Trovare l'inversa.

$f(x) = 3x + 2$

$f: \mathbb{R} \to \mathbb{R}$

scrivo

$y = 3x + 2$

risolvo in x

$y - 2 = 3x$

$\frac{y-2}{3} = x$

$f^{-1}(y) = \frac{y-2}{3}$

$f^{-1}(f(x)) = f^{-1}(3x+2) =$

$\frac{(3x+2) - 2}{3} = \frac{3x}{3} = x$

$f^{-1} \circ f = Id =$ identità

$$\mathrm{Id}: A \to A$$

$$\mathrm{Id}(x) = x$$

$$f(f^{-1}(y)) = y$$

Funzioni monotòne

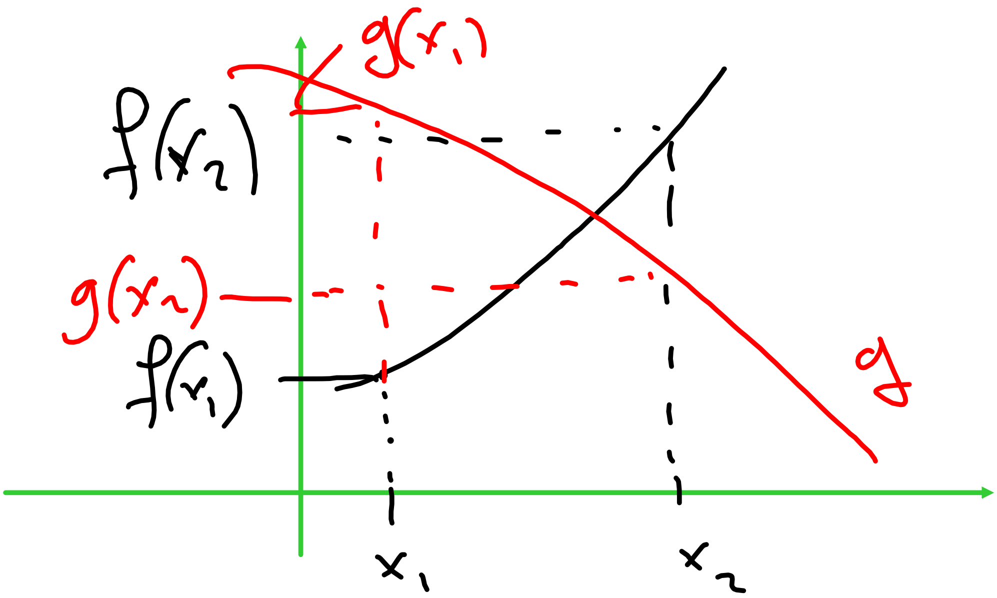
Def: $A, B \subset \mathbb{R}$

Se $\forall x_1, x_2 \in A$ con $x_1 < x_2$ risulta

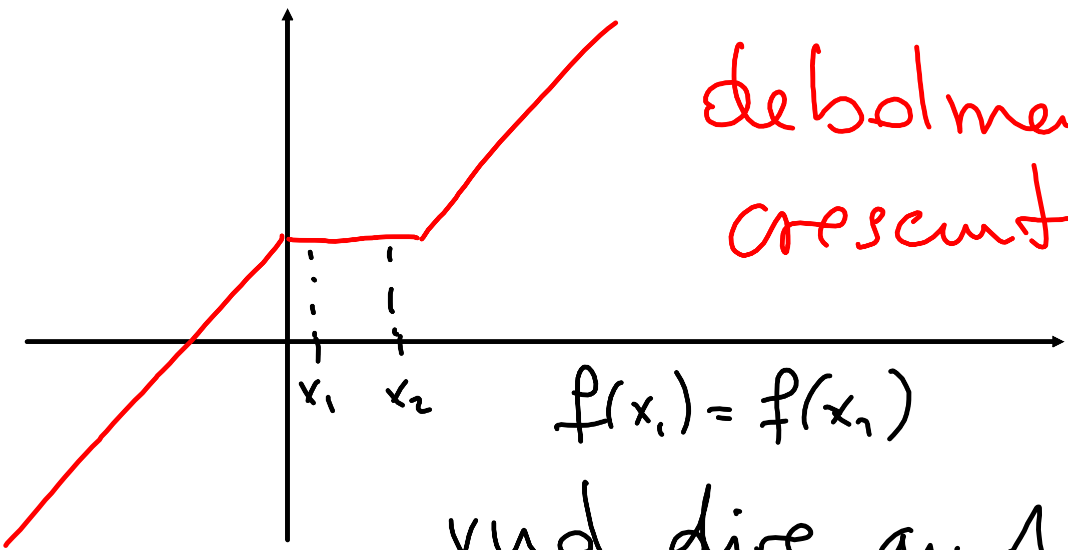
1) $f(x_1) < f(x_2)$ f si dice strettamente crescente
2) $f(x_1) \leq f(x_2)$ f si dice debolmente crescente
3) $f(x_1) > f(x_2)$ strettamente decrescente
4) $f(x_1) \geq f(x_2)$ debolmente decrescente

