ESERCIZI GEOMETRIA

Titolo nota 25/05/2016

Riflessione

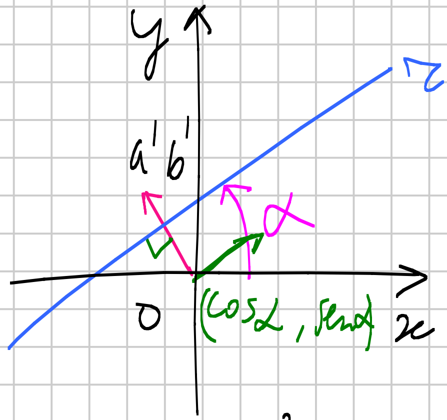
$r: ax+by+c=0$

$$\begin{pmatrix} x_{p'} \\ y_{p'} \end{pmatrix} = \frac{1}{a^2+b^2}\begin{pmatrix} b^2-a^2 & -2ab \\ -2ab & a^2-b^2 \end{pmatrix}\begin{pmatrix} x_p \\ y_p \end{pmatrix} - \frac{2c}{a^2+b^2}\begin{pmatrix} a \\ b \end{pmatrix}$$

divido per a^2+b^2 $(\neq 0)$ $\quad (a,b)\neq(0,0)$

$$r: \underbrace{\frac{a}{a^2+b^2}}_{a'}x + \underbrace{\frac{b}{a^2+b^2}}_{b'}y + \underbrace{\frac{c}{a^2+b^2}}_{c'}$$

$$r: \underbrace{a'}_{-\sin\alpha}x + \underbrace{b'}_{\cos\alpha}y + c' = 0 \qquad a'^2+b'^2=1$$



$r: -\sin\alpha\, x + \cos\alpha\, y + c' = 0$

$$\begin{pmatrix} x_p' \\ y_p' \end{pmatrix} = \begin{pmatrix} \cos^2\alpha-\sin^2\alpha & 2\sin\alpha\cos\alpha \\ 2\sin\alpha\cos\alpha & \sin^2\alpha-\cos^2\alpha \end{pmatrix}\cdot\begin{pmatrix} x_p \\ y_p \end{pmatrix}$$

$$\begin{pmatrix} x_p' \\ y_p' \end{pmatrix} = \begin{pmatrix} \cos(2\alpha) & \sin(2\alpha) \\ \sin(2\alpha) & -\cos(2\alpha) \end{pmatrix}\begin{pmatrix} x_p \\ y_p \end{pmatrix} + 2c'\begin{pmatrix} \sin\alpha \\ -\cos\alpha \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \qquad \theta = 2\alpha \qquad \alpha = \frac{\theta}{2}$$

p 61

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65 $\qquad f(x) = \underline{\underline{A}}\,x + \underline{b}$

Se A non ha autovalore 1, allora f ha un punto fisso x_0

$$A x_0 + b = x_0 \longrightarrow A x_0 - x_0 = -b$$

$A x_0 - I x_0 = -b \qquad (A-I)\, x_0 = -b$

Se $A-I$ è surgettivo, allora siamo a posto

$\exists$ almeno un x_0 tale che $(A-I)\, x_0 = -b$

$\operatorname{Im}(A-I) = \mathbb{R}^n$

$$\boxed{\begin{array}{l} T: \mathbb{R}^n \to \mathbb{R}^n \\ \dim \operatorname{Ker}(T) + \dim \operatorname{Im}(T) = n \end{array}}$$

$\dim \operatorname{Im}(A-I) = n \rightarrow \dim \operatorname{Ker}(A-I) = 0$

$\operatorname{Ker}(A-I) = \{0_{\mathbb{R}^n}\}$ (nucleo banale)

$\operatorname{Ker}(T) = \{ v \in \mathbb{R}^n : T(v) = 0_{\mathbb{R}^n} \}$

$(A-I)(v) = 0 \qquad Av = v \qquad v$ è autovalore di A con autovalore 1

Se A ha autovalore 1 non posso concludere che $\operatorname{Ker}(A-I) = \{0_{\mathbb{R}^n}\}$

Se so che $\nexists\, v : Av = v$ (cioè so che A non ha autovalore 1) allora certamente posso dire che $\dim(\operatorname{Ker}(A-I)) = 0$ e quindi $A-I$ è surgettivo

