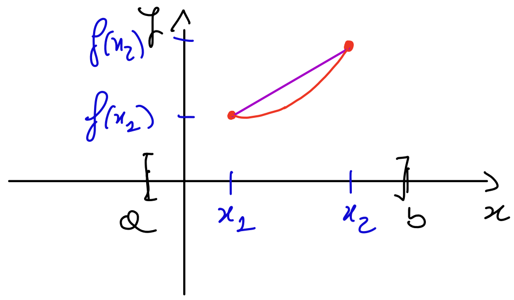
$f: \mathbb{R} \to \mathbb{R}$, D dom naturale

Def f si dice CONVESSA IN $[a,b] \subset D$ se

$\forall\, x_1, x_2 \in [a,b]$ vale che

$$f((1-t)x_1 + t\,x_2) \leq (1-t)f(x_1) + t\,f(x_2) \quad \forall\, t \in [0,1]$$

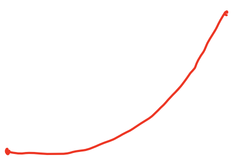


f si dice STRETTAMENTE CONVESSA IN $[a,b] \subset D$ se

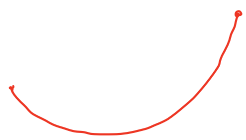
$\forall\, x_1, x_2 \in [a,b]$ vale che

$$f((1-t)x_1 + t\,x_2) < (1-t)f(x_1) + t\,f(x_2) \quad \forall\, t \in (0,1)$$

ES



CONVESSA



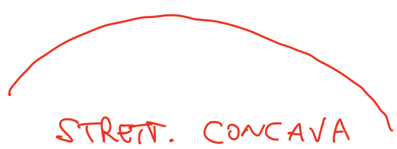
STRETT. CONVESSA

Def f si dice (STRETTAMENTE) CONCAVA IN $[a,b] \subset D$ se

$-f$ è (STRETTAMENTE) CONVESSA

ES

CONCAVA



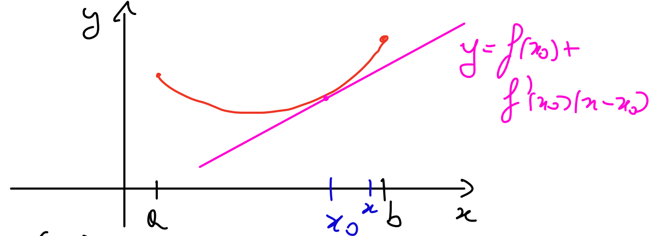
STRETT. CONCAVA

PROP Dato $[a,b] \subset D$ le seguenti condizioni equivalenti:

- f è convessa in $[a,b]$;

- f' è crescente in $[a,b]$;
- Per ogni $x_0 \in (a,b)$,

$$f(x) \geq f(x_0) + f'(x_0)(x-x_0) \quad \forall x \in (a,b).$$



<u>ES</u> $f(x) = e^x - \frac{1}{2}x^2$, $D = \mathbb{R}$, continua e derivabile

- $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \left(e^x - \frac{1}{2}x^2\right) = +\infty$

$+\infty - \infty$

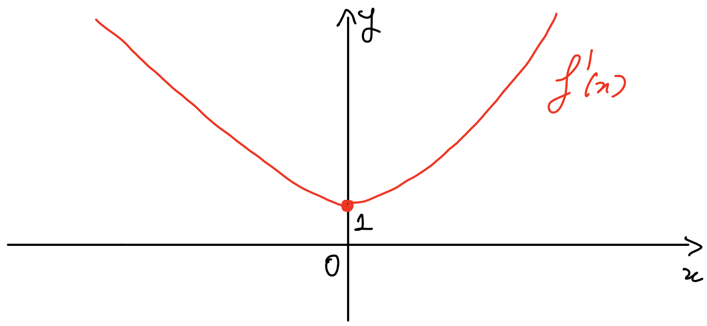
non ci sono asintoti obliqui.

