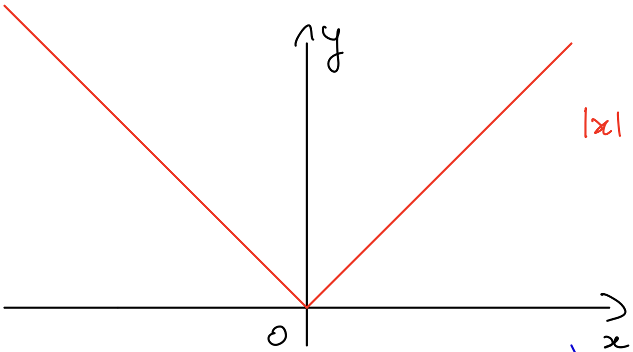
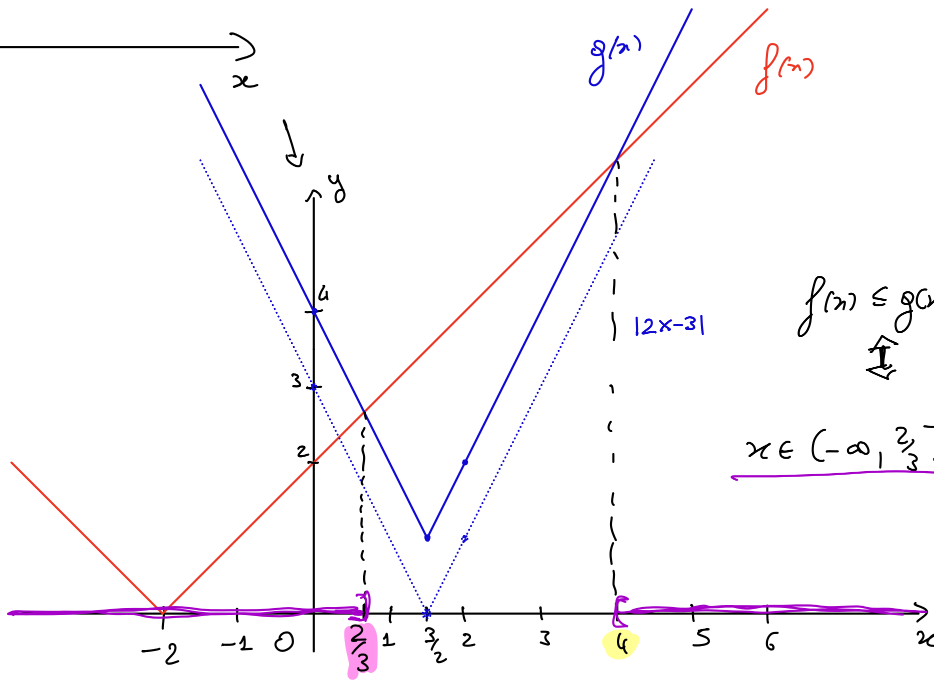
Esercizi

− $|x+2| \le |2x-3|+1$.

