

How to find all roots of complex polynomials by Newton's method

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Abstract. We investigate Newton's method to find roots of polynomials of fixed degree d , appropriately normalized: we construct a finite set of points such that, for every root of every such polynomial, at least one of these points will converge to this root under Newton's map. The cardinality of such a set can be as small as $1.11 d \log^2 d$; if all the roots of the polynomial are real, it can be $1.30 d$.

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1 Introduction

This paper concerns the global dynamics of Newton's method for finding roots of a polynomial $p(z)$ in one variable. We show how to find all roots of such a polynomial without recourse to deflation (which has serious