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \left(e^x - \frac{1}{2}x^2\right) = -\infty$

$0 - \infty$

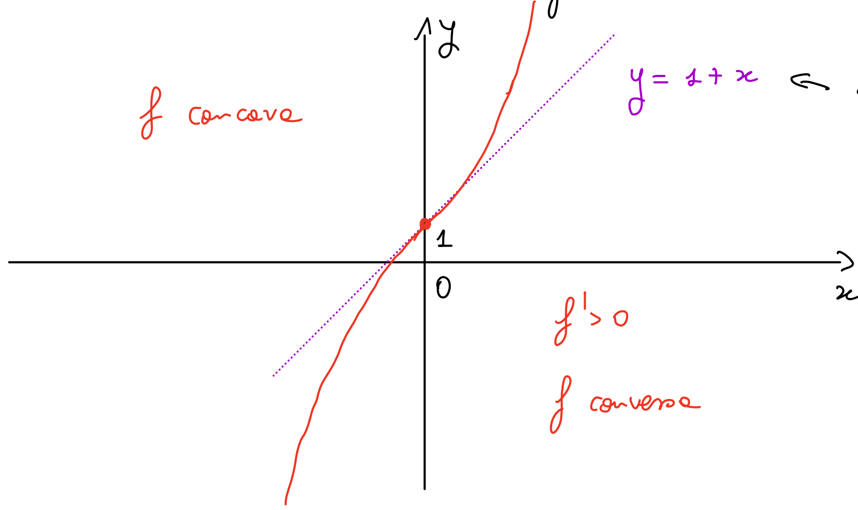
- $f(0) = 1$, $f(x) > 0 \iff e^x > \frac{1}{2}x^2$ ...... (?)

- $f'(x) = \left(e^x - \frac{1}{2}x^2\right)' = (e^x)' + \left(-\frac{1}{2}x^2\right)' = e^x + \left(-\frac{1}{2}\right) \cdot 2x = e^x - x$



$f'(x) \geq 1 > 0 \quad \forall x \in \mathbb{R}$, f' è strettamente crescente in $(0,+\infty)$

