ESERCIZI GEOMETRIA

Titolo nota 24/05/2016

Martedì - Mercoledì - Venerdì Esercizi

Martedì 31 Maggio "ricevimento"

Coniche

curva nel piano $f(x,y)=0$

con opportune ipotesi di regolarità su f

f polinomio di 2° grado in 2 variabili

$$ax^2+bxy+cy^2+dx+ey+f=0$$

$$\begin{pmatrix} x & y & 1 \end{pmatrix} \begin{pmatrix} a & \frac{b}{2} & d/2 \\ \frac{b}{2} & c & e/2 \\ d/2 & \frac{e}{2} & f \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$ coordinate affini

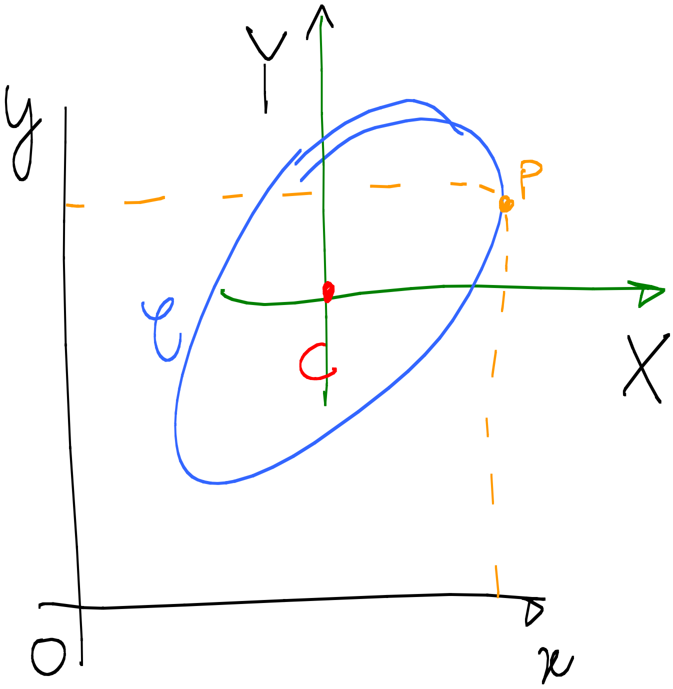
coordinate omogenee $x' \quad y' \quad t'$

$$x = \frac{x'}{t'} \qquad y = \frac{y'}{t'}$$

$$ax'^2 + bx'y' + cy'^2 + \boxed{dx't'} + \boxed{ey't'} + ft'^2 = 0$$

polinomio omogeneo di 2° grado

Se la conica ha un centro, con una traslazione si tolgono i pezzi "di 1° grado" si azzerano d ed e



$P \equiv (x_p, y_p)$ in (Oxy)

$P \equiv (X_P\, Y_P)$ in (CXY)

$$x_p = X_p + x_c \qquad \forall P \in \mathcal{C}$$
$$y_p = Y_p + y_c$$

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

$$a(X+x_c)^2 + b(X+x_c)(Y+y_c) + c(Y+y_c)^2$$
$$+ d(X+x_c) + e(Y+y_c) + f = 0$$

$$aX^2 + bXY + cY^2 + \boxed{(2ax_c + by_c + d)X}$$
$$+ \boxed{(bx_c + 2cy_c + e)Y} + ax_c^2 + bx_cy_c + cy_c^2$$
$$+ dx_c + ey_c + f = 0$$

$$\begin{cases} 2ax_c + by_c = -d \\ bx_c + 2cy_c = -e \end{cases} \qquad x_c = \frac{\begin{vmatrix} -d & b \\ -e & 2c \end{vmatrix}}{4ac - b^2} =$$

$4ac - b^2 \neq 0$

$$= \frac{-2dc + eb}{-\Delta} = \frac{2dc - eb}{\Delta}$$

$$y_c = \frac{\begin{vmatrix} 2a & -d \\ b & -e \end{vmatrix}}{-\Delta} = \frac{-2ae + bd}{-\Delta} = \frac{2ae - bd}{\Delta}$$

15.2 Abate - De Fabritiis

$a'x^2+b'xy+c'y^2+d'x+e'y+f'=0 \quad \mathcal{C}'$

Combinaz. lineare

$$\lambda(ax^2+bxy+\dots)+\mu(a'x^2+b'xy+\dots)=0$$

$$(\lambda a+\mu a')x^2+(\lambda b+\mu b')xy+(\lambda c+\mu c')y^2+$$
$$+(\lambda d+\mu d')x+(\lambda e+\mu e')y+(\lambda f+\mu f')=0$$

