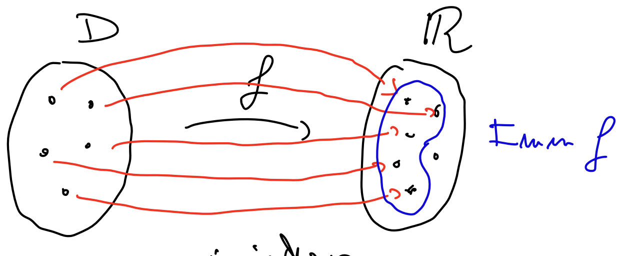
Funzione inversa

$f: D \subseteq \mathbb{R} \to \mathbb{R}$, D dominio naturale

La funzione inversa di f esiste se ad ogni $y \in \mathrm{Imm}\, f$ esiste uno ed un solo $x \in D$ tale che $f(x) = y$.

$\mathrm{Imm}\, f = \{ y \in \mathbb{R} : \exists\, x \in D \text{ per cui } f(x) = y \}$

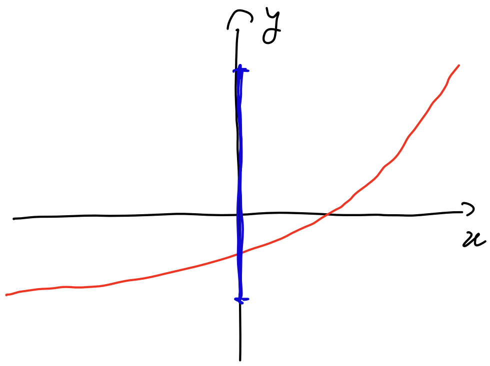
Def Una funzione f si dice <u>iniettiva</u> se $\forall\, x, x' \in D$, $x \neq x'$, $f(x) \neq f(x')$.



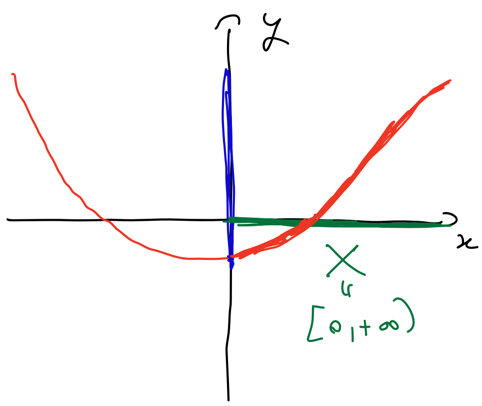
iniettiva

Def Una funzione f si dice <u>surgettiva</u> se $\mathrm{Imm}\, f$ = codominio.

ES



iniettiva



non iniettiva

$f: X \to \mathbb{R}$ iniettiva

Def Data $f: X \subseteq \mathbb{R} \to \mathbb{R}$ iniettiva sul dominio $X \subseteq D$ dominio naturale. Si chiama funzione inversa, f^{-1} $\left((f|_X)^{-1} \right)$, la funzione $f^{-1}: \mathrm{Imm}\, f|_X \subseteq \mathbb{R} \to X$, tale che
$$(f^{-1} \circ f)(x) = x \quad \forall\, x \in X.$$

Def
$$\begin{tikzcd} D \subseteq \mathbb{R}_x \arrow[r, "f"] \arrow[rr, bend right, "g \circ f"'] & \mathbb{R}_y \supseteq D' \arrow[r, "g"] & \mathbb{R}_z \end{tikzcd}$$

$g \circ f$ esiste se $\mathrm{Imm}\, f \subseteq D'$

ES
- $\sin(x^2)$, $f(x) = x^2$, $g(y) = \sin y$

  $(g \circ f)(x) = g(f(x)) = \sin(f(x)) = \sin(x^2)$

  $\mathrm{Imm}\, f = [0, +\infty)$, $D' = \mathrm{Dom}(g) = \mathbb{R}$

- $\log(x+1)$ è definita per $x > -1$

  $f(x) = x+1$, $g(y) = \log y$. , $g(f(x)) = \log(x+1)$.

