

Funzione reale di variabile reale

$$f: \underset{\text{dominio}}{D \subseteq \mathbb{R}} \rightarrow \underset{\text{codominio}}{\mathbb{R}} \quad D \text{ dominio naturale o un suo sottoinsieme}$$

$$\text{Immagine di } f = f(D) \subseteq \mathbb{R} = \{y \in \mathbb{R} : \exists x \in D \text{ t.c. } y = f(x)\}$$

$$\text{Grafico di } f \subseteq \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$$

dominio codominio

$$\text{Graf } f := \{(x, y) \in \mathbb{R}^2 : x \in D, y = f(x)\}$$

Def Massimo di $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$, $\max_D f := \max f(D)$

Minimo di $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$, $\min_D f := \min f(D)$

Estremo superiore di $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$, $\sup_D f := \sup f(D)$

" inferiore di $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$, $\inf_D f := \inf f(D)$

Funzioni elementari

$$f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$$

• Potenze: $f(x) = x^a$, $a \in \mathbb{R}$.

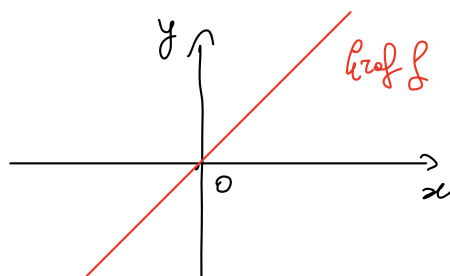
$a=1$ $f(x) = x$, $D = \mathbb{R}$ dom naturale

$$f(D) = \{y \text{ t.c. } \exists x \in D \text{ per cui } f(x) = y\}$$

$$f(D) = \mathbb{R}$$

$$\begin{matrix} \updownarrow \\ x=y \end{matrix}$$

$$\text{Graf } f = \{(x, y) \in \mathbb{R}^2 : x \in D, y = f(x)\}$$



$$\begin{matrix} \updownarrow \\ y=x \end{matrix}$$

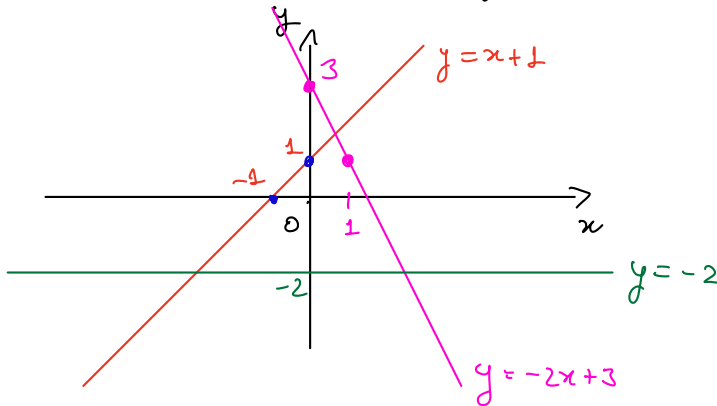
$$\max_{\mathbb{R}} f = \max \mathbb{R} \quad \nexists$$

$$\min_{\mathbb{R}} f = \min \mathbb{R} \quad \nexists$$

$$\sup_{\mathbb{R}} f = \sup \mathbb{R} = +\infty, \quad \inf_{\mathbb{R}} f = \inf \mathbb{R} = -\infty$$

oss $f(x) = c \cdot x + k, \quad c, k \in \mathbb{R}.$

Eq. cartesiane del grafico di f $\bar{x}$ $y = cx + k, \quad x \in \mathbb{R}$



x	y
0	1
-1	0

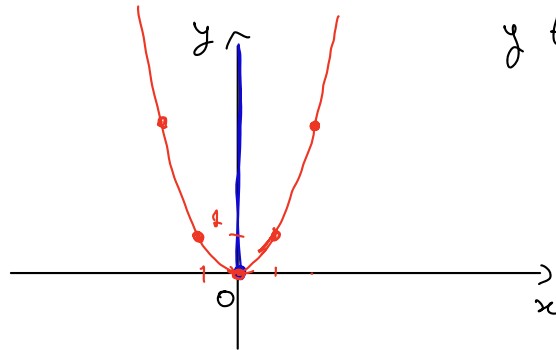
$$y = x + 1$$

x	y
0	3
1	1

$$y = -2x + 3$$

$\alpha = 2$ $f(x) = x^2, \quad D = \mathbb{R}, \quad \underline{f(D) = [0, +\infty)}$

