

145AA Advanced Mechanics
(Meccanica Superiore)
First semester 2023-24
6 CFU, 42 hours

Instructor:
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Information on the course and the timetable can be found at the webpage
<https://pagine.dm.unipi.it/bonanno/meccsup-2324.html>

1 Overview

The aim of the course is to give an introduction to the study of chaotic dynamical systems, both from the qualitative and the quantitative point of view. The course “Sistemi dinamici” provides an introduction to the notion of chaotic system from the point of view of the topological properties of a system. In this course we aim at studying the statistical properties of the orbits of a system, constructing a theory of the behaviour of the orbits also for chaotic systems.

In the first part of the course we give a review of “ergodic theory”, the statistical approach to dynamical systems which uses the methods of measure theory. Then we introduce some examples of chaos indices which induce a classification of chaotic systems. In particular, we study the notion of “metric entropy” and its relation with information theory and the modern notion of compression algorithm. It is thanks to this relation that we can study the applications to the classification of real-world time series as for example series of biomedical origin.

The second part of the course provides an introduction to the modern techniques for the classification of chaotic systems which are based on a functional-analytic approach to the space of measures. This approach has been recently introduced and has been used to classify some of the most interesting chaotic systems such as the billiard maps and the geodesic flow on negatively-curved surfaces.

2 Course outline

- Basic notions of ergodic theory.
- Quantifying chaotic systems: topological entropy; metric entropy; Lyapunov exponents; information content and complexity. Applications to real-world time series.
- Mixing and decay of correlations.
- Transfer operators and the spectral approach.

3 Bibliography

- V. Baladi, “Positive transfer operators and decay of correlations”, World Scientific, 2000

- M. Brin, G. Stuck, "Introduction to dynamical systems", Cambridge University Press, 2002

4 Assessment

The exam is oral. The student which has attended the course may decide to give a seminar on an advanced topic chosen with the instructor. A list of topics is provided during the course. The seminar is followed by a short discussion on the main results of the course.

5 Prerequisites

Basic notions of dynamical systems, measure theory and functional analysis.