$f(x) = |x+2|$, $g(x) = |2x-3|+1$, $f(x) \le g(x)$





$f(x) \le g(x)$

$\Updownarrow$

$x \in (-\infty, \frac{2}{3}] \cup [4, +\infty)$

$|4+2| = |2 \cdot 4 - 3| + 1$

$x \in (0,1)$ tale che $|x+2| = |2x-3|+1$

$\Longleftrightarrow$ $x \in (0,1)$ tale che $x+2 = -2x+3+1$

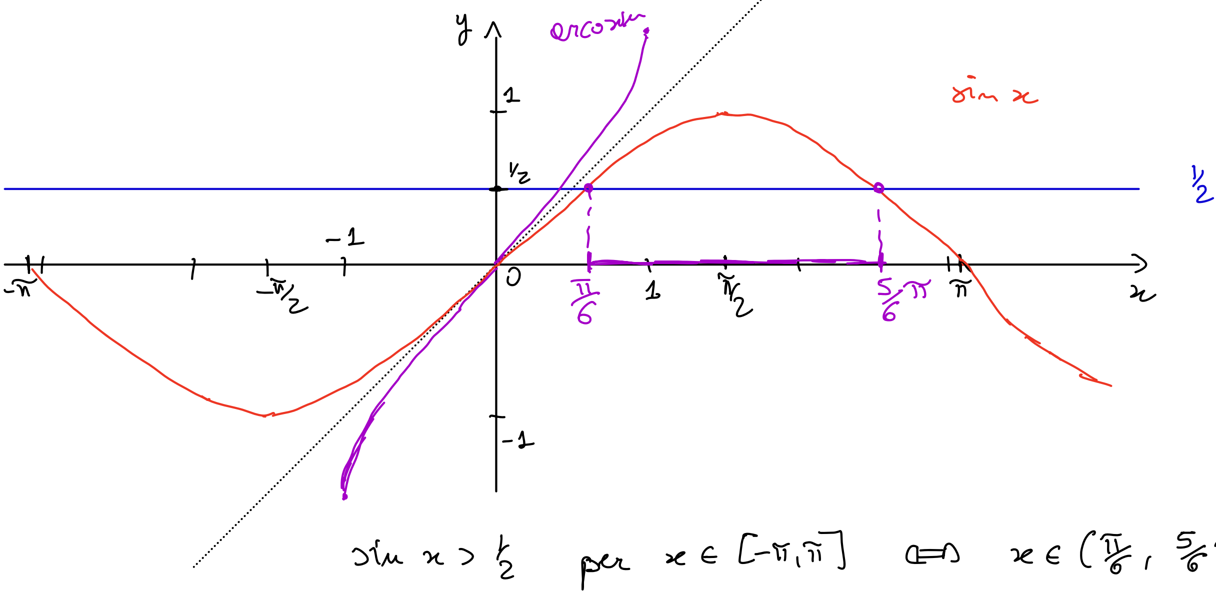
$3x = 2$

$x = 2/3$

$|\frac{2}{3}+2| = |2 \cdot \frac{2}{3} - 3| + 1$

$8/3 = 5/3 + 1$

− $\sin x > 1/2$



$\sin x > \frac{1}{2}$ per $x \in [-\pi, \pi] \iff x \in \left(\frac{\pi}{6}, \frac{5}{6}\pi\right)$

$\sin x > \frac{1}{2} \iff x \in \bigcup_{k \in \mathbb{Z}} \left(\frac{\pi}{6} + 2k\pi, \frac{5}{6}\pi + 2k\pi\right)$

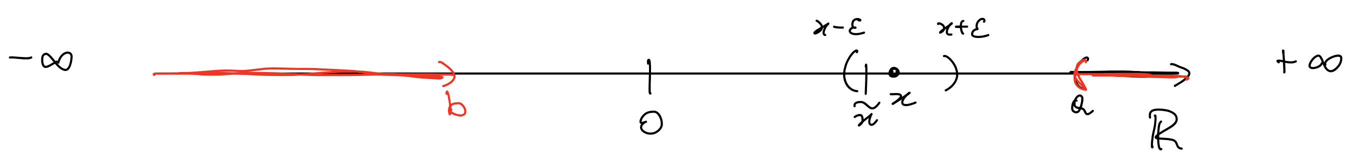
OSS

$\arcsin : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\arcsin = \left(\sin\big|_{[-\frac{\pi}{2}, \frac{\pi}{2}]}\right)^{-1}$

$\sin x > \frac{1}{2} \iff \arcsin(\sin x) > \arcsin\left(\frac{1}{2}\right) \iff x > \frac{\pi}{6}$ su $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

LIMITI



$\mathbb{R}^* = \mathbb{R} \cup \{\pm\infty\}$

Def Si chiama intorno di $x \in \mathbb{R}$ un intervallo della forma $(x-\varepsilon, x+\varepsilon)$, $\varepsilon > 0$

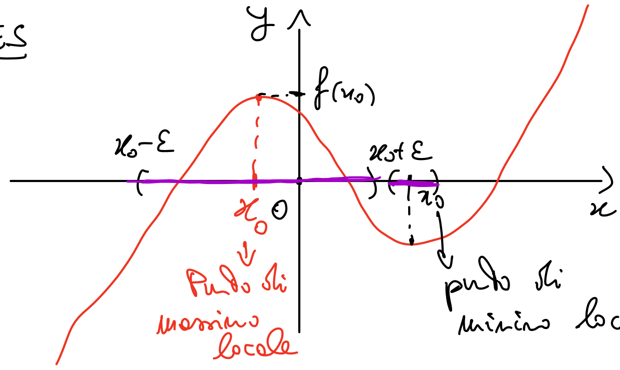
$\left[\ \tilde{x} \in (x-\varepsilon, x+\varepsilon) \iff d(x, \tilde{x}) < \varepsilon\ \right]$

Si chiama intorno di $+\infty$ un intervallo della forma $(a, +\infty]$, $a \in \mathbb{R} \cup \{-\infty\}$

" intorno di $-\infty$ " " " $[-\infty, b)$, $b \in \mathbb{R} \cup \{+\infty\}$

Def $f : \mathbb{R} \to \mathbb{R}$, D dominio naturale. Un punto $x_0 \in D$ si dice PUNTO DI MASSIMO LOCALE se $\exists$ intorno U di x_0 tale che $\forall x \in U \cap D$

ES



vale $f(x) \le f(x_0)$

$[\exists\, \varepsilon > 0$ tale che $\forall x \in (x_0-\varepsilon, x_0+\varepsilon) \cap D$ vale $f(x) \le f(x_0)]$

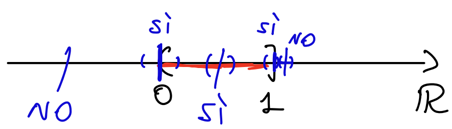
Un punto $x_0 \in D$ si dice PUNTO DI MINIMO LOCALE se $\exists$ intorno U di x_0 tale che $\forall x \in U \cap D$ vale $f(x) \ge f(x_0)$