Se si verificano 1) o 3) f si dice strettamente monotòna

Se si verificano 2) o 4) debolmente monotòna



g(x1)
f(x2)
g(x2)
f(x1)
g
x1
x2



debolmente crescente.

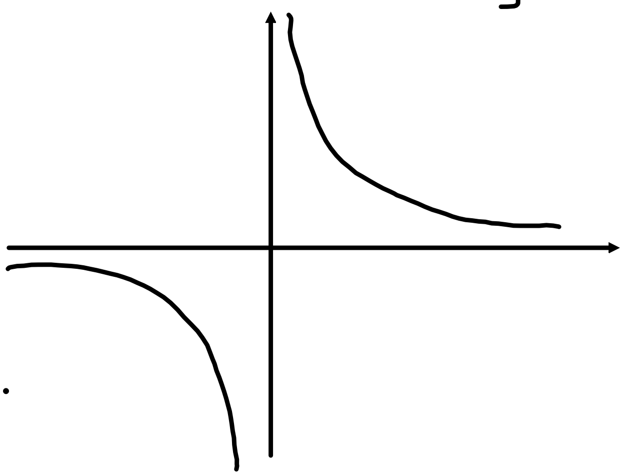
$$f(x_1) = f(x_2)$$

vuol dire anche

$$f(x_1) \leq f(x_2)$$

Es: $f : \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{0\}$

$f(x) = \frac{1}{x}$



f è strett. decresc. in $(-\infty, 0)$

f è strett. decresc. in $(0, +\infty)$

f è decrescente in $\mathbb{R} \setminus \{0\}$?

NO in fatti

$x_1 = -1 \qquad x_2 = 3 \qquad x_1 < x_2$

$f(x_1) = \frac{1}{-1} = -1 \qquad f(x_2) = \frac{1}{3}$

$f(x_1) < f(x_2)$

Composizione di funzioni monotone.

<u>Prop</u>: $A, B, C \subset \mathbb{R}$

$f : A \to B \qquad g : B \to C$

Allora

1) Se f è crescente e g è crescente $\Rightarrow g \circ f$ è cresc.

2) Se f ē cresente e g
ē decresente $\Rightarrow$ $g \circ f$ ē decr.
(e viceversa)
3) Se f ē decresente e
g ē decresente $\Rightarrow$ $g \circ f$ ē cresc.

Es: $h(x) = e^{\operatorname{arctg} x}$

$f : \mathbb{R} \longrightarrow \mathbb{R} \quad f(x) = \operatorname{arctg} x$

$g(t) : \mathbb{R} \longrightarrow \mathbb{R} \quad g(t) = e^{t}$

$$h = g \circ f$$

$$x \overset{f}{\longmapsto} \operatorname{arctg} x \overset{g}{\longrightarrow} e^{\operatorname{arctg} x}$$

$$\underbrace{x \longrightarrow e^{\operatorname{arctg} x}}_{h}$$

f e g sono crescenti

$\Rightarrow$ h è crescente.