$\forall$ t.c. $\exists x \in D$ p.c. cui $y = x^2$



x	y
0	0
1	1
-1	1
2	4
-2	4

$$\nexists \max_{\mathbb{R}} f, \quad \sup_{\mathbb{R}} f = +\infty$$

$$\min_{\mathbb{R}} f = \min [0, +\infty) = 0$$

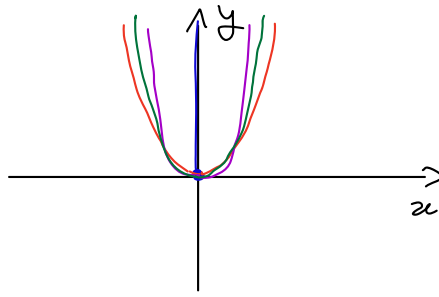
Def Una funzione x dice pari se $\forall x \in D, \quad -x \in D$ e

$$f(-x) = f(x)$$

Una funzione pari ha grafico simmetrico rispetto all'asse y .

$\alpha \in \mathbb{N}$, pari $f(x) = x^\alpha, \quad D = \mathbb{R}, \quad f(D) = [0, +\infty)$

$$\sup_{\mathbb{R}} f = +\infty, \quad \min_{\mathbb{R}} f = 0, \quad f \text{ pari}$$



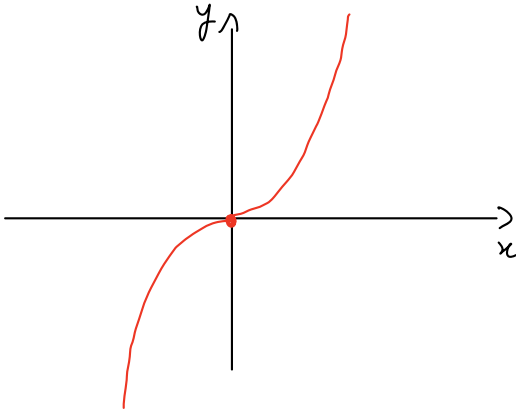
$\alpha = 3$

$f(x) = x^3$, $D = \mathbb{R}$, $f(D) = \mathbb{R}$

$y = x^3 \Leftrightarrow x = \sqrt[3]{y}$

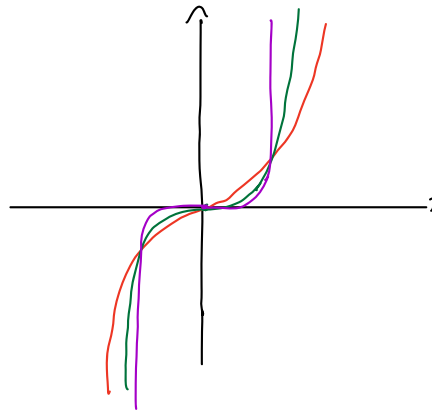
f dispari $\Leftrightarrow f(-x) = -f(x)$

$\sup_{\mathbb{R}} f = +\infty$, $\inf_{\mathbb{R}} f = -\infty$



$\alpha \in \mathbb{N}$, dispari

$f(x) = x^\alpha$



$\alpha \in \mathbb{Z}$, $\alpha < 0$

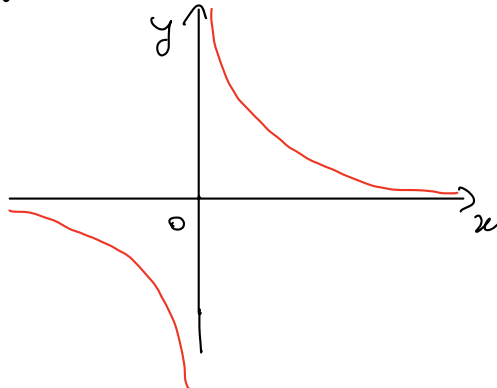
$\alpha = -1, -2, -3, \dots$

$\alpha = -1$

$f(x) = \frac{1}{x}$, $D = \mathbb{R} \setminus \{0\}$, $f(D) = \mathbb{R} \setminus \{0\}$

$y = \frac{1}{x} \Leftrightarrow x = \frac{1}{y}$

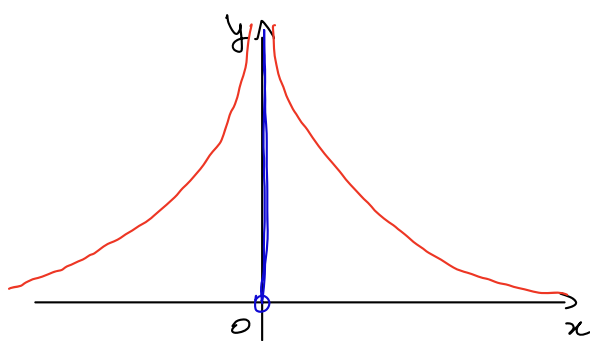
$f(-x) = \frac{1}{-x} = -\frac{1}{x} = -f(x)$
dispari