  Dominio naturale di $g \circ f = \{x \in \mathbb{R} : f(x) \in D' = \mathrm{Dom}(g)\}$

  $\mathrm{Dom}(f) = \mathbb{R}$, $\mathrm{Imm}\, f = \mathbb{R}$, $\mathrm{Dom}(g) = \{y > 0\}$, $\mathrm{Imm}\, g = \mathbb{R}$

  $\mathrm{Imm}\, f \subsetneq \mathrm{Dom}(g)$

  $X = \{x > -1\}$, $\mathrm{Imm}(f|_X) = \{y > 0\} = \mathrm{Dom}(g)$

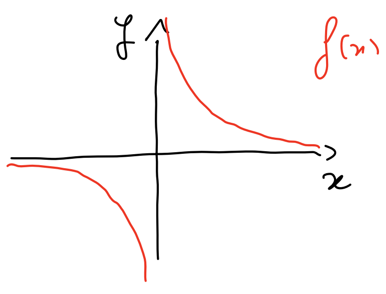
  $\mathrm{Dom}(g \circ f) = \{x > -1\}$

- $\sqrt{\frac{1}{x}} = (g \circ f)(x)$, $\mathrm{Dom}(g \circ f) = \{x \in \mathbb{R} : \frac{1}{x} \geq 0,\ x \neq 0\}$

  $f(x) = \frac{1}{x}$, $g(y) = \sqrt{y}$

  $\mathrm{Dom}(f) = \{x \neq 0\}$, $\mathrm{Imm}\, f = \mathbb{R} \setminus \{0\}$, $\mathrm{Dom}(g) = [0, +\infty)$, $\mathrm{Imm}\, g = [0, +\infty)$

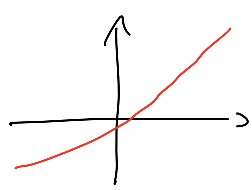
  $\mathrm{Imm}\, f \not\subseteq \mathrm{Dom}(g)$



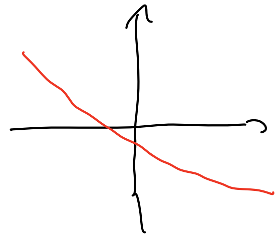
  $X = (0, +\infty)$, $\mathrm{Imm}(f|_X) = (0, +\infty) \subsetneq \mathrm{Dom}(g) = [0, +\infty)$

  $\mathrm{Dom}(g \circ f) = (0, +\infty)$

OSS f monotona strettamente crescente

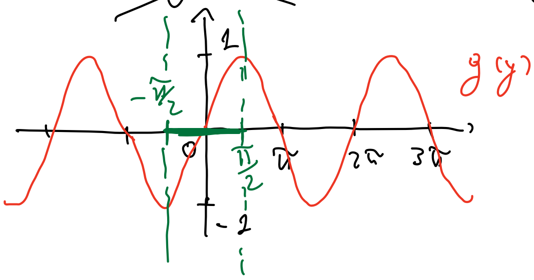


f monotona strettamente decrescente



ES $f(x) = \arcsin x$ è l'angolo y compreso tra $-\frac{\pi}{2}$ e $\frac{\pi}{2}$ tale che $\sin(y) = x$.

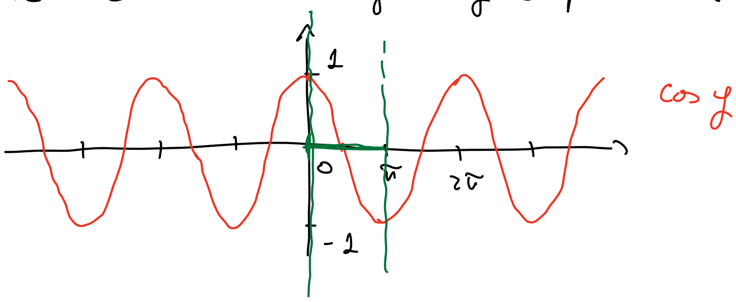
$g(y) = \sin y$, $\cancel{\text{Dom}(g) = \mathbb{R}}$, $\text{Imm}\, g = [-1, 1]$



$X = [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\text{Imm}\, g|_X = [-1, 1]$, $g|_X$ è iniettiva

$f(x) = (g|_X)^{-1}(x) = \arcsin x$, $f : [-1, 1] \subseteq \mathbb{R} \longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

ES $f(x) = \arccos x$ è l'angolo y compreso tra 0 e π per cui $\cos y = x$.