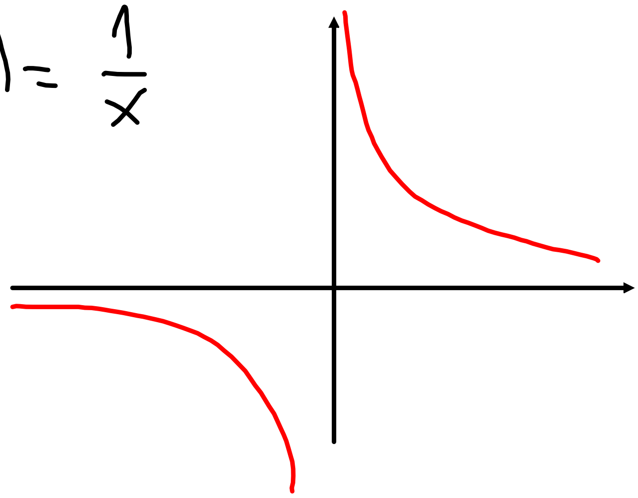
Oss: Se f è strettamente monotona allora f è iniettiva.

Il viceversa è vero?

Se f è iniettiva

$\Rightarrow$ f è monotona? NO

Es: $f(x) = \frac{1}{x}$



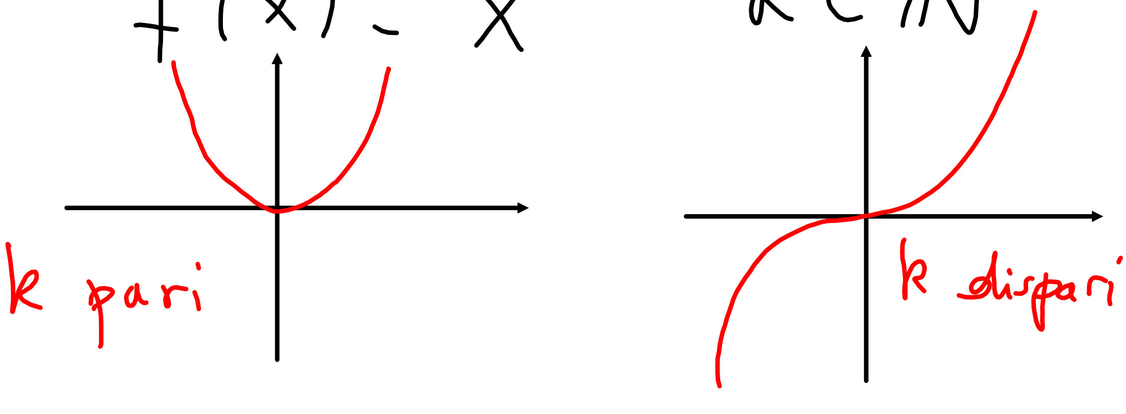
è iniettiva
ma non
monotona

Funzioni elementari

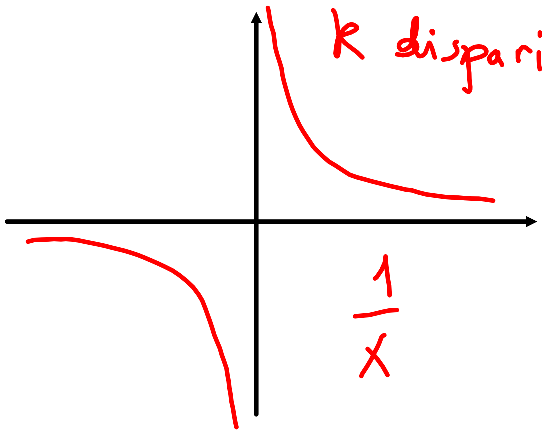
$$f(x) = ax + b \quad , a, b \in \mathbb{R}$$

