ESERCIZI - P. SCAL - TRASF - GEOM. AFFINE

Titolo nota 09/05/2017

3.15 Prod. scalare su $M(n)$ $\langle A, B\rangle = tr({}^tAB)$

$n = 2$ $\boxed{e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}$ $e_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $e_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ $\boxed{e_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}$

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Cerchiamo una base ortonormale

${}^te_1 e_1 = e_1$

${}^te_4 e_4 = e_4$

$$S' = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$ ← in modo da ottenere

$$v = \begin{pmatrix} x & y \\ z & t \end{pmatrix} \quad 0 = tr\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x & y \\ z & t \end{pmatrix}\right] = tr\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} = x$$

$\Rightarrow x = 0$

$$0 = tr\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x & y \\ z & t \end{pmatrix}\right] = tr\begin{pmatrix} 0 & 0 \\ z & t \end{pmatrix} = t \Rightarrow t = 0$$

$$v = \begin{pmatrix} 0 & y \\ z & 0 \end{pmatrix} \qquad tr\left[\begin{pmatrix} 0 & z \\ y & 0 \end{pmatrix}\begin{pmatrix} 0 & y \\ z & 0 \end{pmatrix}\right] = 1 \Rightarrow z^2 + y^2 = 1$$

$$v' = \begin{pmatrix} x' & y' \\ z' & t' \end{pmatrix} \quad x' = 0 \;\; t' = 0 \quad z'^2 + y'^2 = 1 \qquad tr\left[\begin{pmatrix} 0 & z' \\ y' & 0 \end{pmatrix}\begin{pmatrix} 0 & y \\ z & 0 \end{pmatrix}\right] = 0 \Rightarrow$$

$\Rightarrow zz' + yy' = 0$

$$\begin{cases} z^2 + y^2 = 1 \\ z'^2 + y'^2 = 1 \\ zz' + yy' = 0 \end{cases}$$

$z = \cos\alpha$
$y = \operatorname{sen}\alpha$
$z' = -\operatorname{sen}\alpha$
$y' = \cos\alpha$

$\alpha = \frac{\pi}{4}$ $\quad v = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad v' = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Base ortonormale: $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

Caso $n=3$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

posso rimetterli nella base ortonormale

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

n vettori di base hanno 1 nella posiz. i,i con $i = 1 \dots n$ e tutti gli altri zeri

n^2-n vettori di base ottenuti combinando linearmente per somma e differenza le due matrici che hanno 1 nella posizione i,j con $i>j$

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determinare g tale che i vettori $v_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ e $v_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ formino una base ortonormale

$\begin{pmatrix}0\\1\end{pmatrix} = e_2 \qquad \begin{pmatrix}1\\1\end{pmatrix} = e_1 + e_2$

$\langle v_1, v_2 \rangle = \langle e_2, e_1 \rangle + \underbrace{\langle e_2, e_2 \rangle}_{=1} = 0 \Rightarrow \langle e_2, e_1 \rangle = -1$

$\langle v_1, v_1 \rangle = \langle e_2, e_2 \rangle = 1 \qquad \langle e_1, e_2 \rangle = -1$

$\langle v_2, v_2 \rangle = 1 = \langle e_1, e_1 \rangle + \langle \overset{+1}{e_2}, e_2 \rangle + 2 \langle \overset{-1}{e_1}, e_2 \rangle \Rightarrow \langle e_1, e_1 \rangle = 2$

$\cancel{1} = \langle e_1, e_1 \rangle + \cancel{1} - 2 \Rightarrow \langle e_1, e_1 \rangle = 2 \qquad S = \begin{pmatrix}2 & -1\\-1 & 1\end{pmatrix}$

$v_1 = e_2 \qquad e_2 = v_1$

$v_2 = e_1 + e_2 \qquad e_1 = v_2 - v_1 \qquad M = \begin{pmatrix}-1 & 0\\1 & 1\end{pmatrix}$

$$\begin{pmatrix}-1 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}-1 & 0\\1 & 1\end{pmatrix} = \begin{pmatrix}-1 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}-1 & 0\\1 & 1\end{pmatrix} = \begin{pmatrix}2 & 1\\1 & 1\end{pmatrix}$$

c'è un segno sbagliato!