f' è " decrescente in $(-\infty, 0)$



$\Leftarrow$ retta tangente a f in 0

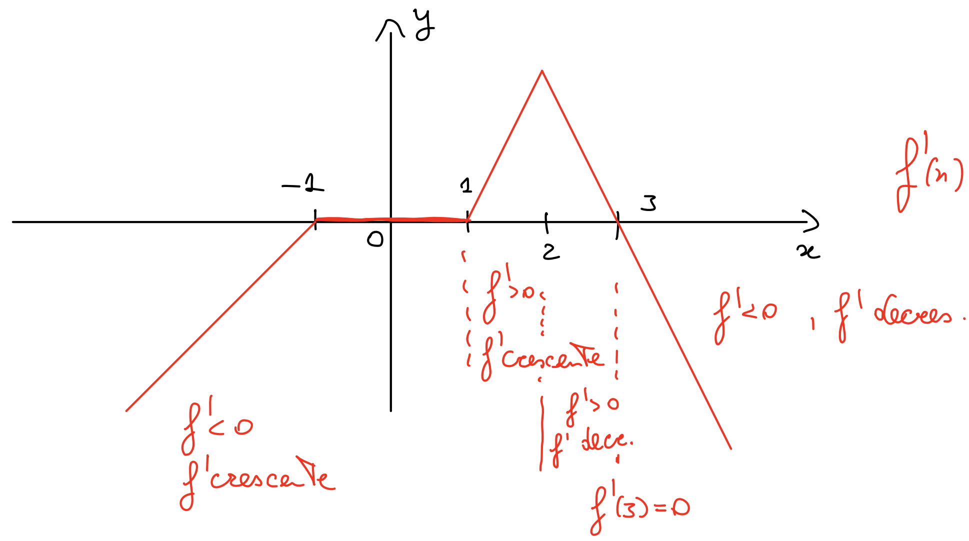
$y = f(0) + f'(0)(x-0)$

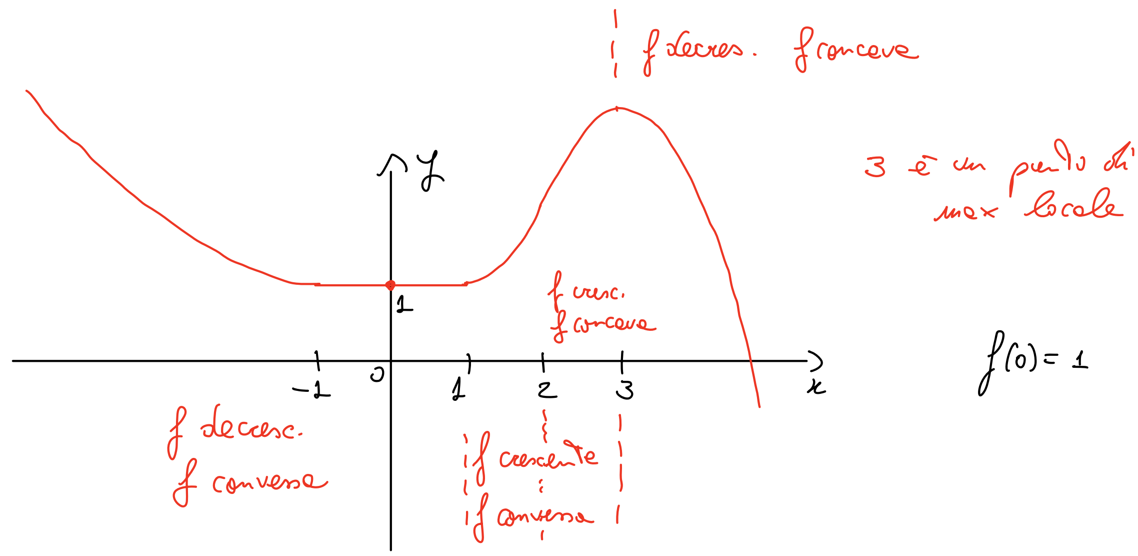
$y = 1 + 1 \cdot (x-0)$

oss $\left(f(x)+c\right)' = f'(x) + (c)' = f'(x)$

$c \in \mathbb{R}$

ES





ES $f(x) = \dfrac{|x|}{\log|x|}$, $D = \{ x \in \mathbb{R} \,/\, |x| > 0 \, , \, \log|x| \neq 0 \}$

$|x|>0$: argomento del log deve essere >0

$\log|x| \neq 0$: denominatore $\neq 0$

– $D = \mathbb{R} \setminus \{0, -1, +1\}$, $Acc(D) = \mathbb{R} \cup \{\pm\infty\}$.

$|0| = 0$ $\qquad \log|\pm 1| = 0$

$-\infty$ $\quad \to \leftarrow \; \to \leftarrow \; \to \leftarrow$ $\quad +\infty$

$-1 \qquad 0 \qquad +1 \qquad \mathbb{R}$

– f è pari , $f(-x) = \dfrac{|-x|}{\log|-x|} = \dfrac{|x|}{\log|x|} = f(x)$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{x}{\log x} = +\infty \qquad (\text{limite notevole})$$

∞/∞

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} \frac{x}{\log x} = +\infty, \quad \frac{1}{0^+}$$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} \frac{x}{\log x} = -\infty, \quad \frac{1}{0^-}$$

asintoto verticale in 1

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{x}{\log x} = 0^-, \quad \frac{0^+}{-\infty} = 0^+\cdot\left(-\frac{1}{\infty}\right) = 0^+\cdot 0^-$$

Asintoto obliquo a $+\infty$? $\quad a := \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\frac{x}{\log x}}{x} = \lim_{x\to+\infty} \frac{1}{\log x} = 0$

$$b := \lim_{x\to+\infty} [f(x) - ax] = \lim_{x\to+\infty} \frac{x}{\log x} = +\infty \notin \mathbb{R}$$

non c'è asintoto.

- Continuità. Sì su D, e $\lim_{x\to0} f(x) = 0$.

- $$f'(x) = \left(\frac{x}{\log x}\right)' = \frac{(x)'\cdot\log x - x\cdot(\log x)'}{(\log x)^2} = \frac{1\cdot\log x - x\cdot\frac{1}{x}}{(\log x)^2} =$$

