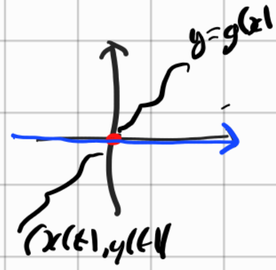
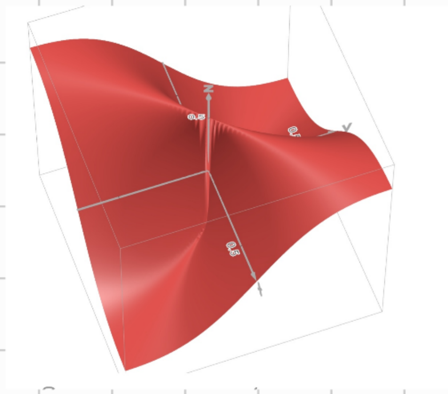
ANALISI MATEMATICA 2
LEZIONE 6 – 7.10.2025



Esercizio $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ non esiste

$$\left[\ \lim_{\underline{x}\to x_0} f(\underline{x}) = \ell \iff \tilde{f}(x) = \begin{cases} f(x) & \text{se } x \neq x_0 \\ \ell & \text{se } x = x_0 \end{cases} \text{ è continua in } x_0 \ \right]$$

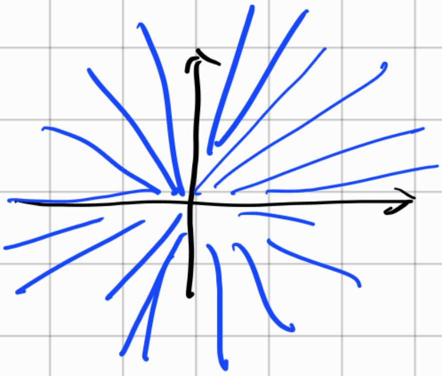


Se $y = 0$ $\quad \dfrac{xy}{x^2+y^2} = \dfrac{0}{x^2} = 0$

Se $y = x$ $\quad \dfrac{xy}{x^2+y^2} = \dfrac{x^2}{2x^2} = \dfrac{1}{2}$

Insiemi di livello: $y = mx$

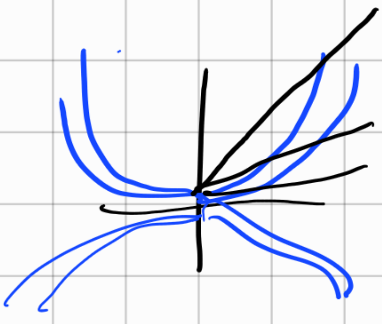
$$\frac{xy}{x^2+y^2} = \frac{mx^2}{x^2+m^2x^2} = \frac{m}{1+m^2}$$



Esempio

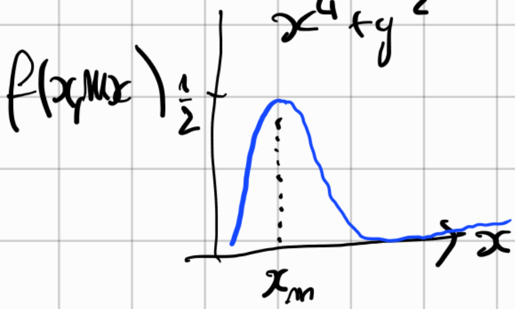
$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} \quad \text{non esiste}$$

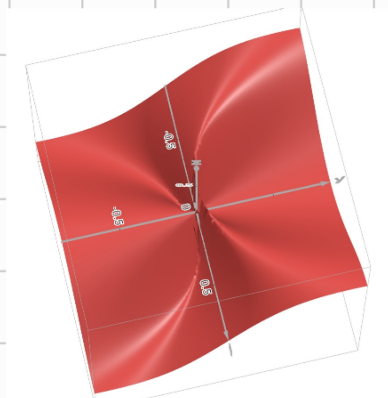
$y = mx^2$



Se $y = mx$

$$\frac{x^2y}{x^4+y^2} = \frac{mx^3}{x^4+m^2x^2} \sim \frac{x}{m} \longrightarrow 0$$