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Matrice simmetrica da studiare $\quad S = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ -1 & 1 & -8 \end{pmatrix}$

$\det(S) = 0$

$$5\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ -8 \end{pmatrix} = \begin{pmatrix} 5-6+1 \\ 10-9-1 \\ -5-3+8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\begin{pmatrix} 5 \\ -3 \\ -1 \end{pmatrix} \in \operatorname{Ker}(S) \qquad S: \mathbb{R}^3 \to \mathbb{R}^3$

Calcolo della segnatura di S $\quad i_0=1 \quad i_+=? \quad i_-=?$

$\lambda_1=0 \quad \lambda_2,\lambda_3\neq 0 \qquad \lambda_2+\lambda_3=-4$

$\cancel{\lambda_1\lambda_2}+\cancel{\lambda_1\lambda_3}+\lambda_2\lambda_3=\lambda_2\lambda_3 \qquad \lambda_2\lambda_3=(3-4)+(-8-1)+(-24-1)$
$=-35$

poichè $\lambda_2\lambda_3<0$, certamente λ_2 e λ_3 sono discordi $\lambda_2<0$
$\lambda_3>0$

$i_+=1 \quad i_-=+1 \qquad (1\ \ 1\ \ 1)$
rette incidenti

$x^2+4xy+3y^2-2x+2y-8=0$ che rette sono?

$$\begin{cases} x_1^2+4x_1x_2+3x_2^2-2x_1x_3+2x_2x_3-8x_3^2=0 \\ x_3=0 \end{cases}$$

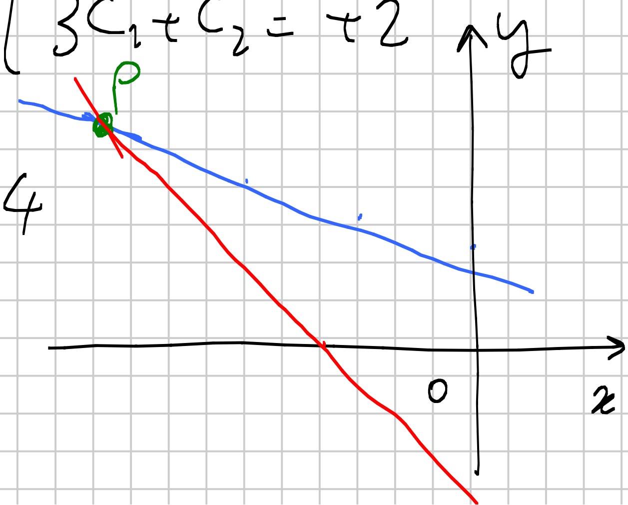
$$\longrightarrow x_1^2+4x_1x_2+3x_2^2=0 \qquad x_1=-2x_2\pm\sqrt{4x_2^2-3x_2^2}=$$

$$= \begin{cases} -2x_2-x_2=-3x_2 \\ -2x_2+x_2=-x_2 \end{cases} \qquad (x_1+x_2)(x_1+3x_2)=0$$

$$(x+y+c_1)(x+3y+c_2)=x^2+4xy+3y^2-2x+2y-8=0$$

principio di identità dei polinomi

$$\begin{cases} (c_1+c_2)x=-2x \\ (3c_1+c_2)y=+2y \end{cases} \quad \forall x, \forall y \qquad \begin{cases} c_1+c_2=-2 \\ 3c_1+c_2=+2 \end{cases}$$



$2c_1=4 \qquad c_1=2 \qquad c_2=-4$

$(x+y+2)(x+3y-4)=0$

Coordinate di P?

$$\begin{cases} x+y=-2 \\ x+3y=4 \end{cases} \qquad y_P = 3 \quad x_P = -5$$

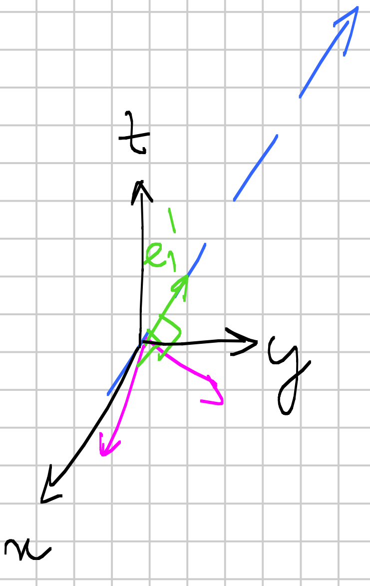
$\begin{pmatrix} 5 \\ -3 \\ -1 \end{pmatrix}$ punto in coordinate proiettive

