ESERCIZI SU SEGNATURA

Titolo nota 18/04/2018

3.18

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$1 > 0$

$1 - 4 = -3 < 0$

$0 - 2 \cdot \det\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} + 1 \cdot \det\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = -2 \cdot 1 + 1 = -1$

$1 \quad 1 \quad -3 \quad -1$

P C P $\qquad (2\ 1\ 0)$

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 8 & -4 \\ 0 & -4 & 2 \end{pmatrix} \qquad 1 \quad 1 \quad 8 \quad 0$$

$i_0 > 0$ perché $\det(S) = 0$

$i_+ \geq 2$

$W = \operatorname{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$ $\quad S|_W$ è definito positivo

$i_+ + i_- + i_0 = 3 \longrightarrow (2, 0, 1)$

$$S = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & \alpha \\ 1 & \alpha & \alpha^2 \end{pmatrix} \qquad \overset{P}{1}, \overset{C}{1}, -4, \overset{?}{\ } -5\alpha^2 + 4\alpha$$

$\det(S) = -2 \det\begin{pmatrix} 2 & 1 \\ \alpha & \alpha^2 \end{pmatrix} - \alpha \begin{pmatrix} 1 & 2 \\ 1 & \alpha \end{pmatrix} = -2(2\alpha^2 - \alpha) - \alpha(\alpha - 2) = -4\alpha^2 + 2\alpha - \alpha^2 + 2\alpha = -5\alpha^2 + 4\alpha$

$-5\alpha^2 + 4\alpha > 0$ se $0 < \alpha < \frac{4}{5}$ $\qquad$ PCC $\quad (1, 2, 0)$

$-5\alpha^2 + 4\alpha < 0$ se $\alpha < 0 \vee \alpha > \frac{4}{5}$ $\qquad$ PCP $\quad (2, 1, 0)$

Per $\alpha = 0$ $\qquad S_0 = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $\qquad (1, 1, 1) \leftarrow$

$1 \quad 1 \quad -4 \quad 0$

P C ?

$i_0 > 0$

$i_+ \geq 1$

$i_- \geq 1$

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad (x\ y\ z)\begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0 \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix}^T \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 0$

$\boxed{x + 2y + z = 0} \longrightarrow \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ ad esempio

$(x\ y\ z)\begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 0 \qquad (x\ y\ z)\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = 0 \quad \boxed{2y + z = 0}$

$x = 0 \quad y = 1, \ z = -2 \qquad \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$

$$M=\begin{pmatrix}1&1&0\\0&0&1\\0&-1&-2\end{pmatrix}\qquad {}^tM=\begin{pmatrix}1&0&0\\1&0&-1\\0&1&-2\end{pmatrix}$$

$${}^tM\,S_0\,M=\underbrace{\begin{pmatrix}1&0&0\\1&0&-1\\0&1&-2\end{pmatrix}\begin{pmatrix}1&2&1\\2&0&0\\1&0&0\end{pmatrix}\begin{pmatrix}1&1&0\\0&0&1\\0&-1&-2\end{pmatrix}}=$$

$$\begin{pmatrix}1&0&0\\1&0&-1\\0&1&-2\end{pmatrix}\begin{pmatrix}1&0&0\\2&2&0\\1&1&0\end{pmatrix}=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$$

$$\operatorname{Span}\left\{\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}=W\qquad \alpha\begin{pmatrix}0\\1\\0\end{pmatrix}+\beta\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}0\\\alpha\\\beta\end{pmatrix}$$

$$(0\ \alpha\ \beta)\begin{pmatrix}1&2&1\\2&0&0\\1&0&0\end{pmatrix}\begin{pmatrix}0\\\alpha\\\beta\end{pmatrix}=(0\ \alpha\ \beta)\begin{pmatrix}2\alpha+\beta\\0\\0\end{pmatrix}=0\quad\forall\alpha,\beta$$

$S|_W$ è il prodotto scalare $\equiv$ nullo

$$\alpha=\frac{4}{5}\qquad S_{4/5}=\begin{pmatrix}\boxed{1}&2&1\\2&0&4/5\\1&\frac{4}{5}&16/25\end{pmatrix}$$

	d_1	d_2	$\det(S_{4/5})$
1	1	-4	0
	P	C	

$$(1,1,1)\Longleftarrow\begin{cases}i_+\geq 1\\ i_-\geq 1\\ i_0\geq 0\end{cases}\qquad i_++i_-+i_0=3$$

$$\begin{pmatrix}1\\0\\0\end{pmatrix}\qquad (x\,y\,z)\left(S_{4/5}\right)\begin{pmatrix}1\\0\\0\end{pmatrix}=0$$