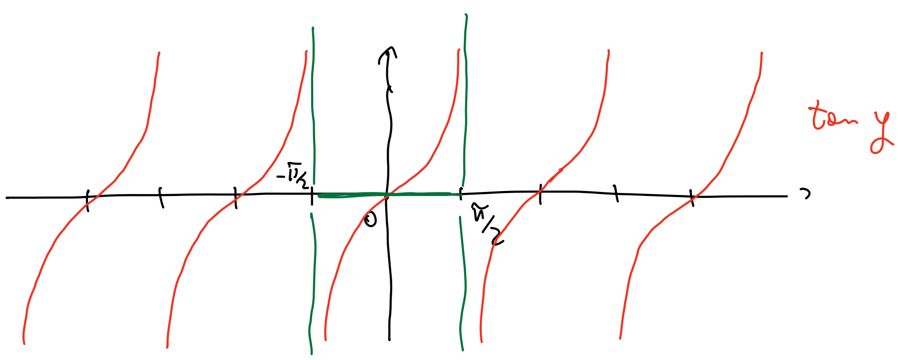
$g(y) = \cos y$

$X = [0, \pi]$, $\text{Imm}\, g|_X = [-1, 1]$, $g|_X$ è iniettiva

$f(x) = (g|_X)^{-1}(x) = \arccos x$

$f : [-1, 1] \subseteq \mathbb{R} \to [0, \pi]$

ES $f(x) = \arctan x$

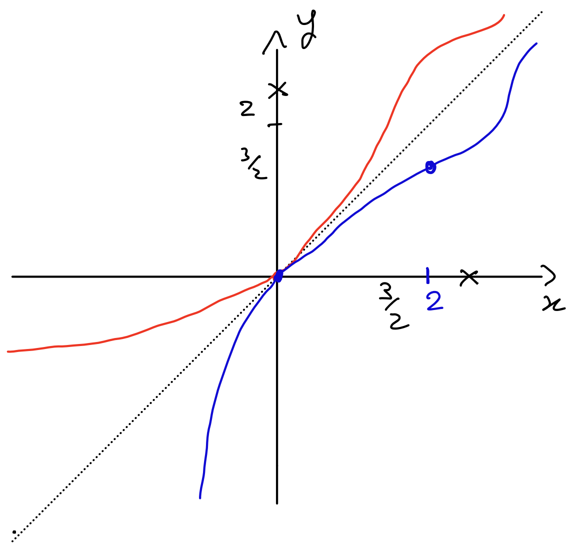


$g(y) = \tan y$, $X = (-\frac{\pi}{2}, \frac{\pi}{2})$, $g|_X$ è iniettiva

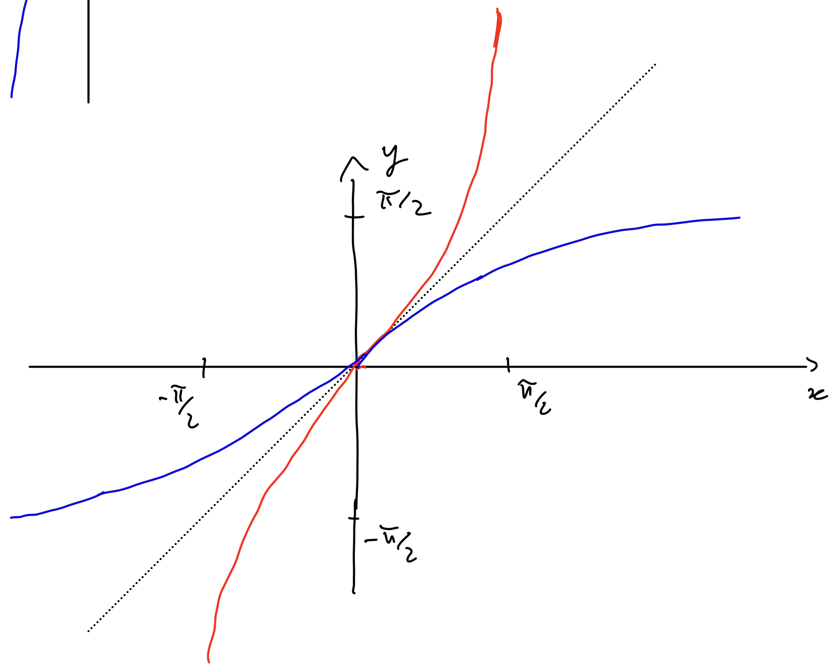
$\mathrm{Imm}\, g|_X = \mathbb{R}$

$f(x) = (g|_X)^{-1}(x) = \arctan x$, $f : \mathbb{R} \longrightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