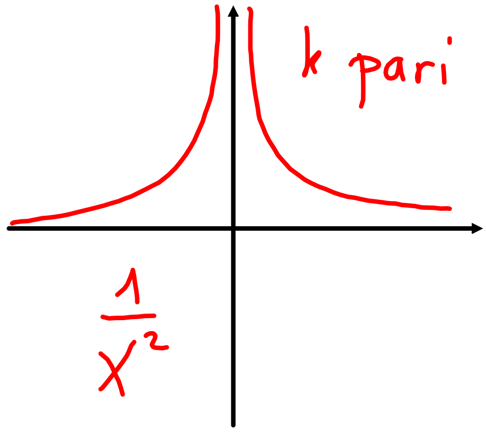
retta.

$$f(x) = x^k \qquad k \in \mathbb{N}$$



$$f(x) = x^k \quad k \in \mathbb{Z},\ k < 0$$





$$f(x) = x^{p/q} \quad p,q \in \mathbb{N}$$

$$q \neq 0$$

p, q non entrambi pari.

Dominio di f?

$$x^{p/q} = \sqrt[q]{x^p}$$

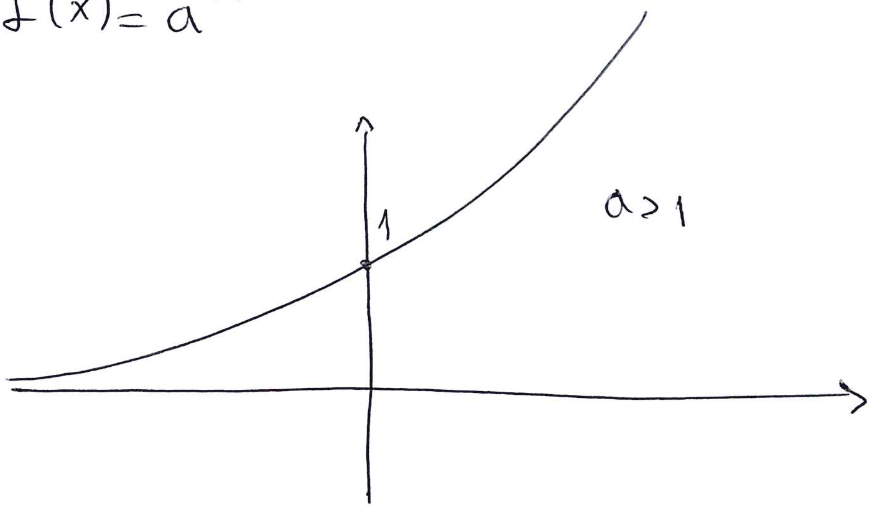
Se q è dispari
dominio $= \mathbb{R}$
Se q è pari
dominio $= [0, +\infty)$

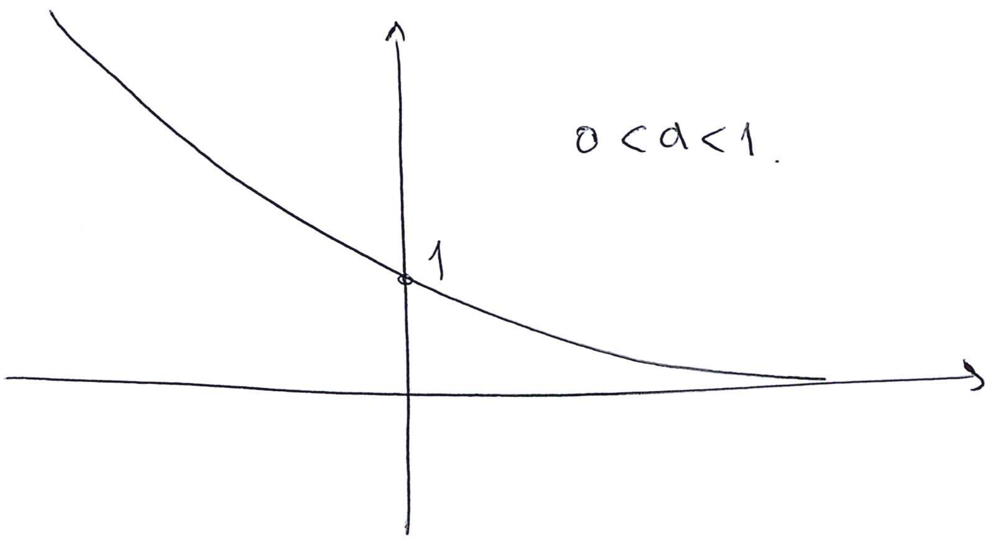
$$\sqrt{0} = 0$$

Funzione esponenziale

$a \in \mathbb{R}$, $a > 0$, $a \neq 1$.

