Funzioni reali di una variabile reale

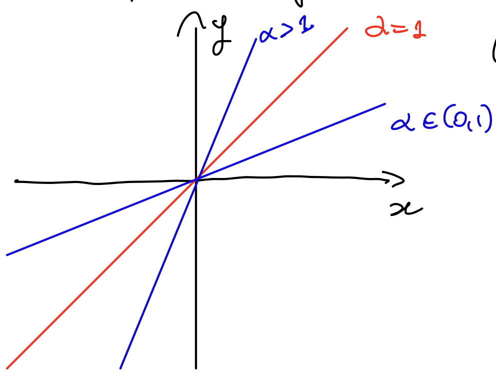
$f : D \subseteq \mathbb{R} \longrightarrow \mathbb{R}$, D dominio naturale, $\text{Imm}\, f = f(D) \subseteq \mathbb{R}$

Grafico di $f \subset \mathbb{R} \times \mathbb{R}$

Funzioni elementari:

- potenze, $f(x) = x^k$, $k \in \mathbb{N}$

<u>$k=1$</u> $\quad f(x) = x$, $D = \mathbb{R}$, $\text{Imm}\, f = \mathbb{R}$

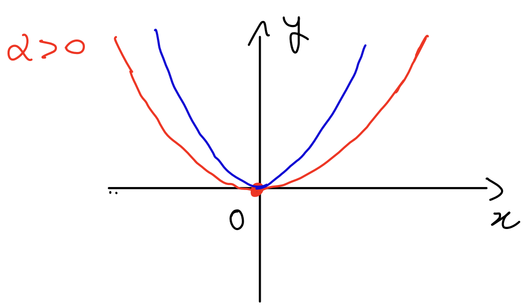


$\text{Graf}\, f = \{ (x,y) \in \mathbb{R}^2 : x \in D, \; y = x \}$

eq. del grafico

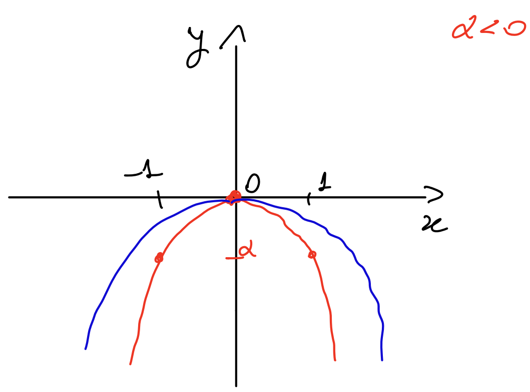
$r : y = \alpha x$, $\alpha \in \mathbb{R}$, $\alpha \neq 0$, è il grafico della funzione $f(x) = \alpha x$

<u>$k=2$</u> $\quad f(x) = \alpha x^2$, $\alpha \in \mathbb{R}$, $\alpha \neq 0$, $D = \mathbb{R}$