$\alpha = -2$

$f(x) = \frac{1}{x^2}$, $D = \mathbb{R} \setminus \{0\}$, $f(D) = (0, +\infty)$

$y = \frac{1}{x^2} \Leftrightarrow x^2 = \frac{1}{y}$



$$\Leftrightarrow x = \pm \sqrt{\frac{1}{y}}$$

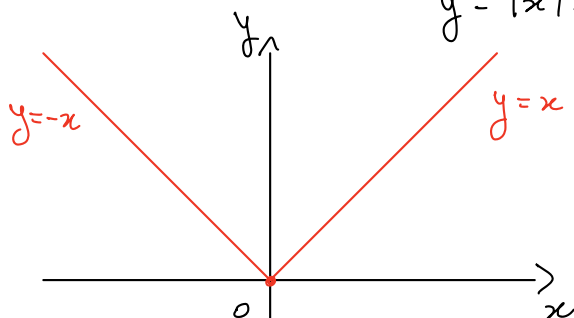
$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$$

peri

• Valore assoluto, $f(x) = |x| = \max\{x, -x\}$

$$D = \mathbb{R}, f(D) = [0, +\infty)$$

$$y = |x| = \max\{x, -x\}.$$

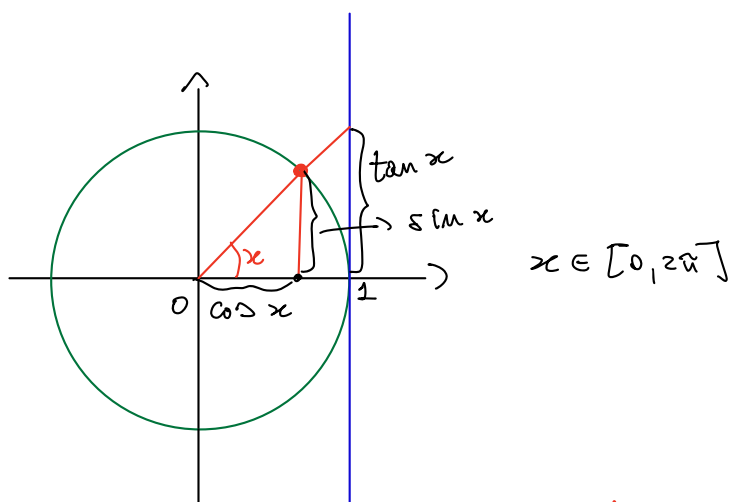


$$f(-x) = |-x| = |x| = f(x)$$

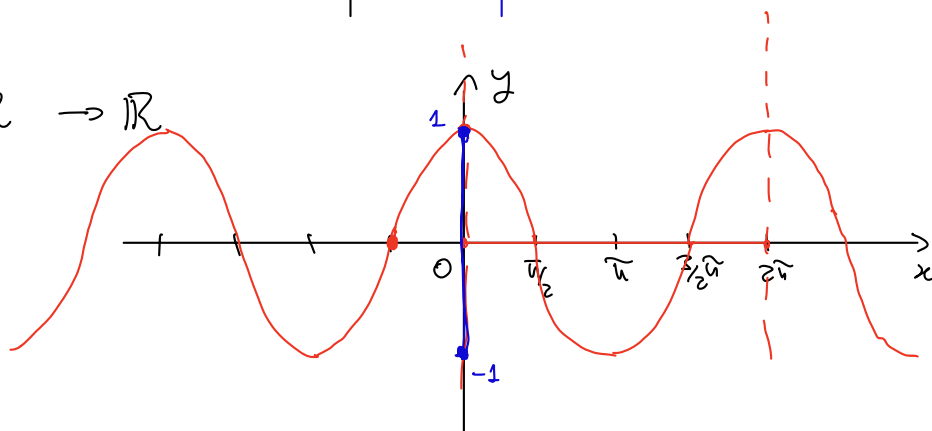
peri

$$\sup_{\mathbb{R}} f = +\infty, \quad \min_{\mathbb{R}} f = 0$$

• Funzioni Trigonometriche



$$\cos : [0, 2\pi] \subseteq \mathbb{R} \rightarrow \mathbb{R}$$

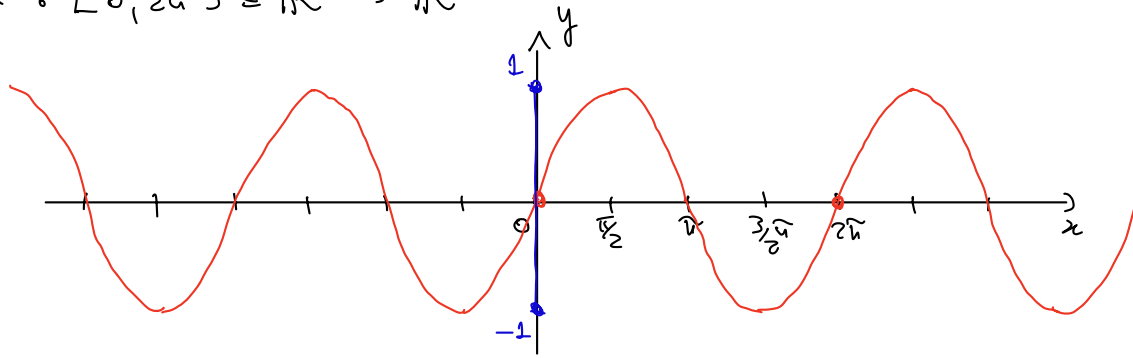


$$\cos : \mathbb{R} \rightarrow \mathbb{R}, \quad \cos(x + 2\pi) = \cos x \quad \forall x \in \mathbb{R}, \quad \text{Im } \cos = [-1, 1]$$