$f(x) = a^x$





strettamente crescente se $a > 1$
strettamente decrescente se $0 < a < 1$.

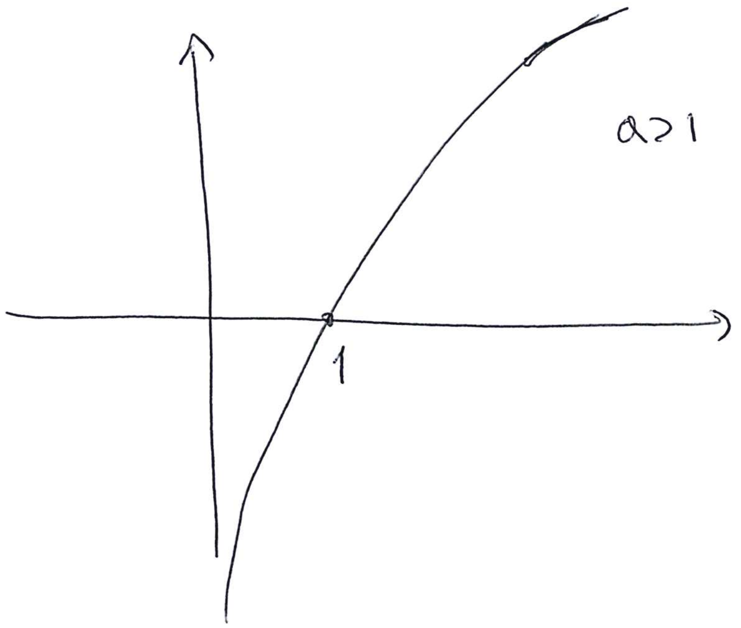
$a^x > 0 \quad \forall x \in \mathbb{R}$.

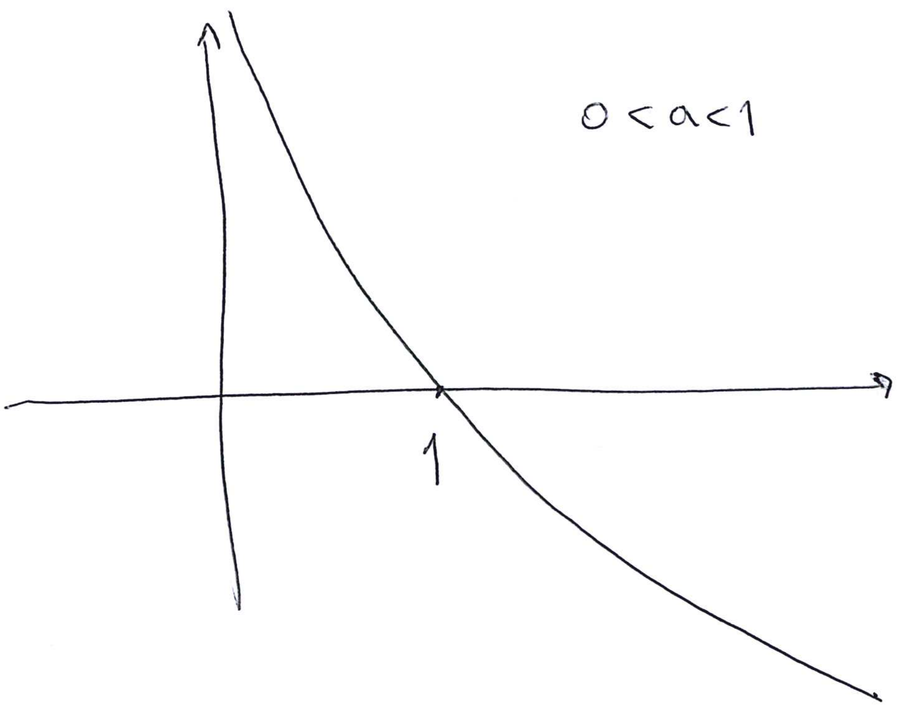
$f(x) = a^x$, $f: \mathbb{R} \to (0, +\infty)$