$y = \alpha x^2$

eq. del grafico



$\text{Imm}\, f = [0, +\infty)$

$f(x) = f(-x)$ funzione pari

$\sup f = \sup(\text{Imm}\, f) = +\infty$

$\min f = \min(\text{Imm}\, f) = 0 = f(0)$

$\text{Imm}\, f = (-\infty, 0]$

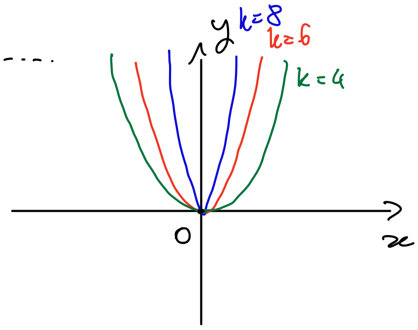
f è pari

$\max f = \max(\text{Imm}\, f) = 0 = f(0)$

$\inf f = \inf(\text{Imm}\, f) = -\infty$

k pari $\quad k = 4, 6, 8, \ldots$

$y = x^k$

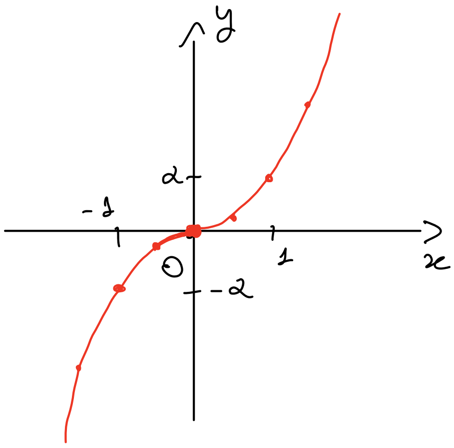


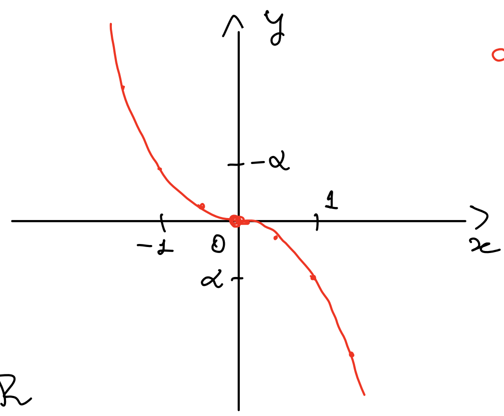
f pari

$\text{Imm } f = [0, +\infty)$

k=3 $\quad f(x) = \alpha x^3, \; \alpha \in \mathbb{R}, \; \alpha \neq 0. \qquad D = \mathbb{R}.$

$\alpha > 0$





$\alpha < 0$

$\text{Imm } f = \mathbb{R}$

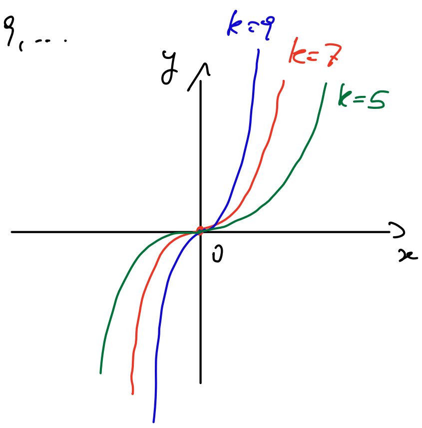
$\sup f = \sup(\text{Imm } f) = +\infty$