$K=\frac{\lambda}{\mu}$ oppure $K'=\frac{\mu}{\lambda}$ per avere un solo parametro (guadagno)

Si perde una delle coniche del fascio

$$(K+1)x^2+(K-1)y^2+2Kx+2y-1=0$$

$$K\underbrace{(x^2+y^2+2x)}_{\text{Circonferenza}}+\underbrace{x^2-y^2+2y-1}_{\text{coppia di rette incidenti}}=0$$

$K=\lambda/\mu$

$x^2-(y^2-2y+1)=0 \qquad x^2-(y-1)^2=0$

$(x-y+1)(x+y-1)=0$

$$\lambda(x^2+y^2+2x)+\mu(x^2-y^2+2y-1)=0$$

$$A = \begin{pmatrix} \boxed{\begin{matrix} k+1 & 0 \\ 0 & k-1 \end{matrix}} & \begin{matrix} k \\ 1 \end{matrix} \\ \begin{matrix} k & 1 \end{matrix} & -1 \end{pmatrix} \qquad A_2$$

$$\det(A) = (k+1)(-k + \cancel{1} - \cancel{1}) + k(-k^2 + k) = \cancel{-k^2} - k - k^3 + \cancel{k^2} =$$
$$= -k(k^2+1)$$

per $k=0$ conica degenere 2 rette incidenti

$$\det A_2 = k^2 - 1$$

ellissi reali   p   ip.   R.I.   ip.p.   ellissi reali

+ + + + + +   • – – X – – •   + + + + +

-1 0 $+1$ k

Ellisse in forma canonica $\quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - 1 = 0$

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0 \qquad \text{stessa cosa!}$$

$$\begin{pmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & \frac{1}{b^2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad > 0$$

$$\mathrm{tr}(A_2) \cdot \det(A) < 0$$

$$\begin{pmatrix} -\frac{1}{a^2} & 0 & 0 \\ 0 & -\frac{1}{b^2} & 0 \\ 0 & 0 & +1 \end{pmatrix} \quad < 0$$

$$2k \cdot [-k(k^2+1)]$$
$$= -2k^2(k^2+1) \leq 0$$

$$x_c = \frac{2dc - eb}{\Delta} = \frac{\cancel{4}k(k-1) - 0}{\cancel{4}(-k^2+1)} =$$

$d = 2k \qquad b = 0$

$e = 2 \qquad a = k+1$

$f = -1 \qquad c = k-1$

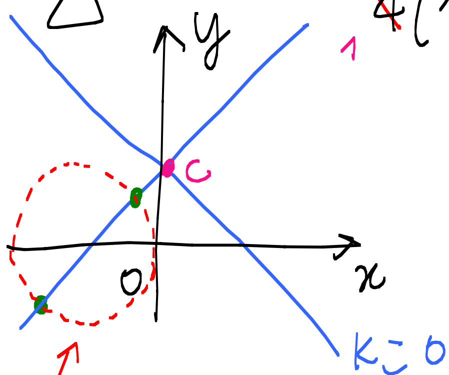
$$x_c = \frac{-K}{1+K}$$

$$= \frac{1}{(1-K)}$$

$$y_c = \frac{2ae-bd}{\Delta} = \frac{\cancel{4}^{\,1}(K+1)}{\cancel{4}_{\,1}(1-K^2)}$$

$y = x+1$

$y = -x+1$



PUNTI BASE (tutte le coniche del fascio passano per questi 2 punti)

