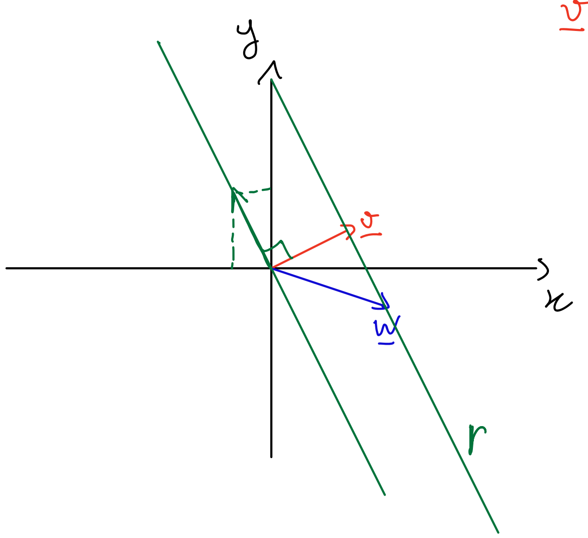
Esercizio

Trovare la retta in $\mathbb{R}^2$ ortogonale a $\underset{\underline{v}}{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}$ e passante per il punto $\underset{\underline{w}}{\begin{pmatrix} 3 \\ -1 \end{pmatrix}}$.



- r sottospazio affine di dim 1.

$$r = \underline{u}_1 + \text{Span}(\underline{u}_2), \quad \underline{u}_1, \underline{u}_2 \in \mathbb{R}^2$$

$$\underline{u}_1 = \underline{w} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \quad \underline{u}_2 \text{ è tale che } \langle \underline{u}_2, \underline{v} \rangle = 0$$

$$\underline{u}_2 = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \langle \underline{u}_2, \underline{v} \rangle = \left\langle \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\rangle = 2x + y = 0$$

$$\Rightarrow \underline{u}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$r = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \text{Span}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

OSS r è sottosp. vettoriale? $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in r$?

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in r \Leftrightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} \in \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \text{Span}\begin{pmatrix} -1 \\ 2 \end{pmatrix} \Leftrightarrow \exists\, \lambda \in \mathbb{R} \text{ tale che}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 - \lambda \\ -1 + 2\lambda \end{pmatrix} \longrightarrow \lambda = 3 \Downarrow -1 + 6 = 5 \neq 0 \qquad \underline{\text{NO}}$$

- r è soluzione di $ax + by = c$, $a, b, c \in \mathbb{R}$.

$$A = (a \ b) \quad C = (a \ b \mid c) \quad \dim W = 2 - \text{rango } A = 1$$

$$\Rightarrow \text{rango } A = 1, \text{ quindi } a \neq 0 \text{ opp. } b \neq 0.$$

W sia sottosp. affine non vettoriale, $\underline{0} \notin W$.

Quindi $c \neq 0$.

$\text{Span}\begin{pmatrix}-1\\2\end{pmatrix}$ = soluzioni di $ax+by=0$

$\begin{pmatrix}3\\-1\end{pmatrix}$ è una soluzione di $ax+by=c$

Scelgo $a,b \in \mathbb{R}$ (non entrambi uguali a 0) tali che

$$-a+2b=0$$

Ad esempio $a=2$, $b=1$ (ossia $\underline{v}=\begin{pmatrix}2\\1\end{pmatrix}$).

Adesso, scelgo $c \neq 0$ tale che $2\cdot 3+1\cdot(-1)=c$

quindi $c=5$.

r è l'insieme delle soluzioni di $2x+y=5$.

Es Trovare il piano in $\mathbb{R}^3$ ortogonale a $\underline{v}=\begin{pmatrix}1\\1\\-1\end{pmatrix}$ e passante per $\underline{w}=\begin{pmatrix}2\\-1\\3\end{pmatrix}$

π: $ax+by+cz=d$

$\underline{w}+\text{Span}(\underline{u}_1,\underline{u}_2)$, $\underline{u}_1,\underline{u}_2 \in \mathbb{R}^3$, lin. indip.

- $\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1\\1\\-1\end{pmatrix}=\underline{v}$, $d = a\cdot 2+b\cdot(-1)+c\cdot 3$

$$= 1\cdot 2+1\cdot(-1)+(-1)\cdot 3$$
$$= -2$$

$$x+y-z=-2$$

- $A=\begin{pmatrix}1 & 1 & -1\end{pmatrix}$ $C=\left(\begin{array}{ccc|c}1 & 1 & -1 & -2\end{array}\right)$

$$\begin{cases} x + y - z = -2 \\ y = \lambda_1 \\ z = \lambda_2 \end{cases} \qquad \begin{cases} x = -2 - \lambda_1 + \lambda_2 \\ y = \lambda_1 \\ z = \lambda_2 \end{cases}$$

$$\pi = W = \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix} + \text{Span}\left(\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right)$$

• $\pi = \underline{w} + \text{Span}(\underline{u}_1, \underline{u}_2)$ dove $\underline{w} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ e trovo

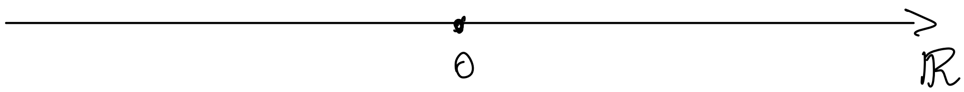
$\langle \underline{u}_1, \underline{v} \rangle = 0$, $\langle \underline{u}_2, \underline{v} \rangle = 0$ con $\underline{u}_1, \underline{u}_2$ lin indip.