periodica di periodo 2π

$\cos$ è peri

$$\sin : [0, 2\pi] \subseteq \mathbb{R} \rightarrow \mathbb{R}$$

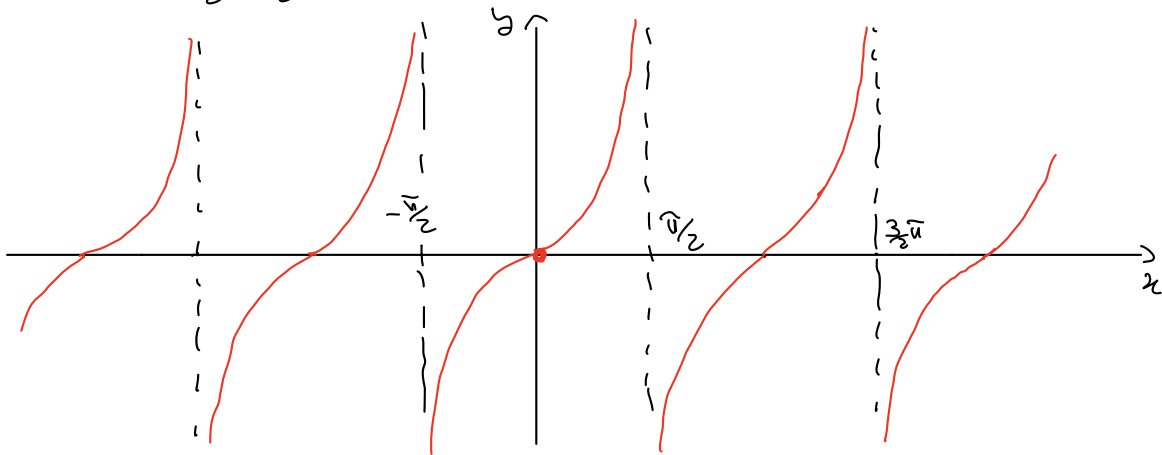


$$\sin : \mathbb{R} \rightarrow \mathbb{R}, \quad \sin(x + 2\pi) = \sin x \quad \forall x \in \mathbb{R}, \quad \text{Im } \sin = [-1, 1]$$

$\sin$ è disperi

$$\text{oss} \quad (\sin x)^2 + (\cos x)^2 = 1 \quad \forall x \in \mathbb{R}.$$

$$\tan : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$$



$$\tan(x + \pi) = \tan x, \quad \text{periodica di periodo } \pi$$

$$\begin{aligned} \tan x &= \frac{\sin x}{\cos x}, \quad \tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\frac{\sin x}{\cos x} \\ &= -\tan x, \quad \text{disperi} \end{aligned}$$

$$\tan : \mathcal{D} \subset \mathbb{R} \rightarrow \mathbb{R}, \quad \mathcal{D} = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi : k \in \mathbb{Z} \right\}, \quad \text{Im } \tan = \mathbb{R}$$