$\begin{pmatrix} -5 \\ +3 \\ +1 \end{pmatrix}$ "coordinate affini"

$\begin{pmatrix} -5 \\ 3 \end{pmatrix}$ coord. cartesiane

2,31 $\quad Ax+b$ di $\mathbb{R}^3$ $\quad$ rotaz. di $\pi/3$ intorno alla retta r

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \underbrace{t\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}$$



$$R' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{pmatrix}$$

$$\begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & 1/\sqrt{2} & 1/\sqrt{6} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & 1/\sqrt{6} \\ \frac{1}{\sqrt{3}} & 0 & -2/\sqrt{6} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \end{pmatrix} =$$

$$= \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix} \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & \frac{1}{\sqrt{3}} \\ \frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} & + \frac{1}{\sqrt{2}} \\ \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{6}} & \frac{-\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{\sqrt{3}} & 1/\sqrt{2} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & -1/\sqrt{2} & +1/\sqrt{2} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{pmatrix} =$$

$$\begin{pmatrix} 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -\frac{1}{3} \\ -1/3 & 2/3 & 2/3 \end{pmatrix} = R \qquad R^6 = I \quad \text{provate a farlo!}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -\frac{1}{3} \\ -1/3 & 2/3 & 2/3 \end{pmatrix} \cdot \begin{pmatrix} x - 1 \\ y - 0 \\ z - 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{3}x - \frac{1}{3}y + \frac{2}{3}z - \frac{4}{3} + 1 \\ \frac{2}{3}x + 2/3\, y - \frac{1}{3}z - 1/3 + 0 \\ -\frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z - \frac{1}{3} + 1 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 2x - y + 2z - 1 \\ 2x + 2y - z - 1 \\ -x + 2y + 2z + 2 \end{pmatrix}$$

$$\begin{matrix} t+1 \\ t \\ t+1 \end{matrix}$$

$$f\begin{pmatrix} t+1 \\ t \\ t+1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2t+2-t+2t+2-1 \\ 2t+2+2t-t-1-1 \\ -t-1+2t+2t+2+2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 3t+3 \\ 3t \\ 3t+3 \end{pmatrix} =$$

$$= \begin{pmatrix} t+1 \\ t \\ t+1 \end{pmatrix}$$ PUNTO FISSO!

$$\mathrm{Tr}(R) = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

$$1 + 2\cos\Phi = 2 \qquad 2\cos\Phi = 1$$

$$\cos\Phi = \frac{1}{2} \qquad \Phi \pm \frac{\pi}{3}$$

Riprendiamo la matrice $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ -1 & 1 & -8 \end{pmatrix}$

$\lambda_1 = 0$

$$\begin{cases} \lambda_2 + \lambda_3 = -4 \\ \lambda_2 \lambda_3 = -35 \end{cases} \qquad t^2 + 4t - 35 = 0$$

$$t = -2 \pm \sqrt{4+35} \qquad \lambda_2 = -2 - \sqrt{39} \qquad \lambda_3 = -2 + \sqrt{39}$$

Si cerca una base di autovettori normalizzata

$$\begin{pmatrix} 5 \\ -3 \\ -1 \end{pmatrix} \frac{1}{\sqrt{25+9+1}} = \frac{1}{\sqrt{35}}\begin{pmatrix} 5 \\ -3 \\ -1 \end{pmatrix}$$ uno dei vettori

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ -1 & 1 & -8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -2-\sqrt{39}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$(3 \pm \sqrt{39})x + 2y - z = 0$$
$$2x + (5 \pm \sqrt{39})y + z = 0$$
$$-x + y + (-6 \pm \sqrt{39})z = 0$$

$$z = (3 \pm \sqrt{39})x + 2y$$
$$2x + (5 \pm \sqrt{39})y + (3 \pm \sqrt{39})x + 2y = 0$$
$$(5 \pm \sqrt{39})x + (7 \pm \sqrt{39})y = 0$$

