$f: \mathbb{R} \to \mathbb{R}$, D dominio naturale

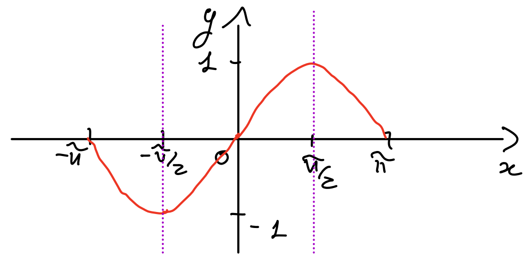
Se $A \subseteq D$ tale che f è iniettiva in A ($\forall x_1, x_2 \in A$, $x_1 \neq x_2$, si ha $f(x_1) \neq f(x_2)$) allora f è invertibile in A, ossia data $f(A)$ = immagine di f ristretta ad A, esiste $g: f(A) \subseteq \mathbb{R} \to \mathbb{R}$ per cui $(g \circ f)(x) = x \quad \forall x \in A$.

E $g = (f|_A)^{-1}$ è la funzione inversa di $f|_A$.

ES $f(x) = \sin x$, $f: \mathbb{R} \to \mathbb{R}$, $D = \mathbb{R}$, $\text{Im} f = [-1, 1]$

$A = [-\frac{\pi}{2}, \frac{\pi}{2}]$ f è iniettiva in A

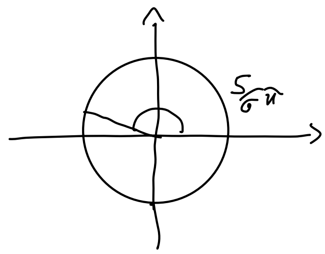
$f(A) = [-1, 1]$



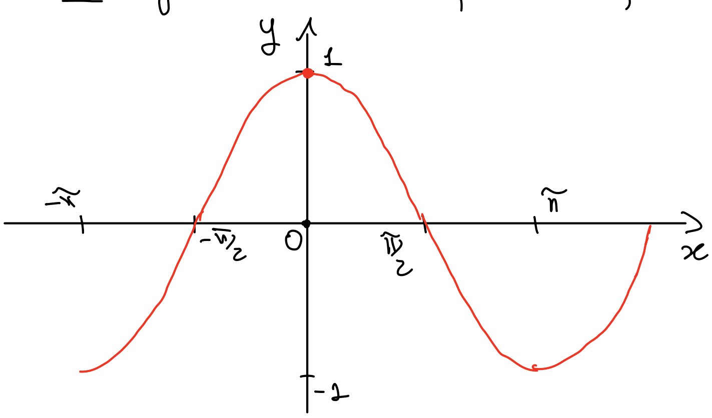
$(f|_A)^{-1}(y) =:$ arcoseno (y)

arcoseno : $[-1, 1] \longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$ tale che arcoseno $(\sin(x)) = x \quad \forall x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

- arcoseno $(\sin(\frac{\pi}{3}))$ = arcoseno $(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$. $(\frac{\pi}{3} \in [-\frac{\pi}{2}, \frac{\pi}{2}])$
- arcoseno $(\sin(\frac{5}{6}\pi))$ = arcoseno $(\frac{1}{2}) = \frac{\pi}{6}$



ES $f(x) = \cos x$, $D = \mathbb{R}$, $\text{Im} f = [-1, 1]$



$A = [0, \pi]$, $f(A) = [-1, 1]$, f invertibile in A.

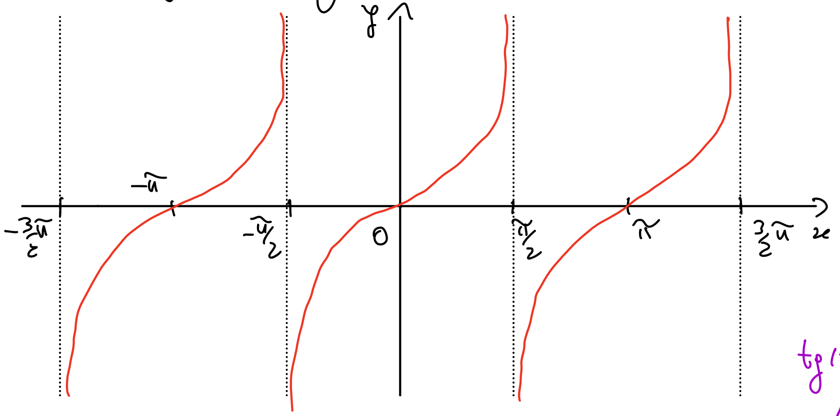
arcocoseno $:= (f|_A)^{-1}$

arcocos : $[-1, 1] \to [0, \pi]$

Vale arcocos $(\cos(x)) = x \quad \forall x \in [0, \pi]$.

