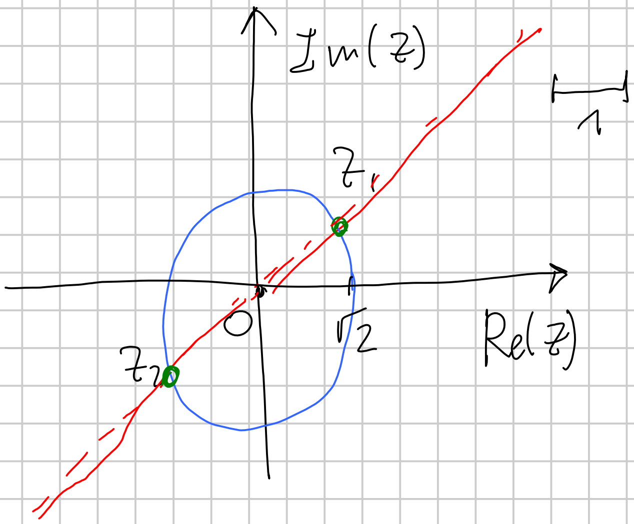
COMPITINO - RISOLUZIONE

Note Title 07/03/2017

TEST 5 domande

$$\begin{cases} |z|^2 = 2 \Rightarrow |z| = \sqrt{2} \\ Re(z) = Im(z) \end{cases}$$



$z = x + iy$

$$\begin{cases} x^2 + y^2 = 2 \\ x = y \end{cases} \rightarrow \quad y^2 + y^2 = 2 \qquad 2y^2 = 2 \qquad y = \pm 1$$

$$x = \pm 1$$

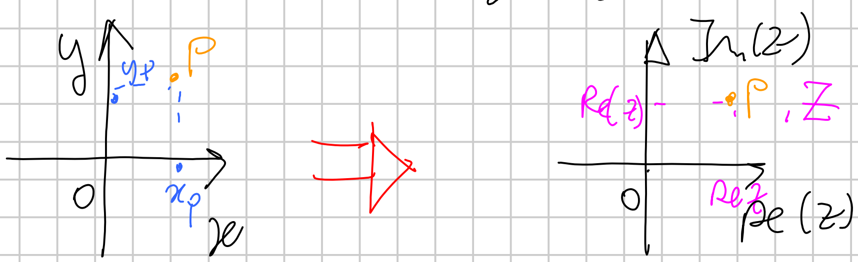
$z_1 = 1 + i \qquad z_2 = -1 - i$

$z = \rho e^{i\theta} \qquad |z| = \rho \qquad \rho = \sqrt{2}$

$Im(z) = \rho \operatorname{sen}\theta \qquad Re(z) = \rho\cos\theta$

$\operatorname{sen}\theta = \cos\theta \qquad tg\,\theta = 1 \qquad \theta = \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}$

$z_1 = \sqrt{2}\, e^{i\pi/4} \qquad z_2 = \sqrt{2}\, e^{5i\pi/4} \qquad k = 0 \quad k = 1$



Piano cartesiano $\qquad$ Piano di Gauss

$z = x + iy$

$\bar{z} = x - iy$

$x^2 + y^2 = 2 \rightarrow |z|^2 = 2$

$x^2 + y^2 + ax + by + c = 0$

$x = \dfrac{z + \bar{z}}{2}$

$iy = \dfrac{z - \bar{z}}{2}$

$y = (z - \bar{z})/2i$

$$|z|^2 + a\frac{(z+\bar z)}{2} + b\frac{z-\bar z}{2i} + c = 0 \qquad C \equiv \left(-\frac{a}{2}, -\frac{b}{2}\right)$$

$$z_c = -\frac{a}{2} - \frac{b}{2} i$$

$$2|z|^2 + a(z+\bar z) - ib(z-\bar z) + 2c = 0$$

$$2|z|^2 + (a-ib)z + (a+ib)\bar z + 2c = 0 \qquad a+ib = -2z_c \qquad a-ib = -2\bar z_c$$

$$2|z|^2 - 2\bar z_c z - 2z_c\bar z + 2c = 0$$

$$|z|^2 - \bar z_c z - z_c \bar z + c = 0$$

2

$$\begin{pmatrix} 2 & 1 & 7 \\ 0 & 1 & 0 \\ 0 & 3 & -1 \end{pmatrix}$$

$$P(\lambda) = \det\begin{pmatrix} 2-\lambda & 1 & 7 \\ 0 & 1-\lambda & 0 \\ 0 & 3 & -1-\lambda \end{pmatrix} = (2-\lambda)\cdot\det\begin{pmatrix} \cancel{1-\lambda \quad 0} \\ 3 \quad -1-\cancel{\lambda} \end{pmatrix}$$

$$= (2-\lambda)(1-\lambda)(-1-\lambda) = \dots *$$

$$-\lambda^3 + (2+1-1)\lambda^2 + \lambda\left[2(1-1) + 1(-2+1) - 1(-2-1)\right] + 2\cdot 1\cdot(-1)$$