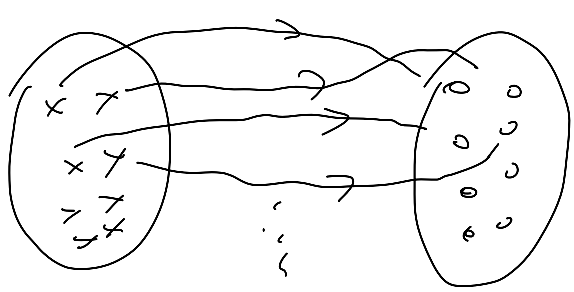
$\underline{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$. $\left\langle \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\rangle = 0$

$x + y - z = 0$ $\quad \underline{u}_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

ANALISI MATEMATICA

$\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$





$\# \mathbb{N} \overset{?}{=} \# \mathbb{Z}$

$1 \longleftrightarrow 2$
$3 \longleftrightarrow 4$
$5 \longleftrightarrow 6$
$7 \longleftrightarrow 8$
$9 \longleftrightarrow 10$
$\vdots \qquad \vdots$

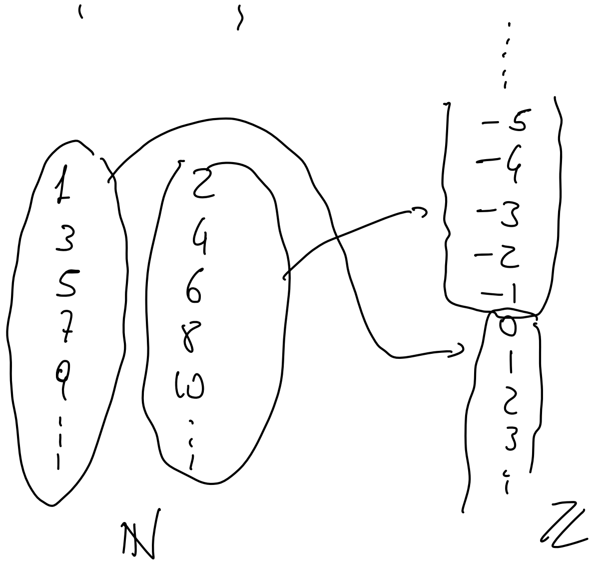
$\# \text{dispari} = \# \text{pari}$

$\# \text{dispari} = \# \mathbb{N}$

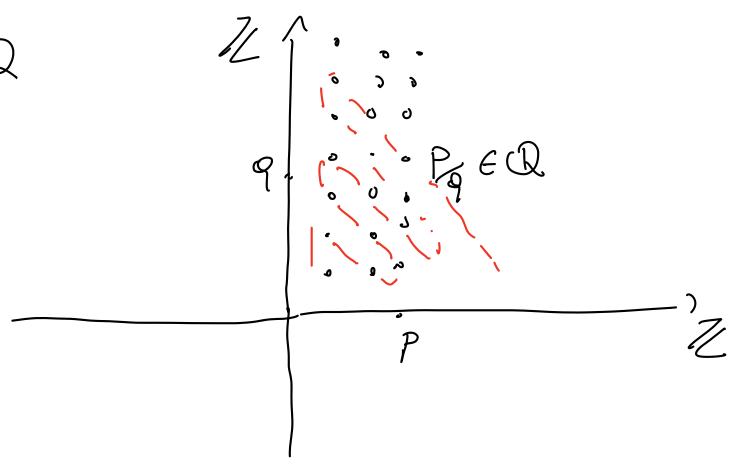
$$\begin{array}{ccc} 1 & \rightarrow & 1 \\ 3 & \rightarrow & 2 \\ 5 & \rightarrow & 3 \\ 7 & \rightarrow & 4 \\ 9 & \rightarrow & 5 \\ 11 & \rightarrow & 6 \\ \vdots & & \vdots \end{array}$$

$2 \cdot n - 1$

$\# \mathbb{N} = \# \mathbb{Z}$



$\# \mathbb{Z} = \# \mathbb{Q}$



$(\# \mathbb{N})^2 = \# \mathbb{Q} \lneq \# \mathbb{R} = 10^{\# \mathbb{N}}$