ES. 1

- $T = \{$curati con farmaci tradizionali$\}$, $N = \{$curati con farmaci nuovi$\}$

$P(T) = 2/3$, $P(N) = 1/3$ T, N alternative

- $Q = \{$pazienti senza sintomi entro 4 giorni$\}$

$P(Q \mid T) = 0.36$, $P(Q) = 0.41$

(a) $P(Q \mid N) = ?$

$$P(Q) = P(Q \mid T)P(T) + P(Q \mid N)P(N)$$

$$\Rightarrow P(Q \mid N) = \frac{P(Q) - P(Q \mid T)P(T)}{P(N)} = \frac{0.41 - 0.36 \cdot 2/3}{1/3} = 0.51$$

(b)

$$P(T \mid Q) = \frac{P(Q \mid T)P(T)}{P(Q)} \quad , \quad P(N \mid Q) = \frac{P(Q \mid N)P(N)}{P(Q)}$$

$$P(T \mid Q) = \frac{0.36 \cdot \frac{2}{3}}{0.41} \sim 0.585 \quad , \quad P(N \mid Q) = \frac{0.51 \cdot \frac{1}{3}}{0.41} \sim 0.415$$

$$P(T \mid Q) > P(N \mid Q)$$

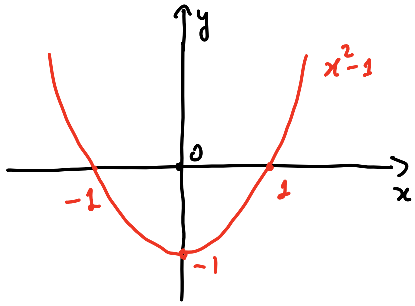
ES. 2

$$F(x) = \begin{cases} 0, & x < a \\ x^2 - 1, & a \le x < b \\ 1, & b \le x \end{cases} \qquad a, b \in \mathbb{R}, \; b > a$$

(a) • $\lim_{x \to -\infty} F(x) = 0$, $\lim_{x \to +\infty} F(x) = 1$ ok

- F debolmente crescente

- F continua da destra ok



F debolmente crescente su (a,b) $\Rightarrow$ $b > a \geq 0$

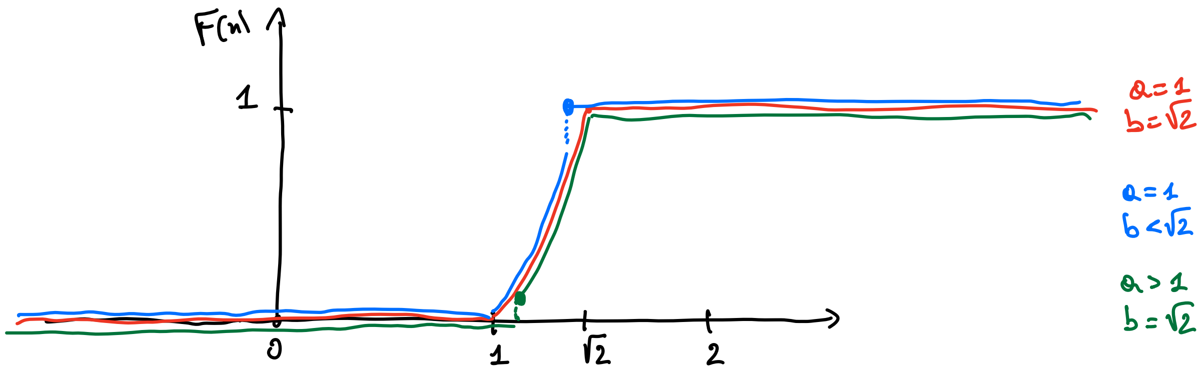
F " " su $(-\infty, a)$ $\Rightarrow$ $F(a) = a^2 - 1 \geq 0$

$\Rightarrow$ $a \geq 1$

F " " su $(b, +\infty)$ $\Rightarrow$ $\lim_{x \to b^-} F(x) = b^2 - 1 \leq 1$

$\Rightarrow$ $b \leq \sqrt{2}$

In conclusione $1 \leq a < b \leq \sqrt{2}$



Se la v.a. deve avere densità, si deve aggiungere che F sia continua, quindi

$$F(a) = \lim_{x \to a^+} F(x) = 0 \quad e \quad \lim_{x \to b^-} F(x) = F(b) = 1$$

$$\Rightarrow \quad a^2 - 1 = 0 \quad e \quad b^2 - 1 = 1 \quad \Rightarrow \quad a = 1,\ b = \sqrt{2}$$