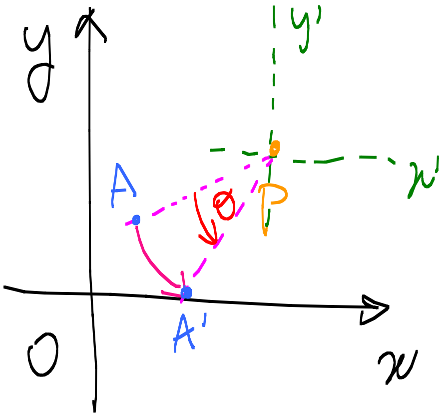
altra conica che genera il fascio

ESERCIZIO 2.32

ROTAZIONI

INTORNO A UN PUNTO QUALSIASI

NON BASTA UN OPERATORE LINEARE

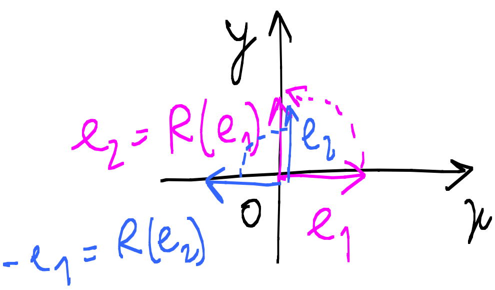


$$\begin{pmatrix} x'_{A'} \\ y'_{A'} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x'_A \\ y'_A \end{pmatrix}$$

$x'_A = x_A - x_p$ | $x'_{A'} = x_{A'} - x_p$

$y'_A = y_A - y_p$ | $y'_{A'} = y_{A'} - y_p$

$$\begin{pmatrix} x_{A'} \\ y_{A'} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_A - x_p \\ y_A - y_p \end{pmatrix} + \begin{pmatrix} x_p \\ y_p \end{pmatrix}$$



$$R_{\frac{\pi}{2}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$f = R_{P,\frac{\pi}{2}}$

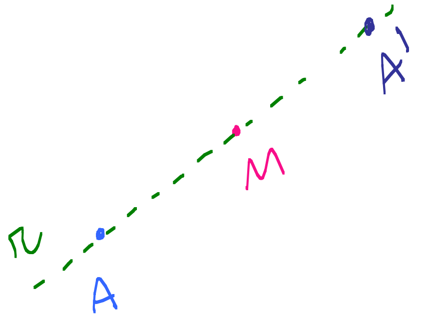
$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x - x_p \\ y - y_p \end{pmatrix} + \begin{pmatrix} x_p \\ y_p \end{pmatrix} = \begin{pmatrix} -y + y_p + x_p \\ x - x_p + y_p \end{pmatrix}$$

$$g = R_{Q,\frac{\pi}{2}} \qquad g\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x - x_Q \\ y - y_Q \end{pmatrix} + \begin{pmatrix} x_Q \\ y_Q \end{pmatrix} = \begin{pmatrix} -y + y_Q + x_Q \\ x - x_Q + y_Q \end{pmatrix}$$

$$(f \circ g)\begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{g} \begin{pmatrix} -y + x_Q + y_Q \\ x - x_Q + y_Q \end{pmatrix} \xrightarrow{f}$$

$$\begin{pmatrix} -x + x_Q - y_Q + y_P + x_P \\ -y + x_Q + y_Q - x_P + y_P \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x_P + x_Q + y_P - y_Q \\ -x_P + x_Q + y_P + y_Q \end{pmatrix}$$

Questa è una simmetria rispetto ad un punto M



r passante per A e M

$A' \in r$ tale che M sia il punto medio di AA'

$$x_M = \frac{x_A + x_{A'}}{2} \qquad y_M = \frac{y_A + y_{A'}}{2}$$

$$x_{A'} = 2x_M - x_A$$

$$y_{A'} = 2y_M - y_A$$

$$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} \to -\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2x_M \\ 2y_M \end{pmatrix}}$$

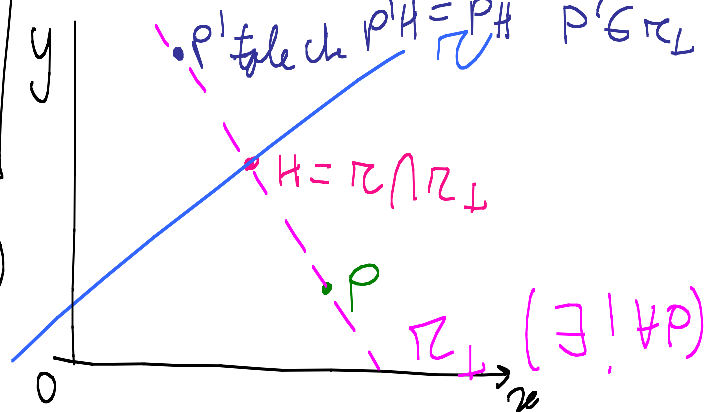
M è l'R del testo!