3.35 iniziato   Rotaz. di $2\pi/3$ intorno a $\begin{pmatrix}1\\1\\1\end{pmatrix}$

$R^3 = I \qquad \frac{2\pi}{3} \times 3 = 2\pi \rightarrow$ identità

$e_1 \xrightarrow{R} e_2 \xrightarrow{R} e_3 \xrightarrow{R} e_1 \qquad e_1 \xrightarrow{R^3} e_1$

$e_2 \rightarrow e_3 \rightarrow e_1 \rightarrow e_2 \qquad e_2 \xrightarrow{R^3} e_2$

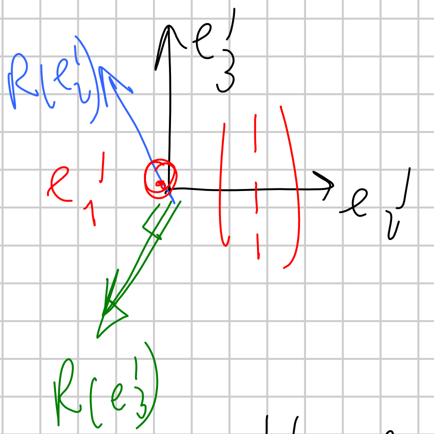
$e_3 \rightarrow e_1 \rightarrow e_2 \rightarrow e_3 \qquad e_3 \xrightarrow{R^3} e_3$

$R = \begin{pmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{pmatrix} \quad tr(R) = 0 \qquad R^3 = I$ provare!

$$\cos\theta = \frac{tr(R) - 1}{2} \qquad \cos\theta = -\frac{1}{2} \qquad \theta = \pm\frac{2\pi}{3}$$

$$R\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$$ ← autovettore di R con autovalore 1

METODO BOVINO



Nella base $\{e_1'\ e_2'\ e_3'\} = \mathcal{B}'$

$$R' = \begin{pmatrix}1&0&0\\0&-1/2&-\sqrt{3}/2\\0&+\sqrt{3}/2&-1/2\end{pmatrix} = [R']^{\mathcal{B}'}_{\mathcal{B}'}$$

$$e_1' = \overbrace{\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}}^{\frac{1}{\sqrt{3}}(e_1+e_2+e_3)} \qquad e_2' = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix} \qquad e_3' = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\1\\-2\end{pmatrix}$$

$$\mathcal{B} = \left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\} \qquad M^{-1} = \begin{pmatrix}1/\sqrt{3}&1/\sqrt{2}&1/\sqrt{6}\\1/\sqrt{3}&-1/\sqrt{2}&1/\sqrt{6}\\1/\sqrt{3}&0&-2/\sqrt{6}\end{pmatrix}$$ ← ortogonale

$$R = M^{-1} R' M$$

$$\begin{pmatrix}1/\sqrt{3}&1/\sqrt{2}&1/\sqrt{6}\\1/\sqrt{3}&-1/\sqrt{2}&1/\sqrt{6}\\1/\sqrt{3}&0&-2/\sqrt{6}\end{pmatrix}\begin{pmatrix}1&0&0\\0&-1/2&-\sqrt{3}/2\\0&\frac{\sqrt{3}}{2}&-1/2\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{3}}&\frac{1}{\sqrt{3}}&\frac{1}{\sqrt{3}}\\\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}&0\\\frac{1}{\sqrt{6}}&\frac{1}{\sqrt{6}}&-\frac{2}{\sqrt{6}}\end{pmatrix} = \ldots$$

- - - - -

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = R$$

rotaz. di $\pi/4$ intorno a $\frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

$$\frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \frac{1}{3}\begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} \quad \frac{1}{3}\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

a meno del segno (!)