- arcocos $(\cos(0))$ = arcocos $(1) = 0$
- arcocos $(\cos(2\pi))$ = arcocos $(1) = 0$

ES $f(x) = tg\, x$, $D = \mathbb{R} \setminus \{ \frac{\pi}{2} + k\pi \,/\, k \in \mathbb{Z} \}$, $Im\, f = \mathbb{R}$

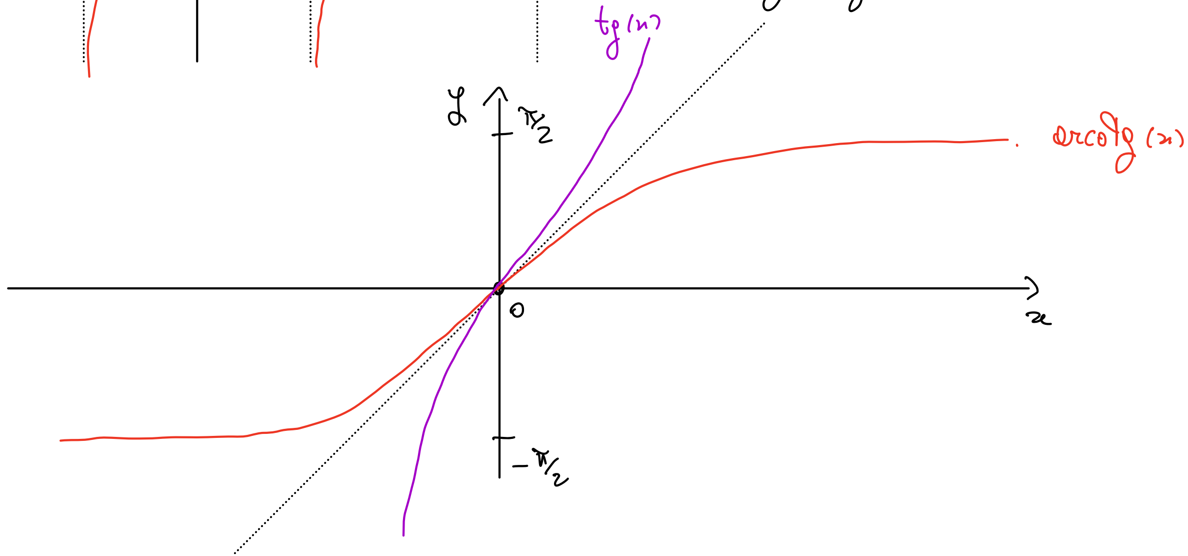


$A = (-\frac{\pi}{2}, \frac{\pi}{2})$, f invertibile in A, $f(A) = \mathbb{R}$

arctg $:= (f|_A)^{-1}$

arctg $: \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$

arctg$(tg(x)) = x \quad \forall x \in (-\frac{\pi}{2}, \frac{\pi}{2})$



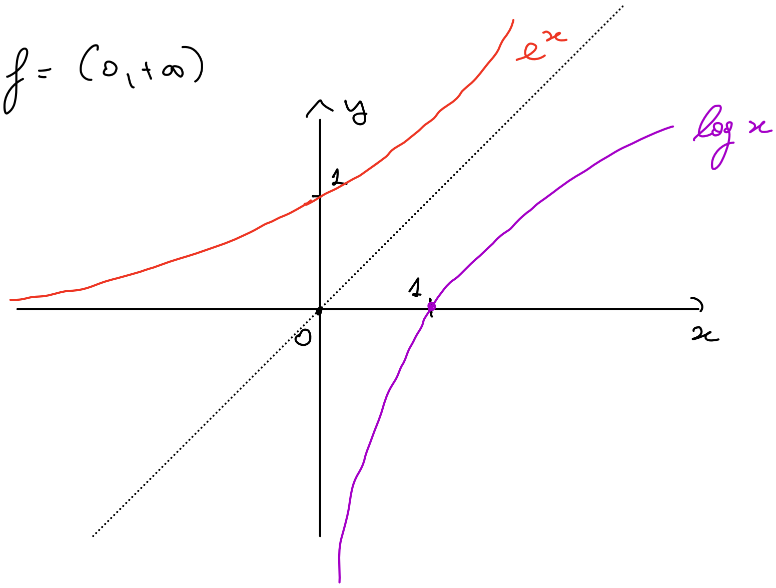
ES $f(x) = e^x$, $D = \mathbb{R}$, $Im\, f = (0, +\infty)$