(b) La v.a. hanno densità

$$f(x) = \begin{cases} 2x, & 1 < x < \sqrt{2} \\ 0, & alt. \end{cases} \quad , \ a = 1,\ b = \sqrt{2}$$

$$E[X^n] = \int_{-\infty}^{+\infty} x^n f(x)\, dx = \int_1^{\sqrt{2}} 2x^{n+1}\, dx = \frac{2}{n+2} x^{n+2} \Big|_1^{\sqrt{2}} = \frac{4 \cdot \sqrt{2}^n - 2}{n+2}$$

$$P\left(108 < X_1 + X_2 + \dots + X_{90}\right) = P\left(\frac{108 - 90 \cdot E[X]}{\sqrt{90 \cdot Var(X)}} < \frac{X_1 + X_2 + \dots + X_{90} - 90 \cdot E[X]}{\sqrt{90\, Var(X)}}\right) \sim$$

$$Var(X) = E[X^2] - \left(\frac{4\sqrt{2} - 2}{3}\right)^2 = \frac{3}{2} - \frac{32 + 4 - 16\sqrt{2}}{9} = \frac{3}{2} - 4 + \frac{16}{9}\sqrt{2}$$

$$= \frac{16}{9}\sqrt{2} - \frac{5}{2}$$

$$90 \cdot E[X] = \frac{4\sqrt{2}-2}{3} \cdot 90 = 60(2\sqrt{2}-1) \sim 109.706$$

$$90 \cdot Var(X) = \left(\frac{16}{9}\sqrt{2}-\frac{5}{2}\right) \cdot 90 = 160\sqrt{2} - 225 \sim 1.274 \qquad \sqrt{90\, Var(X)} \sim 1.129$$

$$\underset{\substack{\downarrow \\ Z \sim N(0,1)}}{\sim} P(-1.511 < Z) \underset{\substack{\downarrow \\ \text{simmetria}}}{=} P(Z < 1.511) \sim 0.9346$$

ES. 3 $\quad X_1, \dots, X_{41}$, $\quad \bar{\sigma}^2 = 0.145$, $\quad n = 41$

(a) $H_0)\ \sigma^2 \leq 0.1 = \sigma_0^2$ al livello $\alpha = 0.05$

$$C = \left\{ (n-1)\frac{S^2}{\sigma_0^2} > \chi^2_{(1-\alpha, n-1)} \right\} \quad \text{regione critica}$$

L'ipotesi va accettata al livello 0.05 $\iff 40 \dfrac{\bar{\sigma}^2}{\sigma_0^2} \leq \chi^2_{(0.95, 40)}$

$$\iff 40 \frac{0.145}{0.1} \leq 55.7585 \iff 58 \leq 55.7585 \quad \text{FALSO}$$

L'ipotesi non si può accettare a questo livello. In particolare $\bar{\alpha} < 0.05$.

(b) Per quali $\bar{\sigma}^2$, l'ipotesi sarebbe stata accettata al livello 0.1?

$$40 \frac{\bar{\sigma}^2}{\sigma_0^2} \leq \chi^2_{(0.9, 40)} \iff 400\, \bar{\sigma}^2 \leq 51.805 \iff \bar{\sigma}^2 \lessapprox 0.13$$

ES. 3 Vecchio programma

$$P = \begin{pmatrix} 1/3 & 2/3 & 0 & 0 \\ 0 & 1/3 & 2/3 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}$$

(a)

$$\begin{cases} (a\ b\ c\ d)\,P = (a\ b\ c\ d) \\ a+b+c+d=1 \\ a,b,c,d \geq 0 \end{cases} \iff \begin{cases} \frac{1}{3}a+\frac{1}{2}c+\frac{1}{4}d=a \\ \frac{2}{3}a+\frac{1}{3}b+\frac{1}{4}d=b \\ \frac{2}{3}b+\frac{1}{4}d=c \\ \frac{1}{2}c+\frac{1}{4}d=d \\ a+b+c+d=1 \\ a,b,c,d\geq 0 \end{cases}$$

$$\iff \begin{cases} (a,b,c,d)=\left(\frac{3}{2}d,\ \frac{15}{8}d,\ \frac{3}{2}d,\ d\right) \\ a+b+c+d=1 \\ a,b,c,d\geq 0 \end{cases} \iff$$

$$\iff (a,b,c,d)=\left(\frac{12}{47},\frac{15}{47},\frac{12}{47},\frac{8}{47}\right)$$

(b)

$$X_0 = (1\ \ 0\ \ 0\ \ 0)$$

$$X_1 = X_0 P = \left(\frac{1}{3}\ \ \frac{2}{3}\ \ 0\ \ 0\right)$$

$$X_2 = X_1 P = \left(\frac{1}{9}\ \ \frac{4}{9}\ \ \frac{4}{9}\ \ 0\right)$$

$$P(X_2 \in \{1,2\} \mid X_0 = 1) = \frac{1}{9}+\frac{4}{9} = \frac{5}{9}.$$