\infty} (e^y - 1) = e^{-\infty} - 1 = \frac{1}{e^{+\infty}} - 1 = 0^+ - 1 = -1^+$$

$$\lim_{x \to 0^-} \frac{x+x^3}{x^2+x^4} = \lim_{x \to 0^-} \frac{x(1+x^2)}{x^2(1+x^2)} = \lim_{x \to 0^-} \frac{1}{x} = \frac{1}{0^-} = -\infty$$

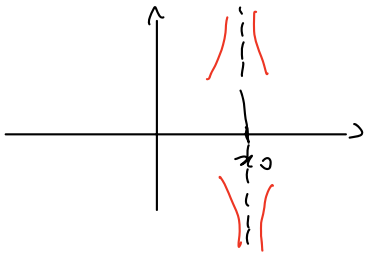
$\exists\, x \in D$ t.c. $f(x) = 0$? $\quad e^{\frac{x+x^3}{x^2+x^4}} - 1 = 0 \iff e^{\frac{x+x^3}{x^2+x^4}} = 1$

$$\iff \frac{x+x^3}{x^2+x^4} = 0 \iff x+x^3 = 0 \iff x \underbrace{(1+x^2)}_{\neq 0} = 0 \quad . \quad \nexists$$

ASINTOTI $\quad f: D \subseteq \mathbb{R} \to \mathbb{R}$

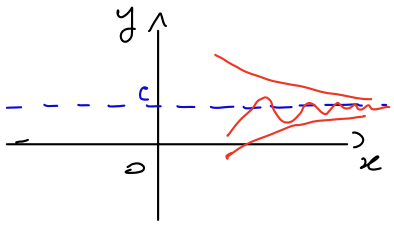
- Sia $x_0 \in \mathbb{R}$, $x_0 \notin D$. con $x_0 \in Acc(D)$.

  f ha un <u>asintoto verticale</u> in x_0 se $\lim_{x \to x_0^+} f$ o $\lim_{x \to x_0^-} f$ sono $\pm\infty$.



- Sia $x_0 + \infty$, punto di accumulazione di D.

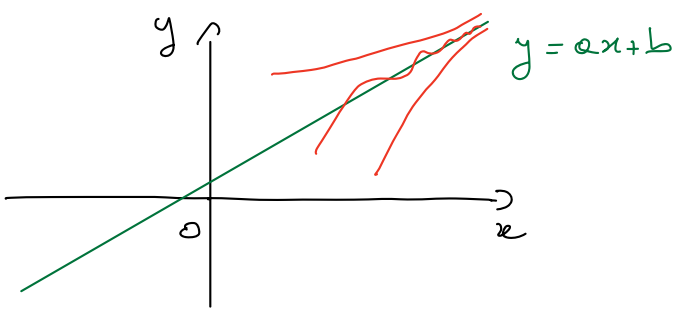
  f ha un <u>asintoto orizzontale</u> a $+\infty$ se esiste $c \in \mathbb{R}$ tale che $\lim_{x \to +\infty} f(x) = c$.



f ha un <u>asintoto obliquo</u> a $+\infty$ se:

- $\exists\, a \in \mathbb{R}$ tale che $\lim_{x \to +\infty} \frac{f(x)}{x} = a$ ($f(x) \sim ax$) (coeff. ang.)
- $\exists\, b \in \mathbb{R}$ tale che $\lim_{x \to +\infty} (f(x) - ax) = b$

Il grafico di $f(x)$ si avvicina a $y = ax + b$



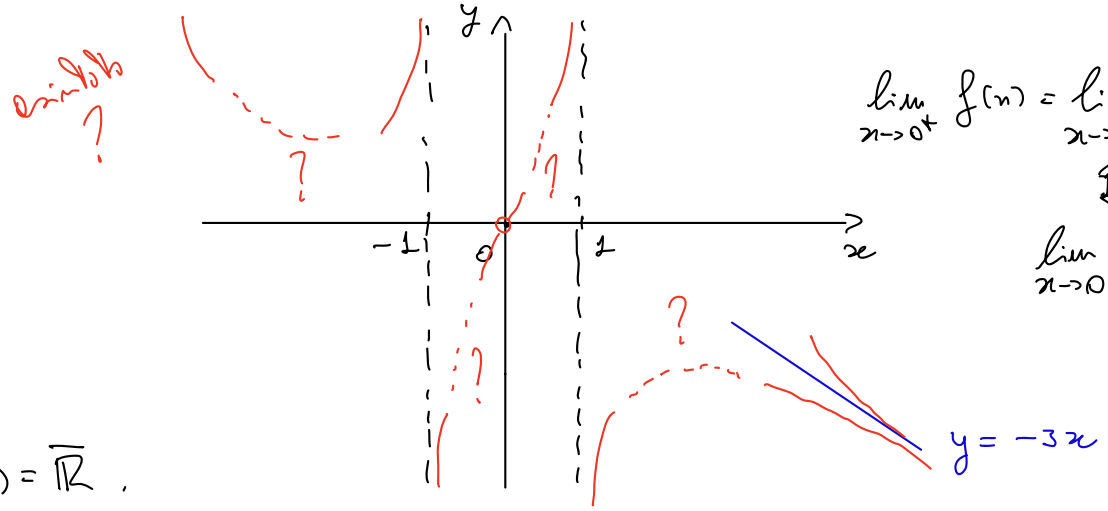
- Sia $x_0 = -\infty$, punto di accumulazione per D.