$g = f^{-1} : (0, +\infty) \longrightarrow \mathbb{R}$

$f^{-1}(x) = \log x$

$g \circ f = \log(e^x) = x \quad \forall x \in \mathbb{R}$

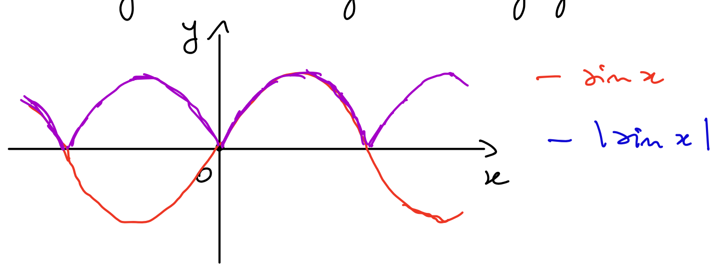
$f \circ g = e^{\log x} = x \quad \forall x \in (0, +\infty)$



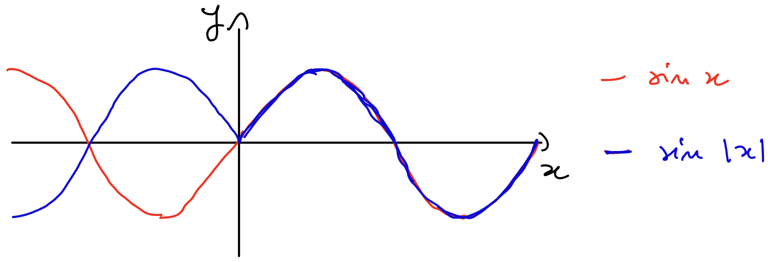
Manipolare i grafici e studio di eq. e diseq.

- Conosciamo il grafico di $f(x)$. Disegnare il grafico di:

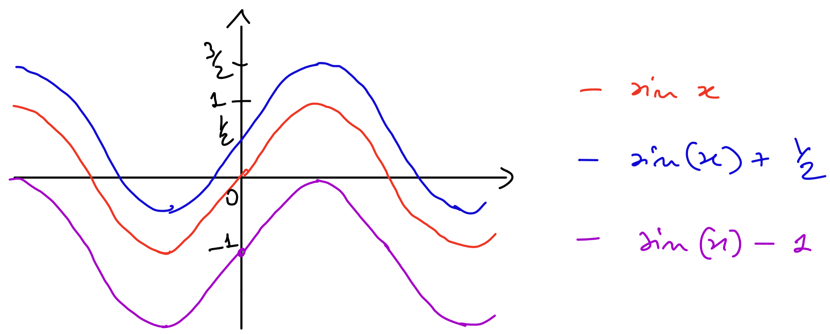
– $|f(x)|$



- $f(|x|)$



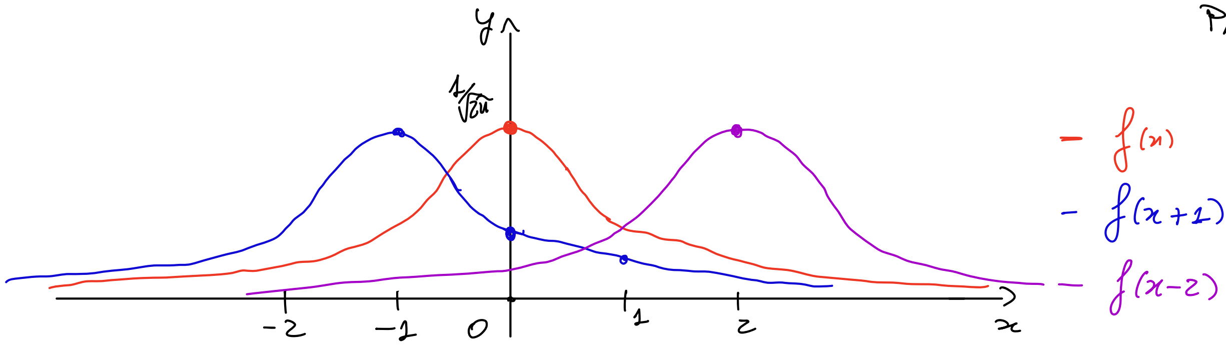
- $f(x)+c$, $c\in\mathbb{R}$



- $f(x+c)$, $c\in\mathbb{R}$ $\qquad f(x)=\frac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}}$, $D=\mathbb{R}$, $\mathrm{Im}\, f \subseteq (0,+\infty)$