4.4

$$r = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \operatorname{span}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \qquad \begin{cases} x = 1 \\ y = 1 \\ z = 0 + 1 \cdot \lambda \end{cases} \qquad \lambda \in \mathbb{R} \text{ parametro}$$

$r \not\parallel s$

$$s = \left\{ \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \operatorname{span}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\} \qquad \begin{cases} x = 2 \\ y = 0 + 1 \cdot \mu \\ z = 3 \end{cases}$$

Dimostrare che sono sghembe (non hanno punti in comune pur non essendo parallele)

Poiché $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0 \quad r \perp s$

$$\begin{cases} x_r = x_s \\ y_r = y_s \\ z_r = z_s \end{cases}$$

$$\begin{cases} 1 = 2 \\ 1 = \mu \\ \lambda = 3 \end{cases} \quad ! \; !! \qquad r \cap s = \emptyset$$

$\exists!$ retta affine t tale che $t \perp r$ $t \perp s$

$t \cap r = P$ $t \cap s = Q$

direzione di t deve essere $\perp$ sia alla direzione di r che alla direzione di s

$$\begin{pmatrix} l \\ m \\ n \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0 \qquad \begin{pmatrix} l \\ m \\ n \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0 \qquad l = \pm 1$$

scelgo $l = 1$

$\Downarrow$ $n = 0$ $\qquad$ $\Downarrow$ $m = 0$

$$t : \begin{cases} x = 1 \cdot \nu + x_0 \\ y = y_0 \\ z = z_0 \end{cases} \qquad \begin{cases} \nu + x_0 = 1 \\ y_0 = 1 \\ z_0 = \lambda \\ \nu' + x_0 = 2 \\ y_0 = \mu \\ z_0 = 3 \end{cases}$$

$y_0 = 1$ $z_0 = 3$ $\lambda = 3$

$\mu = 1$

$P \equiv (1, 1, 3)$ $\qquad Q \equiv (2, 1, 3)$ $\qquad \begin{cases} \nu + x_0 = 1 \\ \nu' + x_0 = 2 \end{cases}$

ad esempio $x_0 = 1$ $\nu = 0$ $\nu' = 1$

oppure $x_0 = 0$ $\nu = 1$ $\nu' = 2$

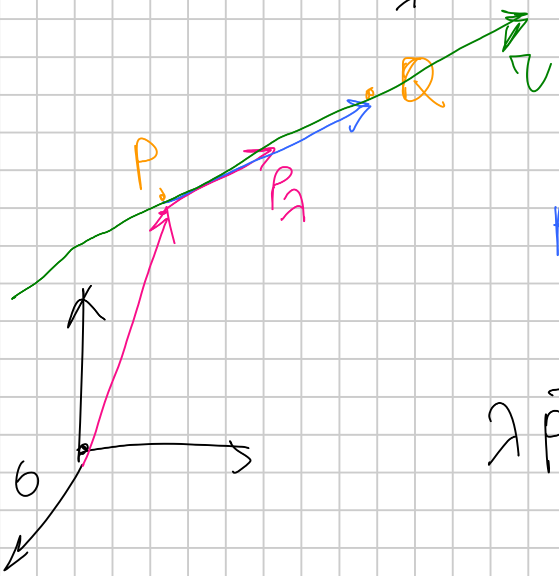
$$t : \begin{cases} x = \nu + 1 \\ y = 1 \\ z = 3 \end{cases}$$

ES. 4.5 rette nello spazio

$P = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\qquad Q = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ (1) trovare la retta r per P e Q

$P_\lambda \in r$ $\quad \vec{OP_\lambda} = \vec{OP} + \vec{PP_\lambda} = \vec{OP} + \lambda \vec{PQ}$

$\vec{OQ}$



$\vec{PQ} = Q - P = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$

$\lambda \vec{PQ} = \lambda \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$