ES

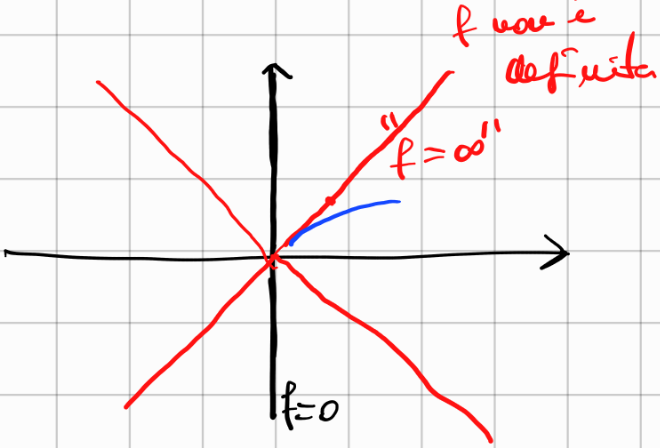
$$\lim_{(x,y)\to 0} \frac{xy^2+x^3}{x^2+y^2} = 0$$

$$|x| \leq \sqrt{x^2+y^2} = \rho$$
$$|y| \leq \quad " \quad = \rho$$

$$\left|\frac{xy^2+x^3}{x^2+y^2}\right| \leq \frac{\rho^3+\rho^3}{\rho^2} = 2\rho = 2\sqrt{x^2+y^2}$$

$$(x,y)\to 0 \iff \sqrt{x^2+y^2} \to 0$$

ES $\lim_{(x,y)\to 0} \frac{x^3}{x^2-y^2}$ non esiste



se $x=0$, $\frac{0}{0-y^2} \to 0$ per $y\to 0$

se $y = x+x^2$ $\frac{x^3}{x^2-y^2} = \frac{x^3}{x^2-(x+x^2)^2} = \frac{x^3}{\cancel{x^2}-\cancel{x^2}-2x^3-x^4}$

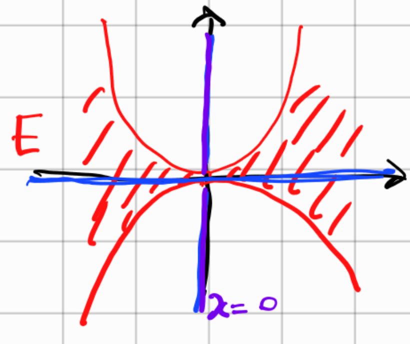
$$= \frac{x^3}{-2x^3+o(x^3)} \xrightarrow[x\to 0]{} \frac{1}{-2}$$

[Se c'è $\sqrt{g(x,y)}$ $\quad g(x,y) \geq 0$
$\uparrow$
RIVEDI es. 8 del test settimanale $\quad$ il dominio si restringe]

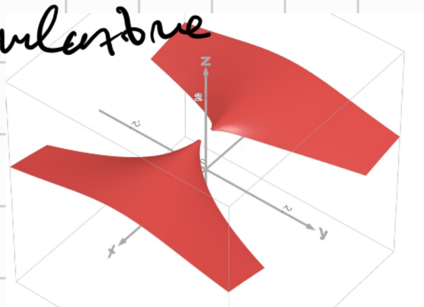
ES Posso considerare una funzione con un dominio "ristretto".

$$E = \{(x,y)\in\mathbb{R}^2 : |y| < x^2\}$$

$$f: E \to \mathbb{R}, \quad f(x) = \frac{x^2}{x^2+y^2}$$



per $y = 0$ (OSS $(0,0)$ è punto di accumulazione per $\{y=0\}\cap E$)



$$\frac{x^2}{x^2+y^2} \overset{y=0}{=} \frac{x^2}{x^2} = 1 \longrightarrow 1.$$

per $x = 0$

$$\frac{x^2}{x^2+y^2} = 0 \to 0$$

MA $(0,0)$ non è punto di accumulazione per $\{x=0\}\cap E$.

$\Rightarrow$ Questo non dice niente!

Provo a dimostrare che il limite esiste. In tal caso deve necessariamente essere 1.