$\inf f = \inf(\text{Imm } f) = -\infty$

$f(-x) = -f(x)$ funzione dispari

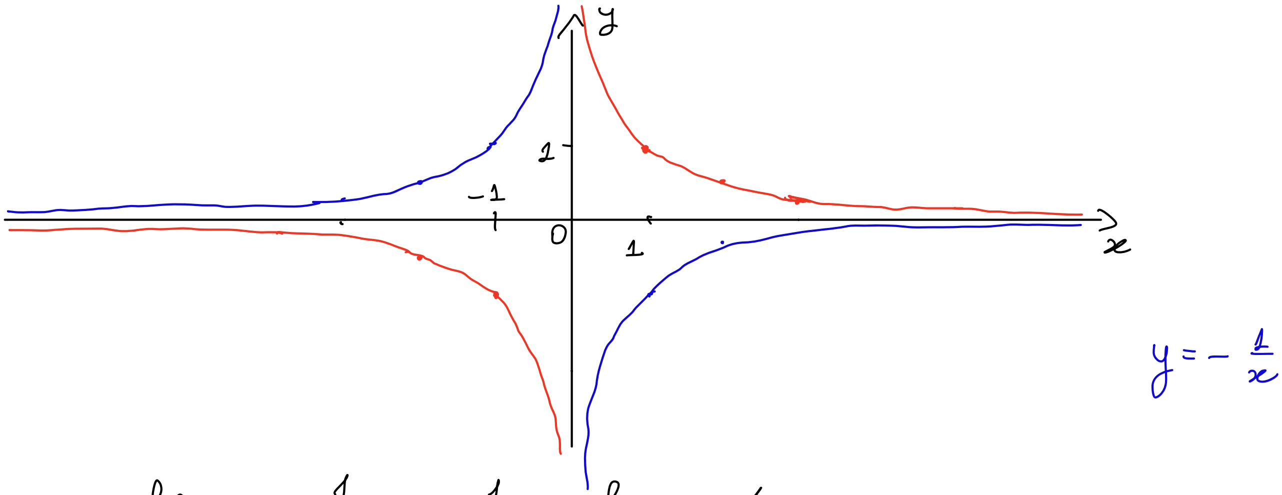
k dispari $\quad k = 5, 7, 9, \ldots$

$y = x^k$



$k \in \mathbb{Z}, \; k < 0$

k=−1 $\quad f(x) = x^{-1} = \frac{1}{x}, \quad D = \mathbb{R} \setminus \{0\}$

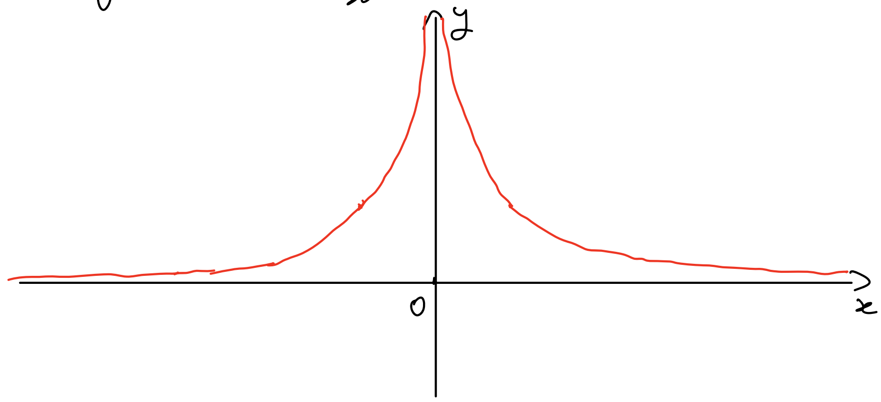


$$f(-x) = \frac{1}{(-x)} = -\frac{1}{x} = -f(x) \quad \text{dispari}$$

$$\mathrm{Imm}\, f = \mathbb{R} \setminus \{0\}, \quad \sup f = +\infty, \quad \inf f = -\infty$$

$k = -3, -5, -7, \dots$ grafici simili

$\underline{k=-2}$ $\quad f(x) = x^{-2} = \frac{1}{x^2}, \quad D = \mathbb{R} \setminus \{0\}$



$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x) \quad \text{pari}$$

$$\mathrm{Imm}\, f = (0, +\infty), \quad \sup f = +\infty, \quad \inf f = 0$$

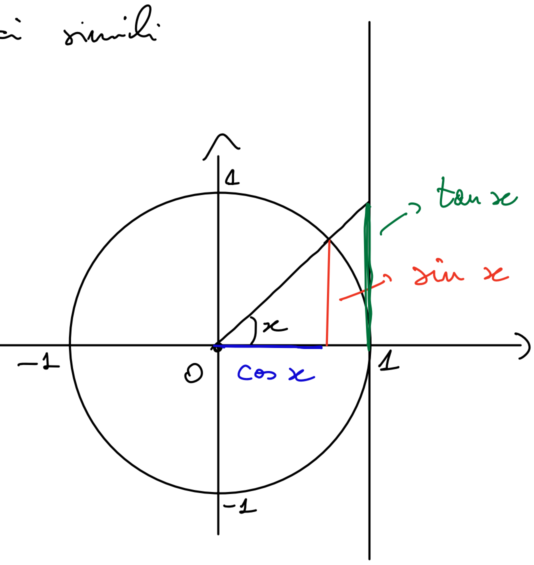
$k = -4, -6, -8, \dots$ grafici simili

Funzioni Trigonometriche

$\sin 0 = \tan 0 = 0, \quad \cos 0 = 1$

$\sin \frac{\pi}{2} = 1, \quad \cos \frac{\pi}{2} = 0, \quad \tan \frac{\pi}{2} \nexists$