$[\exists\, \varepsilon > 0$ tale che $\forall x \in (x_0-\varepsilon, x_0+\varepsilon) \cap D$ vale $f(x) \ge f(x_0)]$

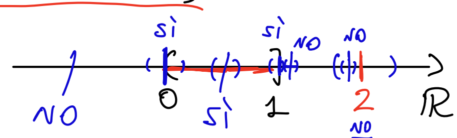
Def Dato $A \subset \mathbb{R}^*$, un punto $x_0 \in \mathbb{R}^*$ si dice di ACCUMULAZIONE per A se $\forall$ intorno U di x_0 $\exists\, x \in U \cap A \setminus \{x_0\}$ (ossia $U \cap A \setminus \{x_0\} \neq \emptyset$).

ES • $A = (0,1]$

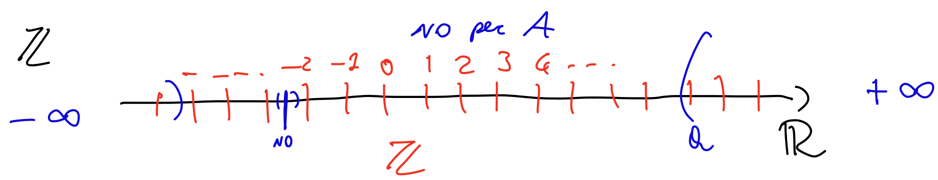
Punti accumulazione di $A = [0,1]$



• $A = (0,1] \cup \{2\}$ Punti di accumulazione di $A = [0,1]$



• $A = \mathbb{Z}$



Punti di accumulazione di $A = \{\pm\infty\}$

Def $f: \mathbb{R} \to \mathbb{R}$, D dominio naturale.

Dato $x_0 \in \mathbb{R}^*$ che sia di accumulazione per D, si dice che "f ha limite $\ell \in \mathbb{R}^*$ per x che tende a x_0", $\lim_{x \to x_0} f(x) = \ell$, se $\forall$ intorno V di ℓ $\exists$ intorno U di x_0 tale che se $x \in U \cap D \setminus \{x_0\}$

si ha che $f(x) \in V$

ES - $x_0 \in \mathbb{R}$, $\ell \in \mathbb{R}$

$$\lim_{x \to x_0} f(x) = \ell \iff \forall \varepsilon > 0 \ \exists \delta > 0 \text{ tale che se } x \in (x_0 - \delta, x_0 + \delta) \cap D \setminus \{x_0\}$$
si ha che $f(x) \in (\ell - \varepsilon, \ell + \varepsilon)$ (ossia $|f(x) - \ell| < \varepsilon$)

- $x_0 \in \mathbb{R}$, $\ell = +\infty$

$$\lim_{x \to x_0} f(x) = +\infty \iff \forall a \in \mathbb{R} \cup \{-\infty\} \ \exists \delta > 0 \text{ tale che se } x \in (x_0 - \delta, x_0 + \delta) \cap D \setminus \{x_0\}$$
si ha che $f(x) > a$ (ossia $f(x) \in (a, +\infty)$).

Esempio $\quad f(x) = \frac{1}{x^2}$, $D = \mathbb{R} \setminus \{0\}$, $Acc(D) = \mathbb{R}^*$.

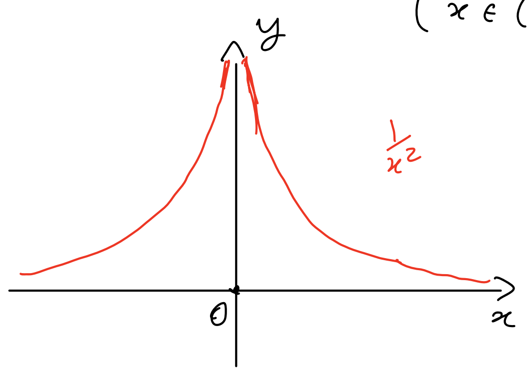
- $x_0 = 0$. $\exists\, \ell \in \mathbb{R}^*$ t.c. $\lim_{x \to 0} f(x) = \ell$?

$$\lim_{x \to 0} f(x) = +\infty \iff \forall a \in \mathbb{R} \cup \{-\infty\} \ \exists \delta > 0 \text{ t.c. se } x \in (-\delta, \delta) \setminus \{0\}$$
si ha $\frac{1}{x^2} > a$.

Prendo $a > 0$ ($a = 10^5$), $\quad \frac{1}{x^2} > a \iff x^2 < \frac{1}{a} \iff x \in \left(-\frac{1}{\sqrt{a}}, \frac{1}{\sqrt{a}}\right) \setminus \{0\}$

$\left(x \in \left(-10^{-5/2}, 10^{-5/2}\right)\right)$

Ok per $\delta = \frac{1}{\sqrt{a}}$.



ES $\quad \lim_{x \to 0} \frac{1}{x} = +\infty$?