$(x,y)\in E \Rightarrow |y|<x^2$

$$\left|\frac{x^2}{x^2+y^2} - 1\right| = \left|\frac{\cancel{x^2}-\cancel{x^2}-y^2}{x^2+y^2}\right| = \frac{y^2}{x^2+y^2} \le \frac{x^4}{x^2} = x^2 \to 0$$

In alternativa:

$$1 \xleftarrow[\text{per } x\to 0]{} \frac{x^2}{x^2+x^4} \underset{(x,y)\in E}{\le} \frac{x^2}{x^2+y^2} \le \frac{x^2}{x^2} = 1$$

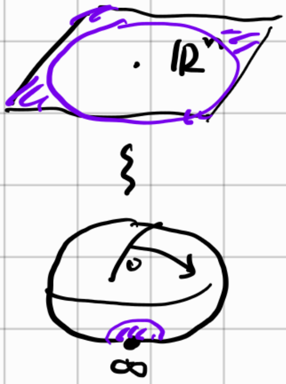
(2 carabinieri)

Cosa significa ① $(x,y)\to\infty$?

② $f(x,y)\to\infty$?

① $(x,y)\to\infty$ significa $|(x,y)|\to+\infty$

Formalmente $X = \mathbb{R}^n \cup \{\infty\}$



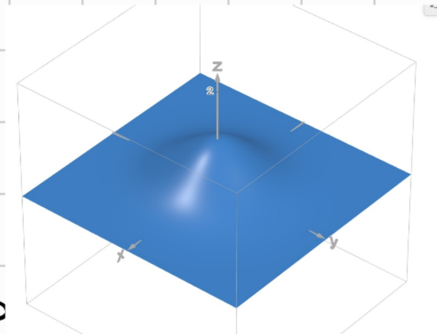
gli intorni (basilari) di ∞ sono: $(\mathbb{R}^n \cup \{\infty\}) \setminus B_R(0)$

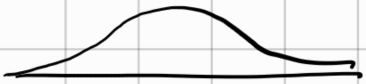
ES

$$\lim_{(x,y)\to\infty} e^{-(x^2+y^2)} = 0$$

perché $e^{-(x^2+y^2)} = e^{-\rho^2} \to 0$

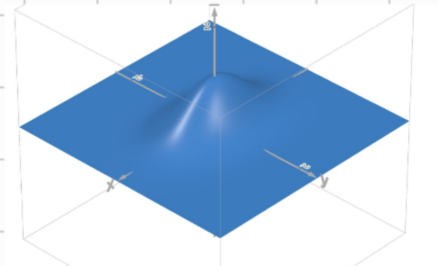
se $\rho \to \infty$



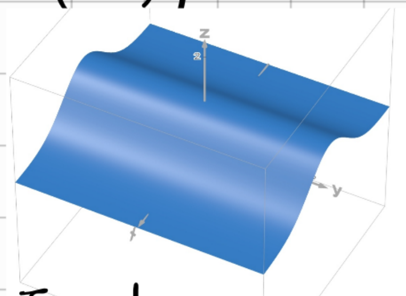


x

$$\lim_{(x,y)\to\infty} e^{-(x^2+3y^2)} = 0$$



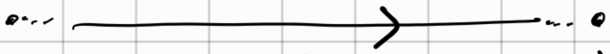
$$\lim_{(x,y)\to\infty} e^{-x^2} \text{ non esiste}$$



$x=0 \quad e^{-x^2} = 1 \to 1$ per $y \to \infty$

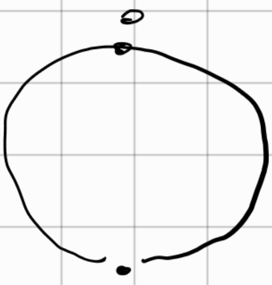
$y=0 \quad e^{-x^2} \to 0$ per $x \to \infty$

Nota In dimensione 1 usualmente si mettono 2 punti all'infinito.



$-\infty$ $\quad$ $\mathbb{R}$ $\quad$ $+\infty$

$$\lim_{x\to 0} \frac{1}{x} \text{ non esiste}$$



∞

MA se non mi interessa l'ordinamento posso mettere un solo punto

$$\lim_{x\to 0} \frac{1}{x} = \infty$$

② $\lim\limits_{(x,y)\to(0,0)} f(x,y) = +\infty \quad \left[+\infty, -\infty, \overset{\pm\infty}{\infty}\right]$

$$\Updownarrow$$

$$\lim_{(x,y)\to(0,0)} \frac{1}{f(x,y)} = 0^+ \quad \left[0^+, 0^-, 0\right]$$