è invertibile.

la funzione inversa si chiama logaritmo in base a

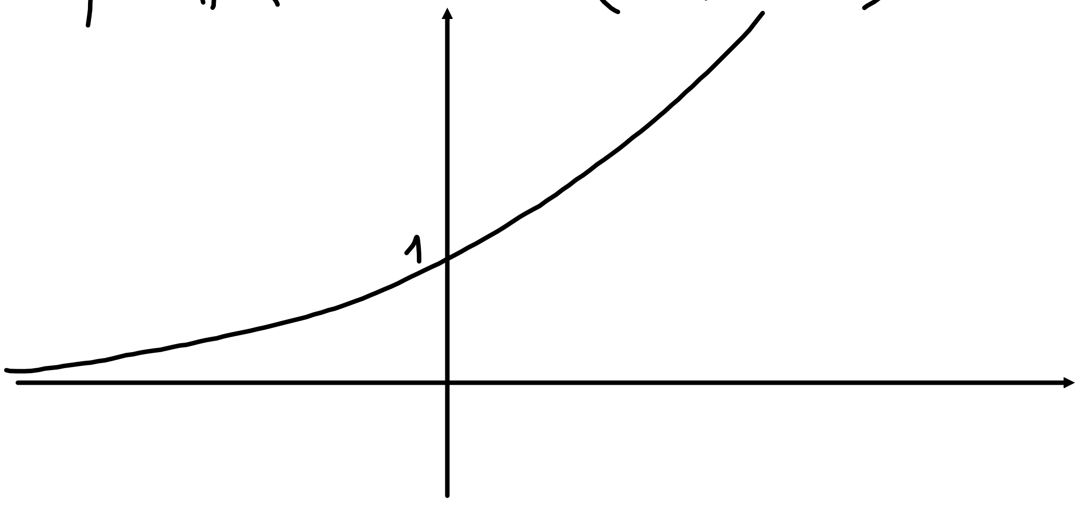
$\log_a : (0,+\infty) \longrightarrow \mathbb{R}$





$$f(x) = e^x$$

$$f : \mathbb{R} \longrightarrow (0, +\infty)$$



e^x è invertibile

la sua inversa è il logaritmo (naturale)