$$\tan x = \frac{\sin x}{\cos x}$$

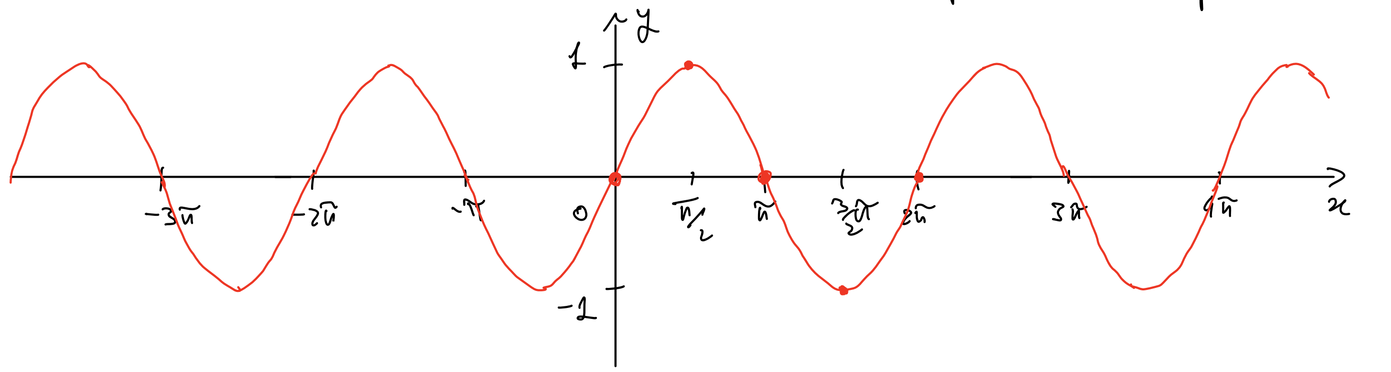


$\sin x$ e $\tan x$ sono funzioni dispari, $\cos x$ funzione pari

$\sin^2 x + \cos^2 x = 1$

$\sin : \mathbb{R} \to \mathbb{R}$, $D = \mathbb{R}$, funzione dispari

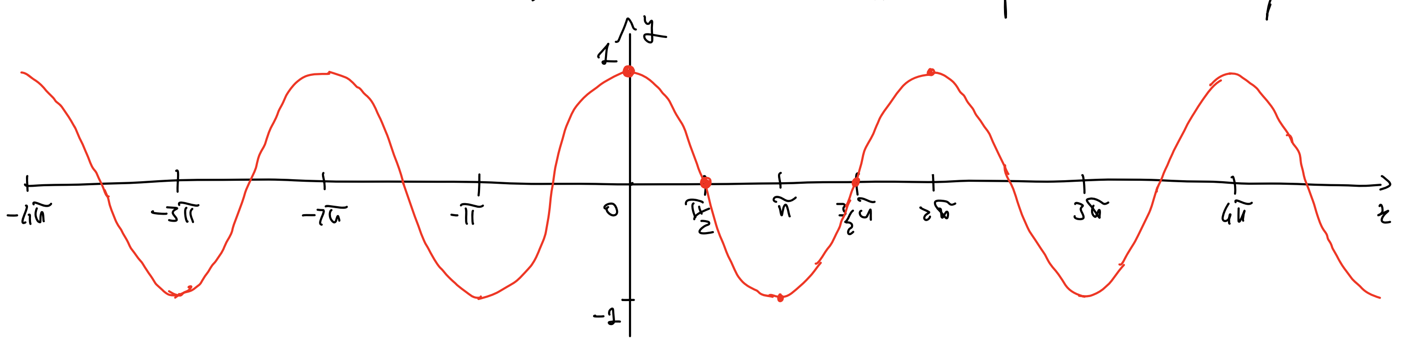
$\sin(x + 2\pi) = \sin x \quad \forall x \in \mathbb{R}$ periodica di periodo 2π.