$r = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \text{span} \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \right\}$ $\quad \begin{cases} x = 1 - 2\lambda \\ y = 1 - \lambda \\ z = \lambda \end{cases}$

per $\lambda = 0$ si ha P $\quad$ per $\lambda = 1$ si ha Q

per $\lambda < 0$ si hanno i punti "prima di P"

per $\lambda > 1$ si ottengono i punti "dopo Q"

per $0 < \lambda < 1$ si hanno i punti fra P e Q

(2) eq. cartesiana del piano $\pi \perp r$ e passante per $R = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

$ax + by + cz = d$ $\quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$

$a = 2 \quad b = 1 \quad c = -1$

$2x + y - z = d$ fascio di piani paralleli fra loro

imponiamo che $R \in \pi$ $\quad 2 \cdot 2 + 1 \cdot 0 - 1 \cdot 0 = d \Rightarrow$

$\Rightarrow d=4 \quad e \ \pi: \quad 2x+y-z=4 \qquad 2x+y-z-4=0$

(3) Trovare l'equazione cartesiana del piano $\perp \pi$ contenente r e $\parallel$ alla retta passante per R e $S=\begin{pmatrix}0\\0\\1\end{pmatrix}$

Come sono fatti tutti i piani che contengono r?

$\begin{cases} x=1-2\lambda \\ y=1-\lambda \\ z=\lambda \end{cases}$ $\lambda \in \mathbb{R}$

Scrivo r come intersez. fra 2 piani togliendo il parametro

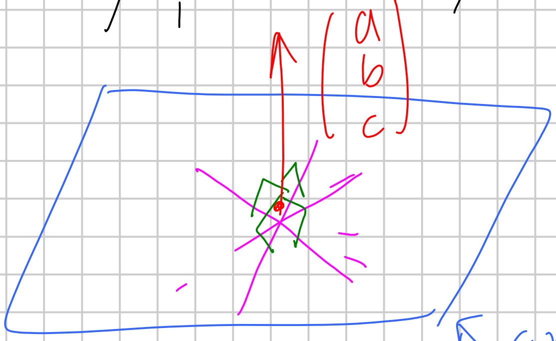
$\begin{cases} x=1-2z \\ y=1-z \end{cases} \qquad \begin{cases} x+2z-1=0 \\ y+z-1=0 \end{cases}$

$\mu\cdot(x+2z-1)+\nu\cdot(y+z-1)=0$ Fascio di piani

i punti di r $\in$ a uno qualunque di questi piani

$\mu(\cancel{1}-\cancel{2\lambda}+\cancel{2\lambda}-\cancel{1})+\nu\cdot(\cancel{1}-\cancel{\lambda}+\cancel{\lambda}-\cancel{1})=\mu\cdot 0+\nu\cdot 0=0$

$\forall\ \mu,\nu \qquad \mu x+\nu y+(2\mu+\nu)z-(\mu+\nu)=0$

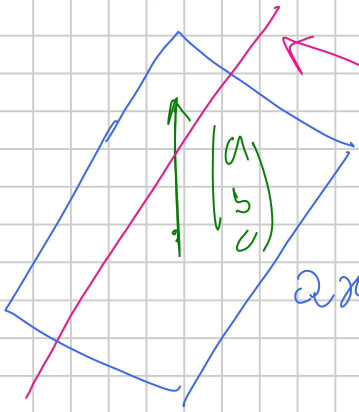


$ax+by+cz=d$

$\begin{pmatrix}-2\\-1\\1\end{pmatrix}\cdot\begin{pmatrix}\mu\\\nu\\2\mu+\nu\end{pmatrix}=0$

$-2\mu-\nu+2\mu+\nu=0$ non si ha nessuna informaz. ulteriore!