$\log : (0, +\infty) \longrightarrow \mathbb{R}$

Cambio di base

$$\log_a x = y \iff a^y = x$$

faccio il logaritmo naturale

$$y \log a = \log x$$

$$\log_a x \cdot \log a = \log x$$

$$\log_a x = \frac{\log x}{\log a}$$

$$f(x) = x^{\alpha} \qquad \alpha \in \mathbb{R}$$

$$\alpha \notin \mathbb{Q}.$$

$$x^{\pi}, \quad x^{\sqrt{2}}, \ldots$$

Def: $x^{\alpha} = e^{\alpha \log x}$

$$e^{\alpha \log x} = \left(e^{\log x}\right)^{\alpha}$$

$$= x^{\alpha}$$

Dominio di x^{α}

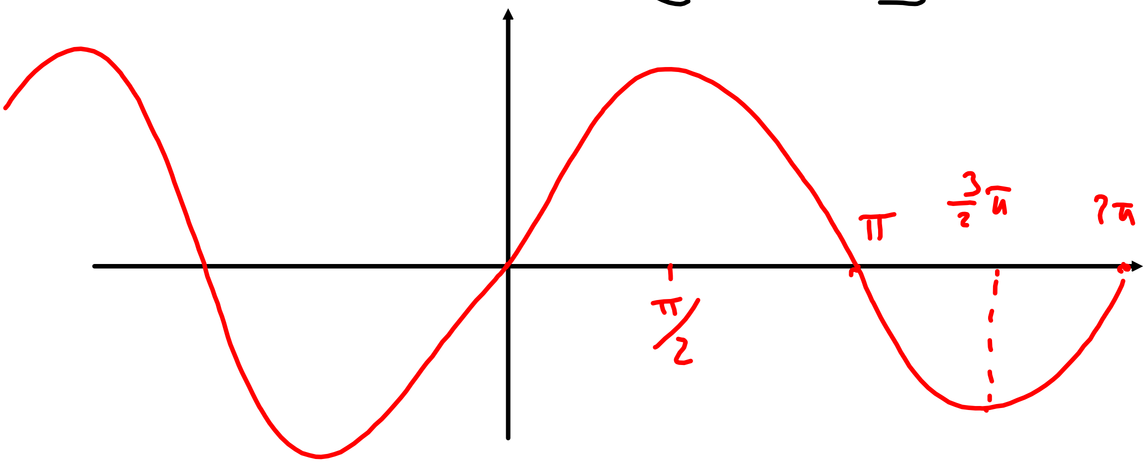
se $\alpha \notin \mathbb{Q}$

dominio $= (0, +\infty)$

per via del logaritmo

$$f(x) = \sin x$$

$$f: \mathbb{R} \longrightarrow [-1, 1]$$



ē periodica di periodo
2π cioē

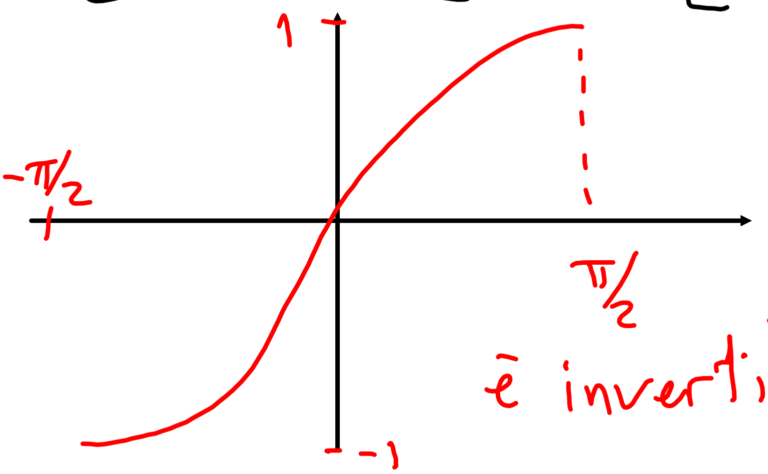
$$f(x+2\pi) = f(x) \quad \forall x$$

$$\sin\left(\frac{\pi}{2}\right) = \sin\left(\frac{5}{2}\pi\right)$$

ē invertibile ?