$$x+2y+z=0\qquad \begin{pmatrix}2\\-1\\0\end{pmatrix}$$ è un'altra scelta possibile

$$(x\,y\,z)\left(S_{4/5}\right)\begin{pmatrix}2\\-1\\0\end{pmatrix}=0\qquad (x\,y\,z)\begin{pmatrix}0\\4\\6/5\end{pmatrix}=0\qquad 4y+\frac{6}{5}z=0$$

$$\boxed{10y+3z=0}$$

$$\begin{pmatrix}-4\\-3\\10\end{pmatrix}$$

$$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}2\\-1\\0\end{pmatrix}\begin{pmatrix}-4\\-3\\10\end{pmatrix}\right\}$$

$$M=\begin{pmatrix}1&2&-4\\0&-1&-3\\0&0&10\end{pmatrix}\qquad {}^tM=\begin{pmatrix}1&0&0\\2&-1&0\\-4&-3&10\end{pmatrix}$$

$${}^tM\, S_{4/5}\, M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \text{Segnatura è } (1,1,1)$$

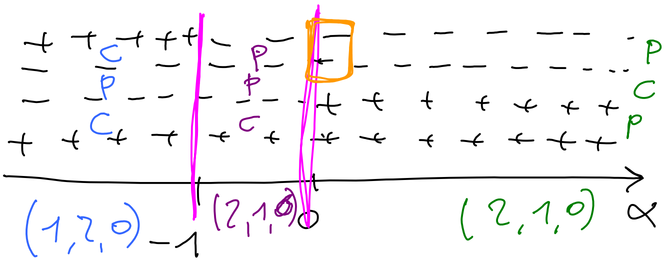
$$S = \begin{pmatrix} \alpha & \alpha+1 & \alpha+2 \\ \alpha+1 & \alpha+2 & \alpha+1 \\ \alpha+2 & \alpha+1 & \alpha \end{pmatrix} \qquad 1 \quad \alpha \quad -1$$

$$\det S = \det \begin{pmatrix} \alpha & \alpha+1 & 1 \\ \alpha+1 & \alpha+2 & -1 \\ \alpha+2 & \alpha+1 & -1 \end{pmatrix} = \det \begin{pmatrix} 2\alpha+1 & 2\alpha+3 & 0 \\ -1 & +1 & 0 \\ \alpha+2 & \alpha+1 & -1 \end{pmatrix} =$$

$$= (-1)\cdot \det \begin{pmatrix} 2\alpha+1 & 2\alpha+3 \\ -1 & +1 \end{pmatrix} = -1\cdot(2\alpha+1+2\alpha+3)$$

$$= -1(4\alpha+4) = -4(\alpha+1)$$

$1,\ \alpha,\ -1,\ -1-\alpha$



$\alpha < -1$ $(1,2,0)$

$\alpha = -1$ a parte

$-1 < \alpha < 0$ $(2,1,0)$

$\alpha = 0$ a parte

$\alpha > 0$ $(2,1,0)$

Caso $\alpha = 0$

$$S_0 = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{pmatrix} \qquad -1 \quad -4$$

$\det(S_0) \neq 0$

$$(-1)\cdot\det\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} + 2\det\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} =$$

$$(-1)\cdot(-2) + 2\cdot(-3) = 2-6 \neq 0$$

$i_0 = 0$

$i_+ \geq 1$

~~$(3,0,0)$~~

$(2,1,0)$

$(1,2,0)$?

lo escludo guardando $\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$

lo escludo guardando il segno del $\det(S_0)$

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ non può essere utilizzato per partire!

$$(x \ \ y \ \ z)\,(S_0)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0 \qquad x+2y+z=0$$

$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ è ok

$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ è il 2° vettore della nuova base

$(x\ y\ z)\,(S_0)\begin{pmatrix}1\\0\\-1\end{pmatrix}=0 \qquad (x\ y\ z)\begin{pmatrix}-2\\0\\2\end{pmatrix}=0 \qquad -2x+2z=0$

$x=z$

$\begin{pmatrix}1\\-1\\1\end{pmatrix}$ 3° vettore di base

$M=\begin{pmatrix}0&1&1\\1&0&-1\\0&-1&1\end{pmatrix} \qquad {}^tM=\begin{pmatrix}0&1&0\\1&0&-1\\1&-1&1\end{pmatrix}$

${}^tM\,S_0\,M=\begin{pmatrix}2&0&0\\0&-4&0\\0&0&2\end{pmatrix} \qquad i_+=2 \quad i_-=1 \quad i_0=0$

$(2,1,0)$ per $\alpha=0$

Caso $\alpha=-1$ $\qquad S_{-1}=\begin{pmatrix}-1&0&1\\0&1&0\\1&0&-1\end{pmatrix} \qquad \det(S_{-1})=0$

$i_0>0$

$i_+\geq 1$

$i_-\geq 1$

unica possibilità $(1,1,1)$

$M=\begin{pmatrix}1&0&1\\0&1&0\\1&0&-1\end{pmatrix}$

${}^tM\,S_{-1}\,M=\begin{pmatrix}0&0&0\\0&1&0\\0&0&-4\end{pmatrix}$

3,20

$S=\begin{pmatrix}1&2&-1\\2&3&1\\-1&1&-8\end{pmatrix}$

1 1 -1 0

P C ?