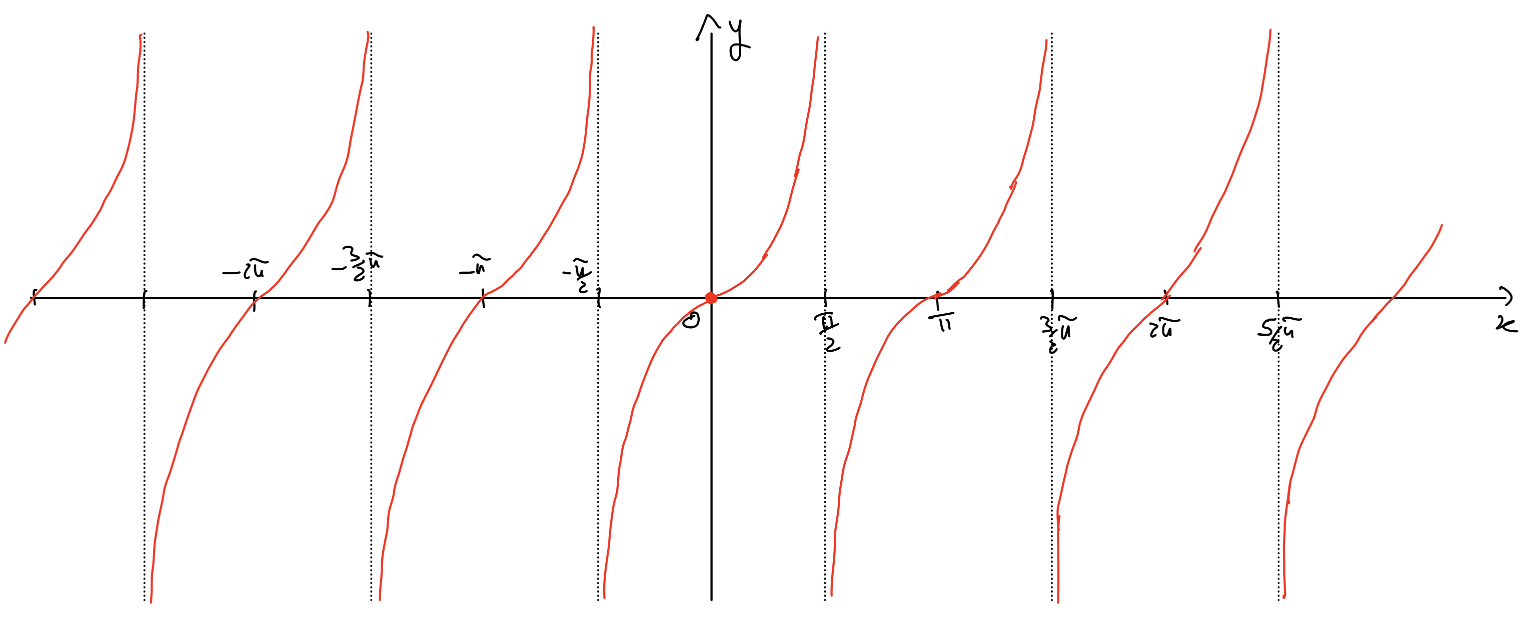
$\mathrm{Imm}(\sin) = [-1, 1]$, $\max(\sin) = +1$, $\min(\sin) = -1$

$\cos : \mathbb{R} \to \mathbb{R}$, $D = \mathbb{R}$, funzione pari, $\mathrm{Imm}(\cos) = [-1, 1]$

$\cos(x + 2\pi) = \cos x \quad \forall x \in \mathbb{R}$ periodica di periodo 2π



$\tan : \mathbb{R} \to \mathbb{R}$, $D = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi : k \in \mathbb{Z} \right\}$, funzione dispari