$$x = (7 \pm \sqrt{39}) \qquad y = -(5 \pm \sqrt{39})$$
$$z = (3 \pm \sqrt{39})(7 \pm \sqrt{39}) - 10 \mp 2\sqrt{39}$$
$$= 21 + 39 - 10 + 10\sqrt{39} - 2\sqrt{39} = 50 + 8\sqrt{39}$$

$$\begin{pmatrix} 7+\sqrt{39} \\ -5-\sqrt{39} \\ 50+8\sqrt{39} \end{pmatrix} \cdot C \qquad \frac{1}{C} = \sqrt{(7+\sqrt{39})^2 + (5+\sqrt{39})^2 + (50+8\sqrt{39})^2}$$

autovettori
$$\left[\underbrace{\frac{1}{\sqrt{35}}\begin{pmatrix} 5 \\ -3 \\ -1 \end{pmatrix} \quad C\begin{pmatrix} 7+\sqrt{39} \\ \vdots \\ \vdots \end{pmatrix}}_{X} \quad \right]$$

MATRICE ORTOGONALE

$$5(7+\sqrt{39}) - 3(-5-\sqrt{39}) - 1(50+8\sqrt{39}) =$$
$$= \cancel{35} + \cancel{5\sqrt{39}} + \cancel{15} + \cancel{3\sqrt{39}} - \cancel{50} - \cancel{8\sqrt{39}} = 0$$

autovettori relativi ad autovalori diversi sono ortogonali

$$\begin{pmatrix} 7-\sqrt{39} \\ -5+\sqrt{39} \\ 50-8\sqrt{39} \end{pmatrix} \cdot C'$$

$$5(7-\sqrt{39})-3(-5+\sqrt{39})-1\cdot(50-8\sqrt{39})=0$$

$$(7+\sqrt{39})(7-\sqrt{39})+(-5-\sqrt{39})(-5+\sqrt{39})+(50+8\sqrt{39})(50-8\sqrt{39})=$$

$$49-39+25-39+2500-64\cdot 39$$

$$10-14+2500-2496$$

$$-4+4=0$$

$$\begin{array}{r} 64 \\ 39 \\ \hline 576 \\ 192 \\ \hline 2496 \end{array}$$

FASCI

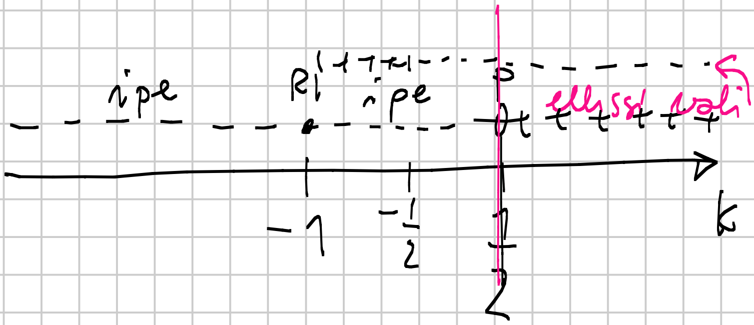
$$x^2-2xy+2ky^2+2kx+2y+1=0$$

$$A=\begin{pmatrix} 1 & -1 & k \\ -1 & 2k & 1 \\ k & 1 & 1 \end{pmatrix}$$

$$\det(A)=(2k-1)+(-1-k)+k(-1-2k^2)=$$

$$=\cancel{2k}-2\cancel{-k}\cancel{-k}-2k^3=$$

$$-2(k^3+1)=-2(k+1)(k^2-k+1)$$



$$A_2=\begin{vmatrix} 1 & -1 \\ -1 & 2k \end{vmatrix}$$

$$\det = 2k-1$$

<0

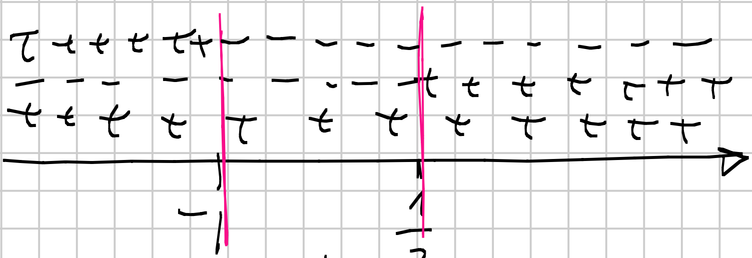
$$-(2k+1)(k+1)$$

$$\mathrm{Tr}(A_2)\cdot\det(A)$$

1

$2k-1$

$-2(k+1)(k^2-k+1)$



Se $k\neq 1/2$ avete sempre un centro

$$(x'+x_c)^2 - 2(x'+x_c)(y'+y_c) + 2K(y'+y_c)^2 + 2K(x'+x_c)$$
$$+2(y'+y_c)+1=0$$