$$x_R = \frac{x_P + x_Q + y_P - y_Q}{2}$$

$$y_R = \frac{-x_P + x_Q + y_P + y_Q}{2}$$

$r: \; ax + by + c = 0 \quad (a,b) \neq (0,0)$

2.33 RIFLESSIONI



$$\begin{pmatrix} x_{p'} \\ y_{p'} \end{pmatrix} = \underbrace{\frac{1}{a^2+b^2}\begin{pmatrix} b^2-a^2 & -2ab \\ -2ab & a^2-b^2 \end{pmatrix}}_{\text{matrice ortogonale con determinante } = -1} \begin{pmatrix} x_p \\ y_p \end{pmatrix} \underbrace{- \frac{2c}{a^2+b^2}\begin{pmatrix} a \\ b \end{pmatrix}}_{\text{VETTORE COSTANTE}}$$

$$\begin{pmatrix} x_{p'} \\ y_{p'} \end{pmatrix} = \begin{pmatrix} S_r \end{pmatrix} \begin{pmatrix} x_p \\ y_p \end{pmatrix} + \begin{pmatrix} V_r \end{pmatrix}$$

$$\begin{pmatrix} x_{p'} \\ y_{p'} \end{pmatrix} = \begin{pmatrix} S_s \end{pmatrix} \begin{pmatrix} x_p \\ y_p \end{pmatrix} + \begin{pmatrix} V_s \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow S_s\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} V_s \end{pmatrix} \longrightarrow S_r\left(S_s\begin{pmatrix} x \\ y \end{pmatrix} + V_s \right) + V_r$$

$$(S_r S_s)\begin{pmatrix} x \\ y \end{pmatrix} + (S_r V_s + V_r)$$

matrice ortog. a det $+1$

$$R\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} V' \end{pmatrix}$$

Se $R = I$ traslazione

Se R è una matrice di rotaz "vera", $R\begin{pmatrix} x \\ y \end{pmatrix} + V'$ è una rotaz (intorno al punto $Q = r \cap s$)

$r : ax + by + c = 0$ trovo l'eq. di $r_\perp$ $\begin{pmatrix} a \\ b \end{pmatrix} \perp r$

$$\begin{cases} x = at + x_p \\ y = bt + y_p \end{cases}$$ eq. parametriche di $r_\perp$

$$H : r \cap r_\perp = \quad a(at_H + x_p) + b(bt_H + y_p) + c = 0$$

$$t_H = -\frac{ax_p + by_p + c}{a^2+b^2} \rightarrow \begin{aligned} x_H &= at_H + x_p \\ y_H &= bt_H + y_p \end{aligned}$$

$$\frac{x_{p'} + x_p}{2} = x_H \qquad x_{p'} = 2x_H - x_p = 2at_H + x_p$$

$$\frac{y_{p'} + y_p}{2} = y_H \qquad y_{p'} = 2y_H - y_p = 2bt_H + y_p$$

$$x_{p'} = \frac{(b^2-a^2)x_p - 2ab\,y_p - 2ac}{a^2+b^2}$$

$$y_{p'} = \frac{-2ab\,x_p + (a^2-b^2)y_p - 2bc}{a^2+b^2}$$

$$\begin{pmatrix} x_{p'} \\ y_{p'} \end{pmatrix} = \frac{1}{a^2+b^2}\begin{pmatrix} b^2-a^2 & -2ab \\ -2ab & a^2-b^2 \end{pmatrix}\begin{pmatrix} x_p \\ y_p \end{pmatrix} - \frac{2c}{a^2+b^2}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$A = \frac{1}{a^2+b^2}\begin{pmatrix} b^2-a^2 & -2ab \\ -2ab & a^2-b^2 \end{pmatrix}$$

$$\det(A) = \frac{1}{(a^2+b^2)^2}\left[(b^2-a^2)(a^2-b^2) - 4a^2b^2\right] =$$

$$= \frac{1}{(a^2+b^2)^2}\left[-a^4-b^4+2a^2b^2-4a^2b^2\right] =$$

$$= \frac{-1}{(a^2+b^2)^2}(a^4+2a^2b^2+b^4) = \frac{-\cancel{(a^2+b^2)^2}}{\cancel{(a^2+b^2)^2}} = -1$$

$$A^2 = I \qquad \begin{pmatrix} a' \\ b' \end{pmatrix} = k\begin{pmatrix} a \\ b \end{pmatrix}$$

se $\operatorname{car}\begin{pmatrix} a & a' \\ b & b' \end{pmatrix} = 2$ allora $AA' = R \neq I$