ISOMETRIE E PRODOTTO VETTORIALE

Titolo nota 03/05/2016

2.17

$$R_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$$

R_1 è ortogonale

$\det(R_1) \quad (-1) \cdot \det\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = +1$

Rotazione

$\lambda = +1$

asse $\rightarrow$ $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{cases} y = x \\ -z = y \\ -x = z \end{cases}$

$y = x$

$z = -x$

$x = +1$ ad esempio $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ direz. asse

$1 + 2\cos(\theta_1) = tr(R_1)$

$$R_1' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix}$$

Traccia è invariante per rotazioni

$0 = 1 + 2\cos\theta_1 \qquad \cos\theta_1 = -\frac{1}{2} \qquad \theta_1 = \pm\frac{2\pi}{3}$

$$R_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$\det(R_2) = +1$

R_2 è ortogonale

$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{cases} y = x \\ x = y \\ -z = z \end{cases}$ $\quad x = y$, $z = 0$

$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\theta_2 = \pi$ $\quad -1 = 1 + 2\cos(\theta_2) \qquad \cos(\theta_2) = -1 \Rightarrow$

$$R_3 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

R_3 è ortogonale

$$\det(R_3) = \frac{1}{27}\det\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} = \frac{1}{27}\det\begin{pmatrix} -1 & 0 & 2 \\ 2 & -3 & 2 \\ 2 & 3 & -1 \end{pmatrix}$$

$$= \frac{1}{27}\det\begin{pmatrix} -1 & 0 & 2 \\ 4 & 0 & 1 \\ 2 & 3 & -1 \end{pmatrix} = \frac{1}{27}\cdot(-3)\det\begin{pmatrix} -1 & 2 \\ 4 & 1 \end{pmatrix}$$

$$= -\frac{1}{9}\cdot(-1-8) = -\frac{1}{9}\cdot(-9) = +1$$

$$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \begin{cases} -x + 2y + 2z = 3x \\ 2x - y + 2z = 3y \\ 2x + 2y - z = 3z \end{cases}$$

$$\begin{cases} -4x + 2y + 2z = 0 \\ 2x - 4y + 2z = 0 \\ 2x + 2y - 4z = 0 \end{cases} \rightarrow \begin{cases} -2x + y + z = 0 \\ x - 2y + z = 0 \\ x + y - 2z = 0 \end{cases}$$

$y = 2x - z$ dalla 1^a eq.

$$\begin{cases} x - 4x + 2z + z = 0 \\ x + 2x - z - 2z = 0 \end{cases}$$

$$\begin{cases} -3x + 3z = 0 \\ 3x - 3z = 0 \end{cases} \rightarrow x - z = 0 \qquad x = z$$

$$y = 2z - z = z$$

$$x = y = z \rightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\operatorname{tr}(R_3) = -1 \qquad -1 = 1 + 2\cos(\theta_3)$

$\theta_3 = \pi$

Esercizio 2.18

$A \in M(3)$ rotaz. intorno alla retta che ha direzione $\begin{pmatrix}1\\1\\1\end{pmatrix}$ e ampiezza $\frac{2\pi}{3}$

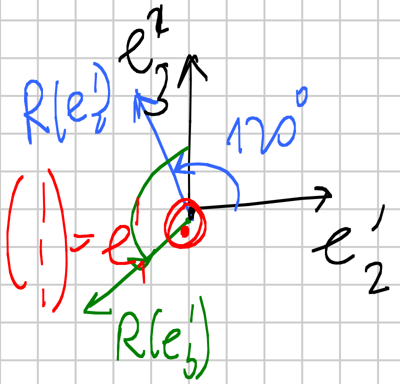
$A^3 = I$

$e_1 \to e_2$
$e_2 \to e_3$
$e_3 \to e_1$

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$tr(A) = 0$ $\qquad 0 = 1 + 2\cos(\theta) \to \theta = \pm \frac{2\pi}{3}$

$\begin{pmatrix}1\\1\\1\end{pmatrix}$ è autovettore di A con autovalore 1



nella base e_1' e_2' e_3'

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\sqrt{3}/2 \\ 0 & +\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = [A]^{B'}_{B'}$$

$$e_1' = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad e_2' = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix} \qquad e_3' = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\1\\-2\end{pmatrix}$$

$$M = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -2/\sqrt{6} \end{pmatrix}$$