$$\begin{cases} (2x_c - 2y_c + 2K)\,x' = 0 \\ (-2x_c + 4Ky_c + 2)\,y' = 0 \end{cases} \qquad \begin{cases} x_c - y_c = -K \\ -x_c + 2Ky_c = -1 \end{cases}$$

$$(2K-1)y_c = -1-K \qquad y_c = \frac{1+K}{1-2K}$$

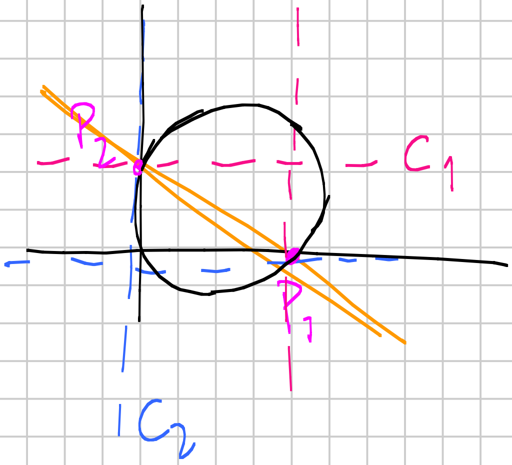
$$x_c = -K + y_c = -K + \frac{1+K}{1-2K} = \frac{\cancel{-K} + 2K^2 + 1 \cancel{+K}}{1-2K} =$$

$$= \frac{2K^2+1}{1-2K} \qquad K \in \mathbb{R}$$

$$\begin{cases} x = \dfrac{2K^2+1}{1-2K} \\ y = \dfrac{1+K}{1-2K} \end{cases} \longrightarrow \quad \begin{aligned} (1-2K)y &= 1+K \\ K(1+2y) &= y-1 \\ K &= \frac{y-1}{1+2y} \end{aligned}$$

$$x = \frac{2\dfrac{(y-1)^2}{(1+2y)^2}+1}{1-\dfrac{2y-2}{1+2y}} = \frac{2(y-1)^2 + (1+2y)^2}{(1+2y)\left[1+\cancel{2y}-\cancel{2y}+2\right]} \ldots$$

PUNTI BASE sono i punti che $\in$ a tutte le coniche del fascio



Costruire il fascio di coniche che passano per P_1 e P_2

$$(x-x_1)(y-y_2)=0$$

$$\cancel{(x-x_2)(y-y_1)=0} \quad P_1 \equiv (x_1, y_1)$$

$$P_2 \equiv (x_2 y_2)$$

$$x^2+y^2+ax+by=0$$

$$x_1 = 2 \qquad (2,0)$$

$$y_2 = 1 \qquad (0,1)$$

$$4+2a=0$$

$$(x-2)(y-1)=0$$

$$1+b=0$$

$$b=-1 \qquad a=-2$$

$$x^2+y^2-2x-y=0$$

$$xy-x-2y+2=0$$

$$\lambda(x^2+y^2-2x-y)+\mu(xy-x-2y+2)=0$$

$$K=\frac{\lambda}{\mu} \quad \text{oppure} \quad K'=\mu/\lambda$$

$$x^2-2xy+2Ky^2+2Kx+2y+1=0$$

$$(x^2-2xy+2y+1)+K(2y^2+2x)=0$$

$$\begin{cases} y^2+x=0 \\ x^2-2xy+2y+1=0 \end{cases} \qquad x=-y^2 \qquad y^4+2y^3+2y+1=0$$

per semplificare la questione è preferibile usare una conica degenere $K=-1$

$x^2 - 2xy - 2y^2 - 2x + 2y + 1 = 0$

$t^2 - 2t - 2 = 0$ $\qquad t = 1 \pm \sqrt{3}$

$\frac{x}{y} = 1 \pm \sqrt{3}$ $\qquad (x - (1+\sqrt{3})y + c_1)(x - (1-\sqrt{3})y + c_2) = 0$

$$\begin{cases} (x - (1+\sqrt{3})y - 1)(x - (1-\sqrt{3})y - 1) = 0 \\ x = -y^2 \end{cases}$$

$$\begin{cases} -y^2 - (1+\sqrt{3})y - 1 = 0 \\ -y^2 - (1-\sqrt{3})y - 1 = 0 \end{cases}$$ e si ottengono le y dei punti base

$\left(\quad S \quad \right)$ diagonalizzata con i vettori della nuova base sono ortogonali rispetto al p.s. descritto così

$\boxed{{}^tMSM = D}$

Attenzione! in generale non ha gli autovalori sulla diagonale!