$$= -\lambda^3 + 2\lambda^2 + 1\cdot\lambda - 2$$

$(-1)^{n(=3)}$ $\quad$ $Tr(A)$ $\quad$ $(-1)^n\cdot(-1) = +1$ $\quad$ $\det(A)$

$$\begin{pmatrix} 2 & 1 & 7 \\ 0 & 1 & 0 \\ 0 & 3 & -1 \end{pmatrix}$$

2 $\quad$ -1 $\quad$ -2

$$2 - 1 - 2 = -1$$

$$*\ (2-\lambda)(-1 - \cancel{\lambda} + \cancel{\lambda} + \lambda^2) = (2-\lambda)(\lambda^2-1) =$$

$$2\lambda^2 - 2 - \lambda^3 + \lambda = -\lambda^3 + 2\lambda^2 + \lambda - 2$$

$$\lambda_1 + \lambda_2 + \lambda_3 = Tr(A) \qquad \lambda_1\cdot\lambda_2\cdot\lambda_3 = \det(A)$$

$(-\lambda+\lambda_1)(-\lambda+\lambda_2)(-\lambda+\lambda_3)$

$\lambda_1\lambda_2\lambda_3 = \det(A)$

$\lambda_1+\lambda_2+\lambda_3 = \operatorname{tr}(A)$

3 $\{v_1 \; v_2 \; \dots \; v_n\}$ base in V

$v \in V$ è esprimibile in maniera unica come comb. lin dei vettori della base

$$V = c_1 V_1 + c_2 V_2 + c_3 V_3 + \dots + c_n V_n = \sum_{i=1}^{n} c_i v_i$$

$$\varphi_i(V) = c_i \qquad \varphi : V \to \mathbb{R} \quad \text{funzionale}$$

Dato V, l'insieme di tutti i possibili funzionali lineari definiti su V è uno spazio vettoriale che si chiama SPAZIO DUALE di V (e talvolta si indica con V^* o V' ...)

Ogni funzionale lineare è esprimibile come combinazione lineare di "opportuni" funzionali che costituiscono una base di V^*

Data $\{v_1 \dots v_n\}$ base di V, la scelta più "opportuna" è di prendere $\{\varphi_1 \dots \varphi_n\}$ come base in V^*

$$v = \sum_{i=1}^{n} c_i v_i = \sum_{i=1}^{n} \underbrace{\varphi_i(v)}_{c_i} v_i$$

$\mathbb{R}^3$ base canonica $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $e_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$V = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ IN QUESTA BASE $\varphi_1^{(c)}(V) = x_1$ $\varphi_2^{(c)}(V) = x_2$ $\varphi_3^{(c)}(V) = x_3$

$V_1 = e_1 + e_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$V_2 = e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $V_3 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = e_2 - e_3$

$V = x e_1 + y e_2 + z e_3$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

quali sono le coordinate di V se uso la base V_1 V_2 V_3 ?

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c_2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c_3 \\ -c_3 \end{pmatrix}$$

$$\begin{cases} x = c_1 \\ y = c_1 + c_2 + c_3 \\ z = -c_3 \end{cases} \quad c_2 = y - c_1 - c_3 \qquad \begin{cases} c_1 = x \\ c_2 \; -x + y + z \\ c_3 = -z \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + (-x+y+z) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + (-z) \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$\varphi_1 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x$ $\varphi_2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -x + y + z$ $\varphi_3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -z$

4) $F: \mathbb{C}^4 \to \mathbb{C}^3 \qquad \dim(N(F)) = 2$

$\uparrow$ Ker (KERNEL)

V

$$\dim(V) = \dim(N(F)) + \dim(Im(F))$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$4 \qquad 2 \quad \Rightarrow \quad 2$$

$\text{Rango}(F) = \dim(Im(F)) = 2$

Rk

Car (Caratteristica)

5 U, W

$U \subset V \qquad W \subset V$

$V = Mat_{2\times 2} \qquad \dim(V) = 4$

$U + W = V \Rightarrow \dim(U+W) = 4$

$\dim(U \cap W) = 1 \qquad \dim(W) = 2$ - Qual è $\dim(U) = ?$

$$\dim(U+W) + \dim(U \cap W) = \dim(U) + \dim(W)$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$4 \qquad 1 \qquad x \qquad 2$$