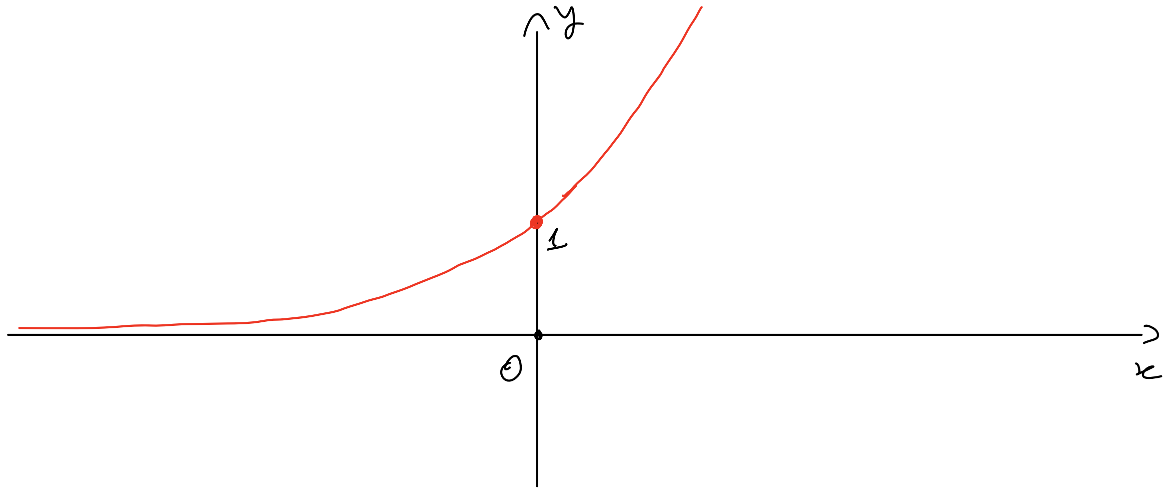


$\tan$ è periodica di periodo π, $\operatorname{Imm}(\tan) = \mathbb{R}$

Funzione esponenziale

$f(x) = e^x$, $e \sim 2.7\ldots$ $\qquad D = \mathbb{R}$

$f(0) = e^0 = 1$, $f(1) = e$, $f(-1) = e^{-1} = \frac{1}{e}$, $e^x > 0 \ \forall x \in \mathbb{R}$.



$\operatorname{Imm}(e^x) = (0, +\infty)$, $\qquad e^{x+y} = e^x \cdot e^y$

$\inf(e^x) = 0$ $\qquad (e^x)^y = e^{xy}$

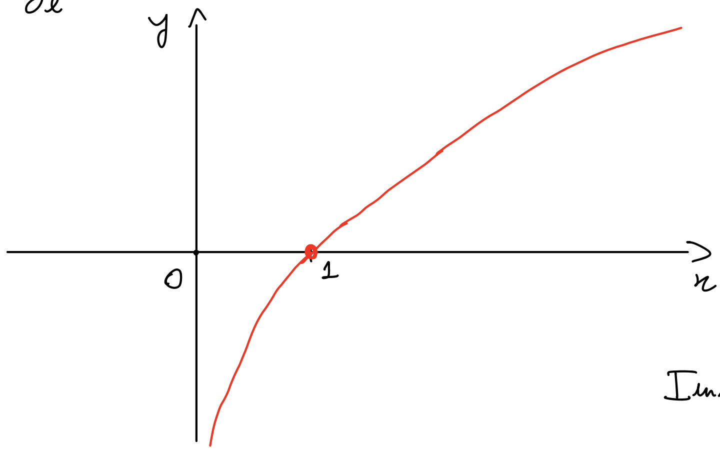
$\sup(e^x) = +\infty$

La funzione esponenziale è monotona strettamente crescente, ovvie se $x \neq y$, $x, y \in D$, $x < y$, allora $e^x < e^y$.

Funzione logaritmo

$f(x) = \log_e x$ $\quad [\log_a x, \ a > 0]$, $D = (0, +\infty)$

$\log_e x$ è il numero reale tale che $e^{(\log_e x)} = x$



$\log_e 1 = 0$, $e^0 = 1$

$\log_e e = 1$, $e^1 = e$

$\log_e \frac{1}{e} = -1$, $e^{-1} = \frac{1}{e}$

$\operatorname{Imm}(\log_e x) = \mathbb{R}$

La funzione logaritmo è strettamente monotone crescente.

$$\log_e(xy) = \log_e x + \log_e y \,, \quad \log_e(x^y) = y \cdot \log_e x$$

$$xy = e^{\log_e(xy)} \,, \quad e^{(\log_e x + \log_e y)} = e^{\log_e x} \cdot e^{\log_e y} = xy$$

$$x^y = e^{\log_e(x^y)} \,, \quad e^{y \cdot \log_e x} = \left(e^{\log_e x}\right)^y = x^y$$