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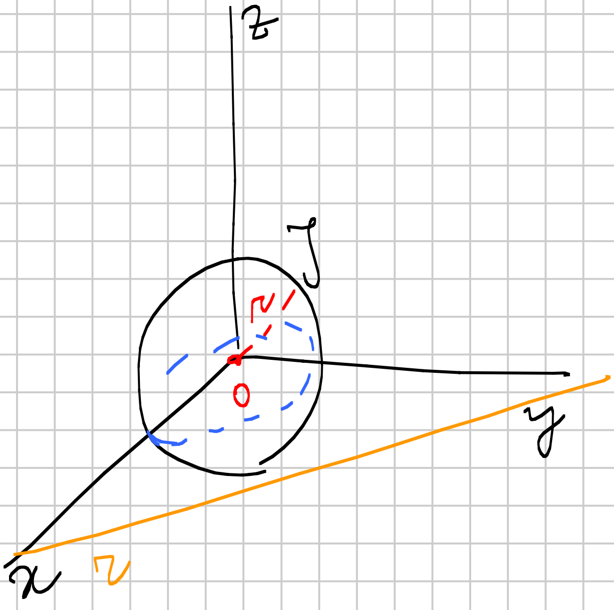
distanza punto-piano



$x^2 + y^2 + z^2 = 1$

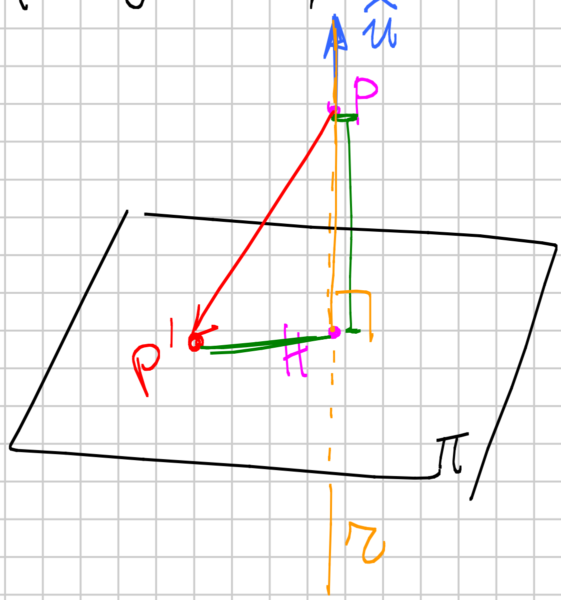
$$\begin{cases} \boxed{x + y = 5} \\ \boxed{z = 0} \end{cases}$$

- fascio di piani passanti per r
- piano π del fascio che dista 1 da O e che passa da r