<u>OSS</u>

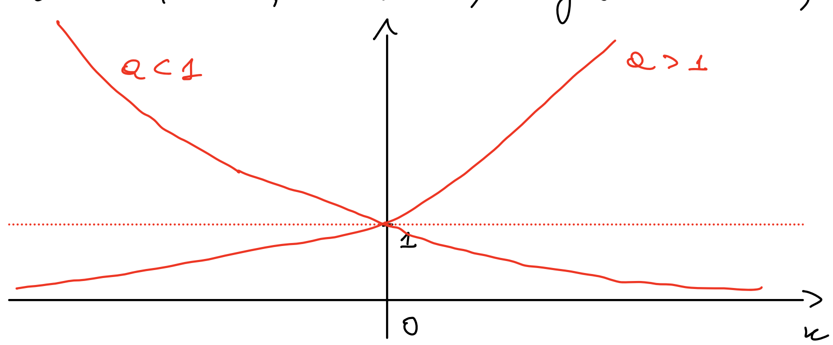


$f^{-1}(f(\frac{3}{2})) = \frac{3}{2}$



<u>ES</u> Funzione esponenziale

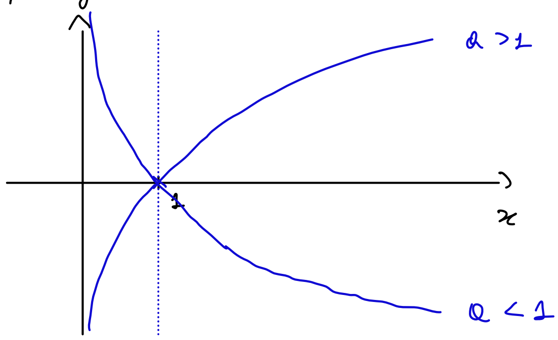
$a \in (0, +\infty)$, $a \neq 1$, $f(x) = a^x$, $D = \mathbb{R}$, $\mathrm{Imm} f = (0, +\infty)$



iniettiva $\forall a \neq 1$

$f^{-1}(x)$ è il numero y tale che $a^y = x$ $=: \log_a x$

$f^{-1}(x) = \log_a(x)$, $f^{-1} : (0, +\infty) \to \mathbb{R}$

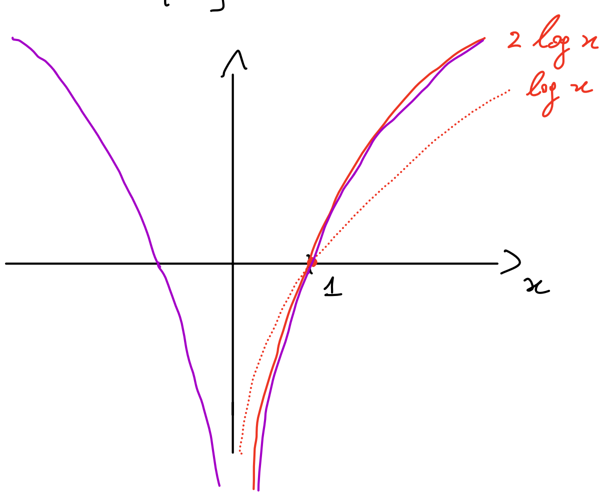


OSS $\log_a(a^x) = x$, $a^{\log_a x} = x$

OSS $\log(x^2) \stackrel{?}{=} 2 \log x$

$D = \{x^2 > 0\}$ $D = \{x > 0\}$

$D = \mathbb{R} \setminus \{0\}$ $D = (0, +\infty)$



$\log(x^2) = 2 \log |x|$

$\sqrt{x^2} \stackrel{?}{=} x$

$\sqrt{x^2} = |x|$