$4 + 1 = x + 2 \qquad x = 3$

1 COMPITO Teorema dim $f: \mathbb{R}^3 \to \mathbb{R}^2$

$g \circ f$ non può essere iniettiva! $g\ \mathbb{R}^2 \to \mathbb{R}$

2 COMPITO $\qquad \mathbb{R}^3 \qquad W \subset \mathbb{R}^3$

$$W = \left\{ w \in \mathbb{R}^3 ,\ w = \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x+y+z=0 \right\}$$

$$\mathbb{R}[t]_{\leq 3} \qquad V \subset \mathbb{R}[t]_{\leq 3} \qquad V = \{ p \in \mathbb{R}[t]_{\leq 3} : p(1)=0 \}$$

$$F : W \to V \qquad F\begin{pmatrix} x \\ y \\ z \end{pmatrix} = x + 2yt + zt^2 + (x+z)t^3$$

a) Scegliere base $\{w_1, w_2\}$ di W e base $\{v_1, v_2, v_3\}$ di V e scrivere la matrice associata a F rispetto a queste basi

$$w_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \qquad w_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$v_1 = t-1 \qquad v_2 = t^2 - 1 \qquad v_3 = t^3 - 1$$

$$F\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 1 + 2\cdot(-1)t + 0t^2 + (1+0)t^3 = \underbrace{1}_{2-1} - 2t + t^3$$

$$= 2 - 2t - 1 + t^3 = -2(t-1) + 1(t^3-1) = -2v_1 + v_3$$

$$\begin{pmatrix} -2 & 2 \\ 0 & -1 \\ 1 & -1 \end{pmatrix}$$

$$F\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 0 + 2(+1)t + (-1)t^2 + (0-1)t^3$$

$$= 2t - t^2 - t^3 = 2t - 2 - t^2 + 1 - t^3 + 1$$

$$= 2(t-1) - (t^2-1) - (t^3-1) = 2v_1 - v_2 - v_3$$

$$G\begin{pmatrix}1\\0\\0\end{pmatrix} = 1+t \qquad G\begin{pmatrix}x\\y\\z\end{pmatrix} = F\begin{pmatrix}x\\y\\z\end{pmatrix} \text{ se } \begin{pmatrix}x\\y\\z\end{pmatrix} \in W$$

$$G: \mathbb{R}^3 \to \mathbb{R}[t]_{\leq 3} \qquad G\begin{pmatrix}1\\0\\0\end{pmatrix} = \boxed{\begin{pmatrix}1\\1\\0\\0\end{pmatrix}}$$

1^a colonna

$$p_0 = 1 \quad p_1 = t \quad p_2 = t^2 \quad p_3 = t^3$$

$$G\begin{pmatrix}0\\1\\0\end{pmatrix} = G\begin{pmatrix}-1+1\\1\\0\end{pmatrix} = G\left[\begin{pmatrix}-1\\1\\0\end{pmatrix} + \begin{pmatrix}1\\0\\0\end{pmatrix}\right] =$$

$$= -\underset{F}{G}\begin{pmatrix}1\\-1\\0\end{pmatrix} + G\begin{pmatrix}1\\0\\0\end{pmatrix} = -\begin{pmatrix}1\\-2\\0\\1\end{pmatrix} + \begin{pmatrix}1\\1\\0\\0\end{pmatrix} = \boxed{\begin{pmatrix}0\\3\\0\\-1\end{pmatrix}}$$

2^a colonna

$$G\begin{pmatrix}0\\0\\1\end{pmatrix} = G\begin{pmatrix}0\\-1+1\\1\end{pmatrix} = G\left[\begin{pmatrix}0\\-1\\1\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix}\right] =$$

$$= \underset{F}{G}\underset{\in V}{\begin{pmatrix}0\\-1\\1\end{pmatrix}} + G\begin{pmatrix}0\\1\\0\end{pmatrix} = -F\begin{pmatrix}0\\1\\-1\end{pmatrix} + G\begin{pmatrix}0\\1\\0\end{pmatrix} =$$

$$= \begin{pmatrix}0\\-2\\1\\1\end{pmatrix} + \begin{pmatrix}0\\3\\0\\-1\end{pmatrix} = \boxed{\begin{pmatrix}0\\1\\1\\0\end{pmatrix}}$$

3^a colonna

$$\begin{pmatrix}1&0&0\\1&3&1\\0&0&1\\0&-1&0\end{pmatrix} = G^{e_1 e_2 e_3}_{1,t,t^2,t^3}$$

$$z^4 \bar{z} = 32 \qquad z^4 = \rho^4 e^{4i\theta}$$

$$\bar{z} = \rho \cdot e^{-i\theta}$$

$$\rho^5 e^{i(4-1)\theta} = 32$$

$$\begin{cases} \rho^5 = 32 \\ e^{i3\theta} = e^0 \end{cases} \qquad \rho = 2$$

$$3i\theta = 0 + 2k\pi i$$

$$\theta = \frac{2k\pi}{3} \qquad k = 0, 1, 2$$

$$z_0 = 2 \qquad z_1 = 2e^{\frac{2\pi i}{3}} \qquad z_2 = 2e^{\frac{4\pi i}{3}}$$

$$e^{2\pi i/3} = \cos\frac{2\pi}{3} + i\,\mathrm{sen}\left(\frac{2\pi}{3}\right) \qquad \text{etc...}$$