$\hookrightarrow$ π è tangente a S

$$\lambda(x+y-5)+\mu z=0$$

$$d(P,\pi)=\frac{|ax_p+by_p+cz_p+d|}{\sqrt{a^2+b^2+c^2}}$$



$$\begin{cases} x=at+x_p \\ y=bt+y_p \\ z=ct+z_p \end{cases}$$

P per noi è l'origine

$H=r\cap P$

$\overline{PH}$

$\overrightarrow{PP'}$ tale che $P'\in\pi$ e poi lo proietto su r

$$\hat{u}=\frac{a\hat{e}_1+b\hat{e}_2+c\hat{e}_3}{\sqrt{a^2+b^2+c^2}}$$

$x_{P'}=0$
$y_{P'}=0$
$z_{P'}=-\frac{d}{c}$

se $c\neq 0$

$$\lambda x+\lambda y+\mu z-5\lambda=0$$

$$\frac{|0+0+0-5\lambda|}{\sqrt{\lambda^2+\lambda^2+\mu^2}}=1$$

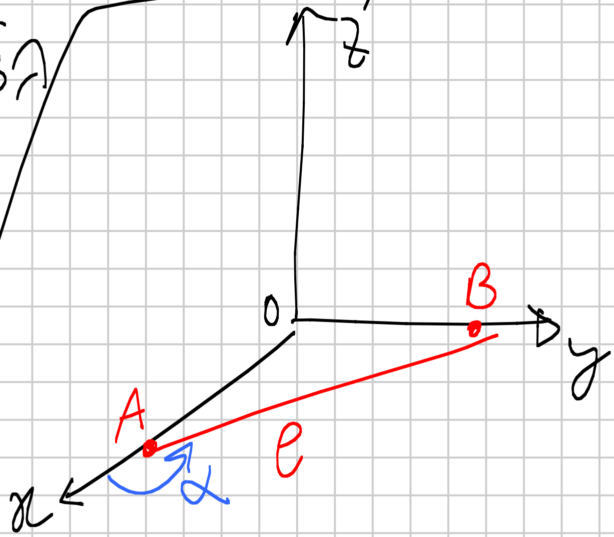
$$|5\lambda|=\sqrt{2\lambda^2+\mu^2}$$

$$25\lambda^2=2\lambda^2+\mu^2$$

$$\mu^2=23\lambda^2 \qquad \mu=\pm\sqrt{23}\lambda$$

$$\lambda(x+y-5)\pm\sqrt{23}\lambda z=0$$

$$x+y\pm\sqrt{23}z-5=0$$



Cosa succede al variare di α? $\qquad \frac{\pi}{2} < \alpha < \pi$

$A \equiv (-\cos\alpha, 0, 0) \qquad B \equiv (0, \operatorname{sen}\alpha, 0)$

$$\begin{cases} \dfrac{x}{-\cos\alpha\, l} + \dfrac{y}{\operatorname{sen}\alpha\, l} = 1 \\ z = 0 \end{cases}$$

$$\begin{cases} x\operatorname{sen}\alpha - y\cos\alpha + l\operatorname{sen}\alpha\cos\alpha = 0 \\ z = 0 \end{cases}$$

$x^2 + y^2 + z^2 = R^2 \qquad \lambda(x\operatorname{sen}\alpha - y\cos\alpha + l\operatorname{sen}\alpha\cos\alpha) + \mu z = 0$

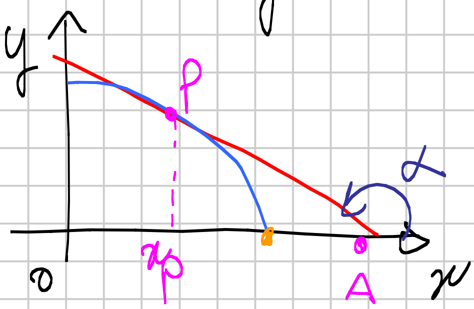
$$\frac{|\lambda l \operatorname{sen}\alpha\cos\alpha|}{\sqrt{\lambda^2 + \mu^2}} = R$$

$$\lambda^2 l^2 \operatorname{sen}^2\alpha\cos^2\alpha = R^2(\lambda^2 + \mu^2) \qquad l > R$$

$$\mu^2 = \lambda^2 \frac{l^2(\operatorname{sen}^2\alpha\cos^2\alpha) - R^2}{R^2}$$

$$\mu = \frac{\lambda}{R}\sqrt{\left(\frac{l}{2}\operatorname{sen} 2\alpha\right)^2 - R^2}$$

$$x\operatorname{sen}\alpha - y\cos\alpha + \frac{1}{R}\sqrt{}\, z + l\operatorname{sen}\alpha\cos\alpha = 0$$



$y = \dfrac{-x^2}{a} + a \qquad a > 0$

retta per A tangente alla parabola

$$\begin{cases} y = m(x - x_A) \\ y = -\dfrac{x^2}{a} + a \end{cases} \rightarrow \Delta = 0$$

$A \equiv (x_A, 0)$

$x_A > a$

$tg\alpha = m$

$y' = -\dfrac{2x}{a}$

$y'(x_P) = -\dfrac{2x_P}{a} = m$

$m = \dfrac{y_P - 0}{x_P - x_A}$

si legano fra di loro le ascisse di P e di A