$\left(0, \frac{1}{\sqrt{2\pi}}\right]$

PARI



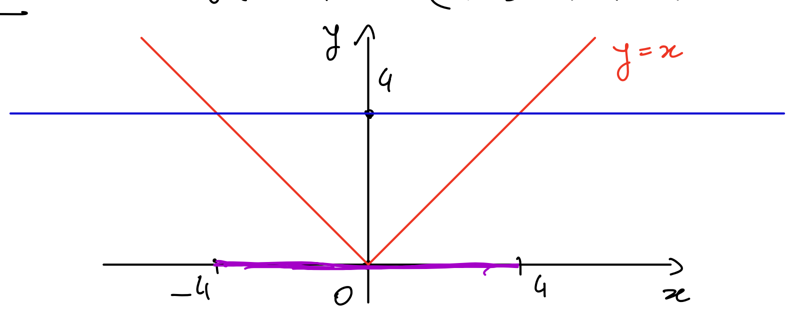
OSS $\quad f(x)=\log(x^2)$, $\quad g(x)=2\log(x)$, $\quad h(x)=2\log(|x|)$

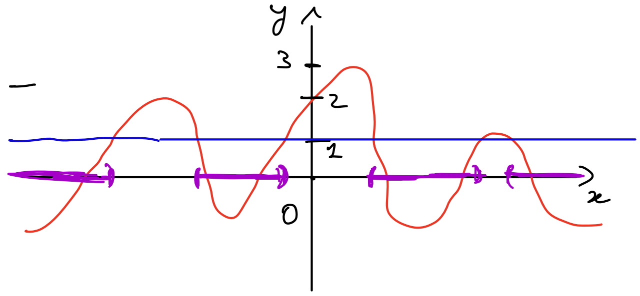
$\left[\text{se } a,b>0,\ \log(a^b)=b\log a\right]$

$D_f=\{x\in\mathbb{R} \,/\, x^2>0\}$ $\qquad D_g=(0,+\infty)$, $\qquad D_h=\mathbb{R}\setminus\{0\}$

$\quad =\mathbb{R}\setminus\{0\}$

Es $\quad$ - $\sqrt{x^2}<4$ $\quad (\iff |x|<4 \iff -4<x<4)$



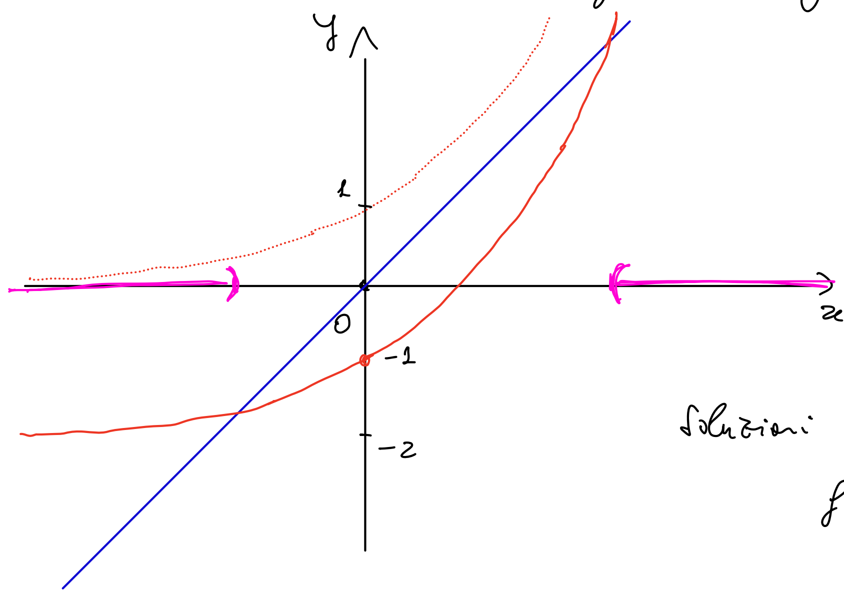


Quante soluzioni ha l'eq. $f(x) = 1$?

$f(x) < 1$

- Quante soluzioni ha l'eq $e^x - 2 = x$? $\quad f(x) = e^x - 2$

$f(x) = g(x)$? $\quad g(x) = x$



— $f(x)$

— $g(x)$

Soluzioni di $e^x - x > 2$

$f(x) = e^x - 2 > x = g(x)$

Esercizi

- $\log(x+3) < 0$
- $e^{x^2 - 3x + 3} \geq 0$
- $|x+2| \leq |2x-3| + 1$
- $\sin x > \frac{1}{2}$