$\det(S)=$

$=-25+30-5$

$=0$

$i_0>0$

$i_+\geq 1$

$i_-\geq 1$

$(1,1,1)$

$W=\{2x+y=z\}$

base di $W^\perp$

Base di $W=\left\{\begin{pmatrix}1\\0\\2\end{pmatrix},\begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$

$w\in W=\alpha\begin{pmatrix}1\\0\\2\end{pmatrix}+\beta\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}\alpha\\\beta\\2\alpha+\beta\end{pmatrix}$

$(x,y,z)\begin{pmatrix}1&2&-1\\2&3&1\\-1&1&-8\end{pmatrix}\begin{pmatrix}\alpha\\\beta\\2\alpha+\beta\end{pmatrix}=(x\ y\ z)\begin{pmatrix}-\alpha+\beta\\4\alpha+4\beta\\-17\alpha-7\beta\end{pmatrix}$

$=(-\alpha+\beta)x+4(\alpha+\beta)y-(17\alpha+7\beta)z=$

$$= \alpha(-x+4y-17z)+\beta(x+4y-7z)=0 \quad \forall \alpha,\beta$$

$$\begin{cases} -x+4y-17z=0 \\ x+4y-7z=0 \end{cases} \qquad 2x+10z=0 \qquad x=-5z$$

$$y=\frac{7z+5z}{4} \qquad y=3z$$

$\begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}$ base di $W^\perp$

$\dim(W^\perp)=1$

3.17 Verifiche: matrici con la stessa segnatura

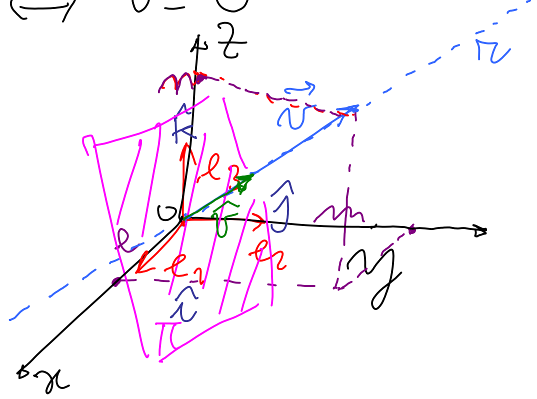
$(2,1,0) \quad (2,1,0) \quad (1,2,0) \quad (2,1,0)$

$1^a, 3^a, 4^a$ congruenti $\qquad 2^a$ sta da sola.

PARTE PIÙ "GEOMETRICA"

prodotto scalare euclideo $\cdot \qquad V = \mathbb{R}^2$ o $\mathbb{R}^3$

$\|v\| = \sqrt{v\cdot v} \geq 0, \quad \|v\|=0 \iff v=0$



$v \neq 0$

$\hat{v} = \frac{\vec{v}}{\|v\|}$ versore

$v=\begin{pmatrix} l \\ m \\ n \end{pmatrix} \qquad \vec{v} = l\hat{i}+m\hat{j}+n\hat{k}$

$$\hat{v} = \frac{l\hat{i}+m\hat{j}+n\hat{k}}{\sqrt{l^2+m^2+n^2}}$$

$r: \begin{cases} x=lt \\ y=mt \\ z=nt \end{cases} \quad t\in\mathbb{R}$ qualsiasi (parametro)

$\pi: \ lx+my+nz=0 \qquad r \perp \pi$

base su π $\quad (-m, l, 0)$
$\quad (0, n, -m) \qquad m \neq 0$

vettore di $\pi \quad \alpha\begin{pmatrix} -m \\ l \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 0 \\ n \\ -m \end{pmatrix} = \begin{pmatrix} -m\alpha \\ +n\beta+l\alpha \\ -m\beta \end{pmatrix} = w$

$$v = \begin{pmatrix} te \\ tm \\ tn \end{pmatrix} \qquad v \cdot w = \cancel{-te\,m\alpha} + \cancel{tmn\beta} \cancel{+ tme\alpha} - \cancel{tmn\beta} = 0 \quad \forall \alpha, \beta$$

$$e^2 + m^2 + n^2 = 1 \qquad e' = \frac{e}{\sqrt{e^2+m^2+n^2}} \quad \text{e così via}$$

e allora certamente $e'^2 + m'^2 + n'^2 = 1$ (coseni direttori)