  Si ripete analogamente al caso $+\infty$.

ES $f(x) = \dfrac{2x^2 + 3x^4}{x - x^3}$, $D = \{ x \in \mathbb{R} : x - x^3 \neq 0 \}$

$= \mathbb{R} \setminus \{ -1, 0, 1 \}$

$x - x^3 = 0 \iff x(1 - x^2) = 0$ $\begin{cases} x = 0 \\ x^2 = 1 \iff x = \pm 1 \end{cases}$



$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 0$

$\Updownarrow$

$\lim_{x \to 0} f(x) = 0$

$Acc(D) = \overline{\mathbb{R}}$.

$x_0 = +\infty$. $\lim_{x \to +\infty} \dfrac{2x^2 + 3x^4}{x - x^3} = \lim_{x \to +\infty} \dfrac{x^4}{x^3} \dfrac{\left(\frac{2}{x^2} + 3\right)}{\left(\frac{1}{x^2} - 1\right)} =$

$= \lim_{x \to +\infty} x \cdot \dfrac{\frac{2}{x^2} + 3}{\frac{1}{x^2} - 1} = +\infty \cdot \dfrac{\frac{2}{+\infty} + 3}{\frac{1}{+\infty} - 1} = +\infty \cdot (-3) =$

$= -\infty$

Asintoto obliquo ?

$q = \lim_{x \to +\infty} \dfrac{f(x)}{x} = \lim_{x \to +\infty} \dfrac{\frac{2x^2 + 3x^4}{x - x^3}}{x} = \lim_{x \to +\infty} \dfrac{1}{x} \cdot \dfrac{2x^2 + 3x^4}{x - x^3} =$

$= \lim_{x \to +\infty} \dfrac{2x^2 + 3x^4}{x^2 - x^4} = \lim_{x \to +\infty} \dfrac{x^4}{x^4} \dfrac{\left(\frac{2}{x^2} + 3\right)}{\left(\frac{1}{x^2} - 1\right)} = \dfrac{\frac{2}{+\infty} + 3}{\frac{1}{+\infty} - 1} = -3$

$b = \lim_{x \to +\infty} \left(f(x) - qx \right) = \lim_{x \to +\infty} \left(f(x) + 3x \right) =$

$= \lim_{x \to +\infty} \left[\dfrac{2x^2 + 3x^4}{x - x^3} + 3x \right] = \lim_{x \to +\infty} \dfrac{2x^2 + 3x^4 + 3x(x - x^3)}{x - x^3} =$

$= \lim_{x \to +\infty} \dfrac{2x^2 + 3x^4 + 3x^2 - 3x^4}{x - x^3} = \lim_{x \to +\infty} \dfrac{5x^2}{x - x^3} = 0$

Asintoto obliquo $y = -3x$

- $x_0 = +1$

$$\lim_{x \to 1^+} \frac{2x^2 + 3x^4}{x - x^3} = \lim_{x \to 1^+} \frac{2x^2 + 3x^4}{x(1-x^2)} = \frac{5}{1 \cdot (1-1^+)} = \frac{5}{1 \cdot 0^-} = \frac{5}{0^-} =$$

$$= 5 \cdot \frac{1}{0^-} = 5 \cdot (-\infty) = -\infty$$

- $x_0 = -1$

$$\lim_{x \to -1^+} \frac{2x^2 + 3x^4}{x(1-x)(1+x)} = \frac{5}{(-1) \cdot 2 \cdot 0^+} = \left(-\frac{5}{2}\right) \cdot \frac{1}{0^+} = \left(-\frac{5}{2}\right) \cdot +\infty = -\infty$$

$$\lim_{x \to -1^-} \frac{2x^2 + 3x^4}{x(1-x)(1+x)} = \frac{5}{(-1) \cdot 2 \cdot 0^-} = \left(-\frac{5}{2}\right) \cdot \frac{1}{0^-} = \left(-\frac{5}{2}\right) \cdot (-\infty) = +\infty$$