$x\in(0,+\infty)\setminus\{1\}$
$$= \frac{\log x - 1}{(\log x)^2}$$

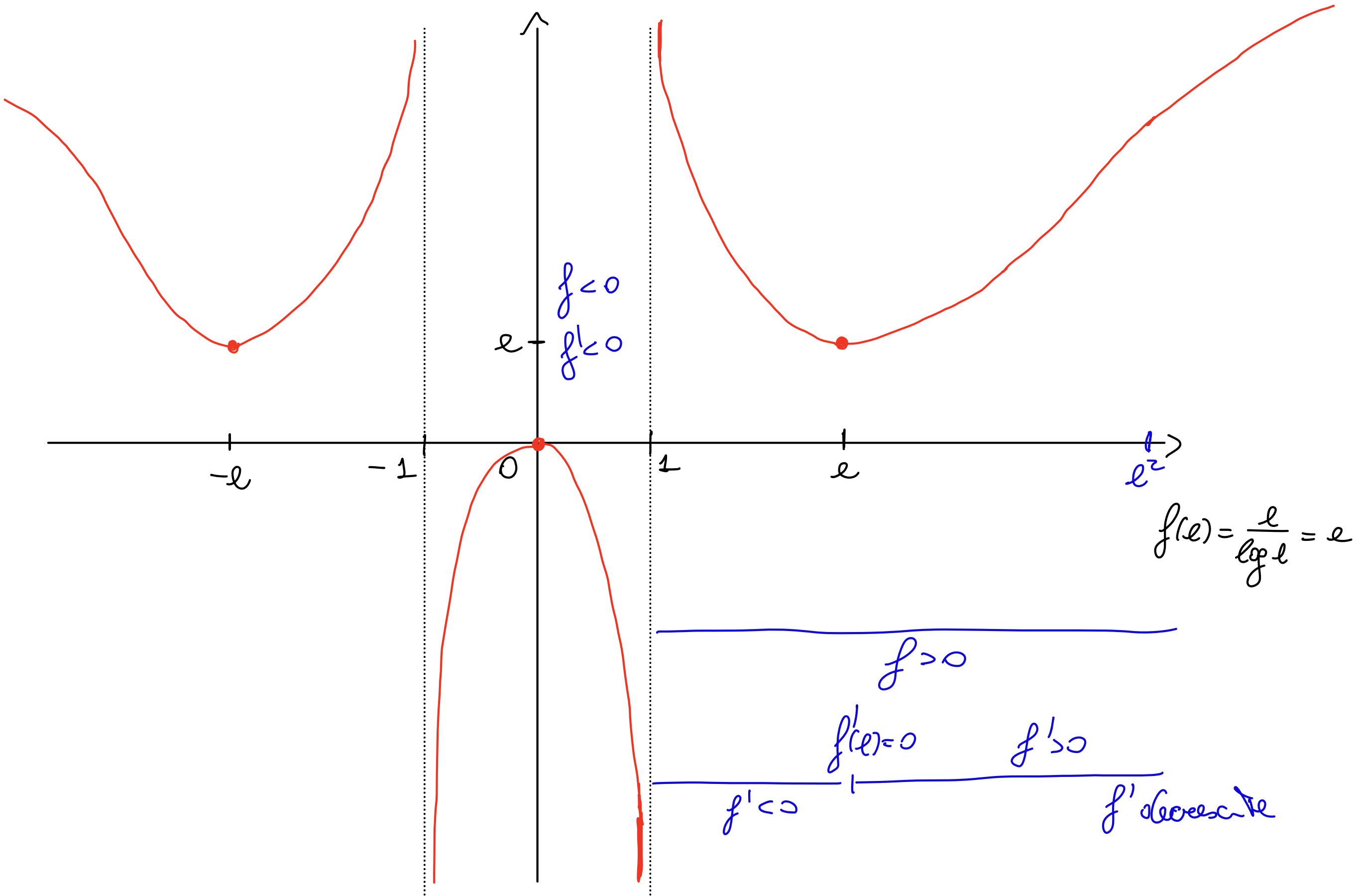
$$f'(x) > 0 \iff \log x - 1 > 0 \iff x > e$$

$$f'(x) \begin{cases} >0, & \text{se } x\in(e,+\infty) \\ =0, & \text{se } x = e \\ <0, & \text{se } x\in(0,e)\setminus\{1\} \end{cases}, \quad f' \text{ decrescente per } x\to+\infty$$

- $$f(x) > 0 \iff \frac{x}{\log x} > 0 \iff \begin{cases} x>0 \\ \log x > 0 \end{cases} \vee \cancel{\begin{cases} x<0 \\ \log x < 0 \end{cases}} \iff x\in(1,+\infty)$$

$x\in(0,+\infty)\setminus\{1\}$

$$f(x) \begin{cases} >0, & x\in(1,+\infty) \\ =0, & \nexists \\ <0, & x\in(0,1) \end{cases}$$



Trovare l'eq. della retta tangente al grafico di $f(x)$ in e^2.

$$y = f(x_0) + f'(x_0)(x-x_0) \quad , \quad x_0 = e^2 \quad , \quad f(x_0) = e$$

$$f'(e^2) = \frac{\log e^2 - 1}{(\log e^2)^2} = \frac{1}{4}$$

$$y = e + \frac{1}{4}(x - e^2)$$

$$= \frac{1}{4}x + e - \frac{1}{4}e^2$$