Cerchiamo tutti i vettori $\perp \pi$ $(e^2+m^2+n^2=1)$

$\vec{v}_1 = m\hat{\imath} - e\hat{\jmath}$

cerco un altro vettore di base per generare π, che sia $\perp \vec{v}_1$ in modo da avere su π una base ortonormale

$$(\alpha\hat{\imath} + \beta\hat{\jmath} + \gamma\hat{k}) = \vec{v}_2 \qquad \alpha^2 + \beta^2 + \gamma = 1$$

$$\begin{cases} \alpha e + \beta m + \gamma n = 0 & \perp \hat{v} \\ \alpha m - \beta e = 0 & \perp v_1 \\ \alpha^2 + \beta^2 + \gamma^2 = 1 & \|\vec{v}_2\| = 1 \end{cases}$$

$$\boxed{\hat{v}_1 = \frac{m\hat{\imath} - e\hat{\jmath}}{\sqrt{m^2+e^2}} = \frac{m\hat{\imath} - e\hat{\jmath}}{\sqrt{1-n^2}}}$$

$$\alpha = \frac{\beta e}{m} \qquad \frac{\beta e^2}{m} + \beta m + \gamma n = 0$$

$$= \beta(e^2+m^2) + \gamma n m = 0$$

$$\gamma = \frac{-\beta(1-n^2)}{nm}$$

$$\beta^2 \frac{e^2}{m^2} + \beta^2 + \frac{\beta^2(1-n^2)^2}{n^2m^2} = 1$$

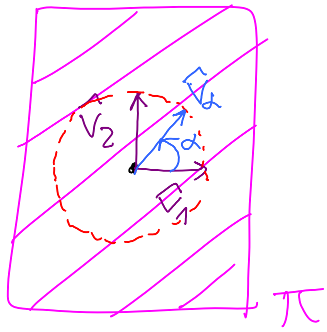
$$\beta^2\left(n^2e^2 + n^2m^2 + (1-n^2)^2\right) = n^2m^2$$

$$\beta^2\left(n^2(1-n^2) + (1-n^2)^2\right) = n^2m^2$$

$$\beta^2(1-n^2)(\cancel{n^2} + 1 - \cancel{n^2}) = n^2m^2 \qquad \beta^2 = \frac{n^2m^2}{1-n^2}$$

$$\beta = \pm\frac{nm}{\sqrt{1-n^2}} \qquad \alpha = \pm\frac{ne}{\sqrt{1-n^2}} \qquad \gamma = \mp\sqrt{1-n^2}$$

$$\hat{v}_1 = \frac{m\hat{\imath} - e\hat{\jmath}}{\sqrt{1-n^2}} \qquad \hat{v}_2 = \frac{ne\hat{\imath} + nm\hat{\jmath} - (1-n^2)\hat{k}}{\sqrt{1-n^2}}$$



Un qualunque altro versore $\perp r$ lo ottengo

$$\hat v_\alpha = \cos\alpha\,\hat v_1 + \operatorname{sen}\alpha\,\hat v_2 =$$

$$= \frac{1}{\sqrt{1-n^2}}\left[\cos\alpha\left(m\hat i - l\hat j\right) + \operatorname{sen}\alpha\left(nl\hat i + nm\hat j + (n^2-1)\hat k\right)\right] =$$

$$\hat v_\alpha = \frac{1}{\sqrt{1-n^2}}\left[(m\cos\alpha + nl\operatorname{sen}\alpha)\hat i + (-l\cos\alpha + nm\operatorname{sen}\alpha)\hat j + (n^2-1)\operatorname{sen}\alpha\,\hat k\right]$$

$$\hat v_\alpha \perp \hat v \quad \forall \alpha$$

$$\begin{pmatrix} l \\ m \\ n \end{pmatrix} \cdot \begin{pmatrix} m\cos\alpha + nl\operatorname{sen}\alpha \\ -l\cos\alpha + nm\operatorname{sen}\alpha \\ +(n^2-1)\operatorname{sen}\alpha \end{pmatrix} = \cancel{ml\cos\alpha} + nl^2\operatorname{sen}\alpha - \cancel{ml\cos\alpha} + n\,m^2\operatorname{sen}\alpha + n(n^2-1)\operatorname{sen}\alpha =$$

$$= n(l^2+m^2)\operatorname{sen}\alpha + n(n^2-1)\operatorname{sen}\alpha =$$

$$= n\operatorname{sen}\alpha\,\underbrace{\left(l^2+m^2+n^2-1\right)}_{0} = 0$$