$$R-S=\begin{pmatrix}2\\0\\0\end{pmatrix}-\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\-1\end{pmatrix}$$



$$\begin{cases}x=l\lambda+x_0\\ y=m\lambda+y_0\\ z=n\lambda+z_0\end{cases}$$

$ax+by+cz=d$

$$\begin{pmatrix}a\\b\\c\end{pmatrix}\cdot\begin{pmatrix}l\\m\\n\end{pmatrix}=0$$

$$\begin{pmatrix}\mu\\ \nu\\ 2\mu+\nu\end{pmatrix}\cdot\begin{pmatrix}2\\0\\-1\end{pmatrix}=0$$

$$\cancel{2\mu}-\cancel{2\mu}-\nu=0\Rightarrow\nu=0$$

$\mu=$ qualsiasi

$$\underset{1}{\mu}x+\underset{0}{\nu}y+(2\underset{1}{\mu}+\underset{0}{\nu})z-(\underset{1}{\mu}+\underset{0}{\nu})=0$$

$$x+2z-1=0$$

$\pi: ax+by+cz=d$

$$r:\begin{cases}x=l\lambda+x_0\\ y=m\lambda+y_0\\ z=n\lambda+z_0\end{cases}$$

Verificare che tutta $r\in\pi$

$$a(l\lambda+x_0)+b(m\lambda+y_0)+c(n\lambda+z_0)=d$$ deve essere vera $\forall\lambda$

$$(al+bm+cn)\lambda+ax_0+by_0+cz_0-d=0\quad\forall\lambda$$

$$\begin{cases}al+bm+cn=0\\ ax_0+by_0+cz_0-d=0\end{cases}$$

$\leftarrow$ parallelismo fra retta e piano

$\leftarrow$ verifichiamo che un punto della retta $\in$ al piano

4.6 $r: \begin{cases} x - y + z = 0 \\ 2x + y + z = 1 \end{cases}$ $s: \begin{cases} x + z = 1 \\ y - z = -2 \end{cases}$

$\begin{cases} x = -\frac{2}{3}\lambda + \frac{1}{3} \\ y = \lambda/3 + 1/3 \\ z = \lambda \end{cases}$

$\begin{cases} 2x - 2y + 2z = 0 \\ 2x + y + z = 1 \end{cases}$

$3y - z = 1$

$\lambda' \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1/3 \\ 1/3 \\ 0 \end{pmatrix}$

$\begin{cases} x = -\mu + 1 \\ y = \mu - 2 \\ z = \mu \end{cases}$

$\mu \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$

r ed s non sono parallele poiché $\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \neq k \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\begin{cases} -2\lambda + 1/3 = -\mu + 1 \\ \lambda + 1/3 = \mu - 2 \\ 3\lambda = \mu \end{cases}$ $\begin{cases} -2\lambda + 1/3 = -3\lambda + 1 \\ \lambda + 1/3 = 3\lambda - 2 \end{cases}$

$\begin{cases} \lambda = 2/3 \\ -2\lambda = -7/3 \end{cases}$ impossibile

r e s non hanno punti in comune

r e s sono sghembe

$\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0$ $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0$ $\begin{cases} -2l + m + 3n = 0 \\ -l + m + n = 0 \end{cases}$

$n = l - m$ dalla 2^a $-2l + m + 3l - 3m = 0$

$l - 2m = 0$

$m = 1$
$l = 2$
$n = 1$

$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

qualunque retta del tipo $\begin{cases} x = 2\nu + x_0 \\ y = \nu + y_0 \\ z = \nu + z_0 \end{cases}$ è $\perp$ sia ad r che ad s

$$\begin{cases} 2\nu + x_0 = -\frac{2}{3}\lambda + \frac{1}{3} \\ \nu + y_0 = \frac{\lambda}{3} + \frac{1}{3} \\ \nu + z_0 = \lambda \end{cases} \quad \begin{cases} 2\nu' + x_0 = -\mu + 1 \\ \nu' + y_0 = \mu - 2 \\ \nu' + z_0 = \mu \end{cases}$$

$\downarrow$ P

$\downarrow$ Q

$d(P, Q)$ è la distanza fra le rette