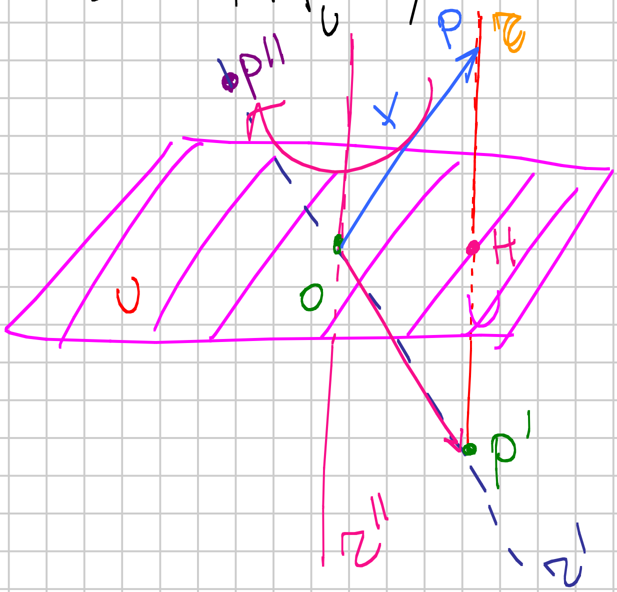
$$B = \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix} \begin{pmatrix}0\\0\\1\end{pmatrix} \right\}$$

base canonica di $\mathbb{R}^3$

$$[A]^{B}_{B} = \begin{pmatrix} \frac{1}{\sqrt3} & \frac{1}{\sqrt2} & \frac{1}{\sqrt6} \\ \frac{1}{\sqrt3} & -\frac{1}{\sqrt2} & \frac{1}{\sqrt6} \\ \frac{1}{\sqrt3} & 0 & -2/\sqrt6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -\sqrt3/2 \\ 0 & +\sqrt3/2 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt3} & \frac{1}{\sqrt3} & \frac{1}{\sqrt3} \\ \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 \\ \frac{1}{\sqrt6} & \frac{1}{\sqrt6} & -\frac{2}{\sqrt6} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt3} & \frac{1}{\sqrt2} & \frac{1}{\sqrt6} \\ \frac{1}{\sqrt3} & -\frac{1}{\sqrt2} & \frac{1}{\sqrt6} \\ \frac{1}{\sqrt3} & 0 & -2/\sqrt6 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt3} & \frac{1}{\sqrt3} & \frac{1}{\sqrt3} \\ \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 \\ \frac{1}{\sqrt6} & -\frac{2}{\sqrt6} & \frac{1}{\sqrt6} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

2.19 data Rif_U, cos'è $-Rif_U$



$r \perp U$

r passa per P

$H = r \cap U$

$P' \in r$ è il simmetrico di P rispetto ad H

$\overline{PH} = \overline{P'H}$ P, H, P' oppure P', H, P

è l'ordine in cui si susseguono i punti

$\vec{OP'} = Rif_U(\vec{OP})$

$(P'-O) = Rif_U(P-O)$

$r' =$ retta per O e P' $P'' \in r'$ tale che

$\overline{OP''} = \overline{OP'}$, ordine $P'OP''$

$\vec{OP}'' = -\mathrm{Rif}_U(\vec{OP})$

$(P''-O) = -\mathrm{Rif}_U(P-O)$

I punti O, P, P', P'' $\in$ tutti ad uno stesso piano, definito dalle due rette r e r'

$\|\vec{OP}\| = \|\vec{OP}'\| = \|\vec{OP}''\|$

$\hat{g}$ ← versore diretto come r'' (o come r)

$\langle \vec{OP}, \hat{g} \rangle_E = \langle \vec{OP}'', \hat{g} \rangle$

$\vec{OP} = \vec{OP}_{\parallel} + \vec{OP}_{\perp}$

parallelo ad U ↑ ⊥ ad U

$-\mathrm{Rif}_U(\vec{OP}_{\parallel}) = \vec{OP}_{\parallel}$

$-\mathrm{Rif}_U(\vec{OP}_{\perp}) = -\vec{OP}_{\perp}$ $\qquad \vec{OP}'' = \vec{OP}_{\parallel} - \vec{OP}_{\perp}$

→ Rotaz. di $180°$ intorno a r''

$U \quad z=0$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \xrightarrow{\mathrm{Rif}_U} \begin{pmatrix} x \\ y \\ -z \end{pmatrix} \xrightarrow{-v} \begin{pmatrix} -x \\ -y \\ z \end{pmatrix}$$

r'' è l'asse z

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$tr(A) = -1$

$-1 = 1 + 2\cos\theta$

$\cos\theta = -1 \qquad \theta = \pi$