$f(x) = \sin x$

$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$

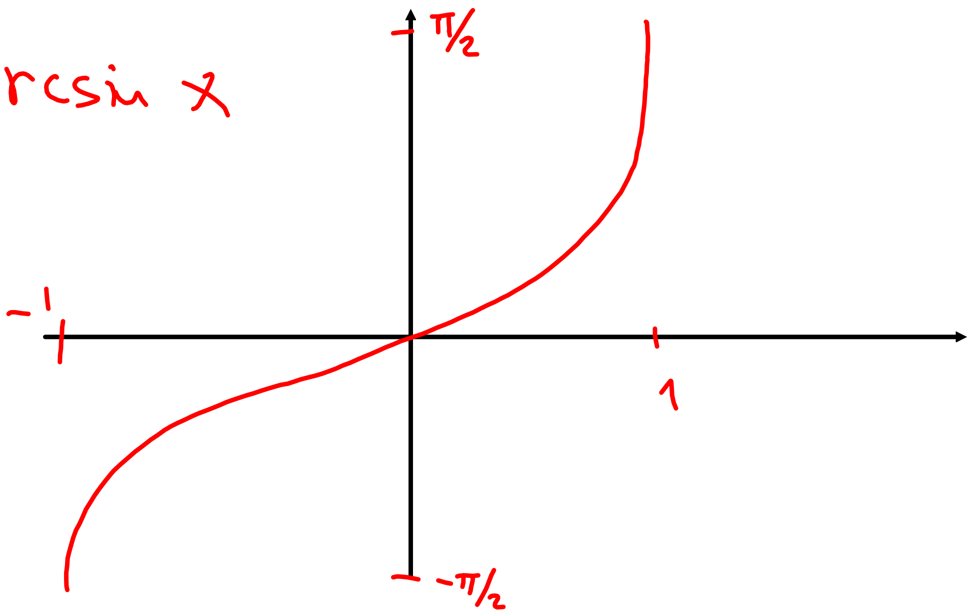


è invertibile

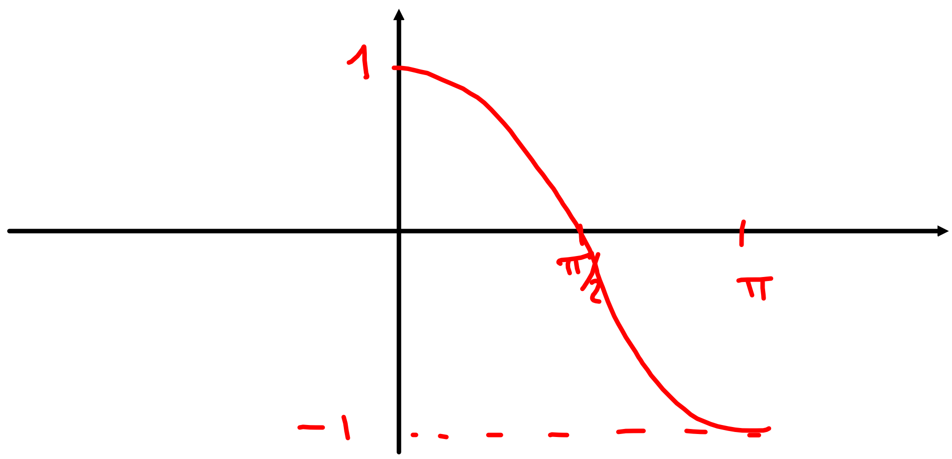
$$\arcsin : [-1,1] \longrightarrow [-\pi/2, \pi/2]$$

La funzione arcoseno è l'inversa della funzione seno definita da $[-\frac{\pi}{2}, \frac{\pi}{2})$ a valori in $[-1,1]$

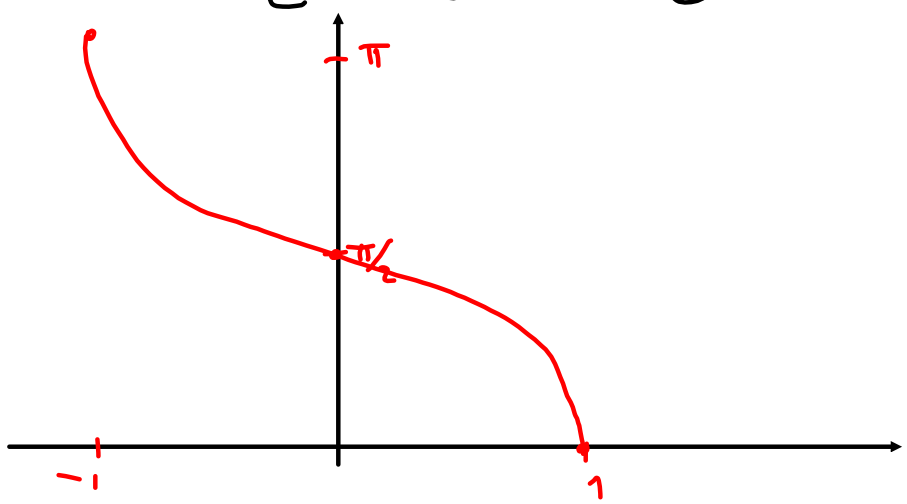
$\arcsin x$



$$\cos x : [0, \pi] \longrightarrow [-1, 1]$$



$$\arccos : [-1, 1] \longrightarrow [0, \pi]$$



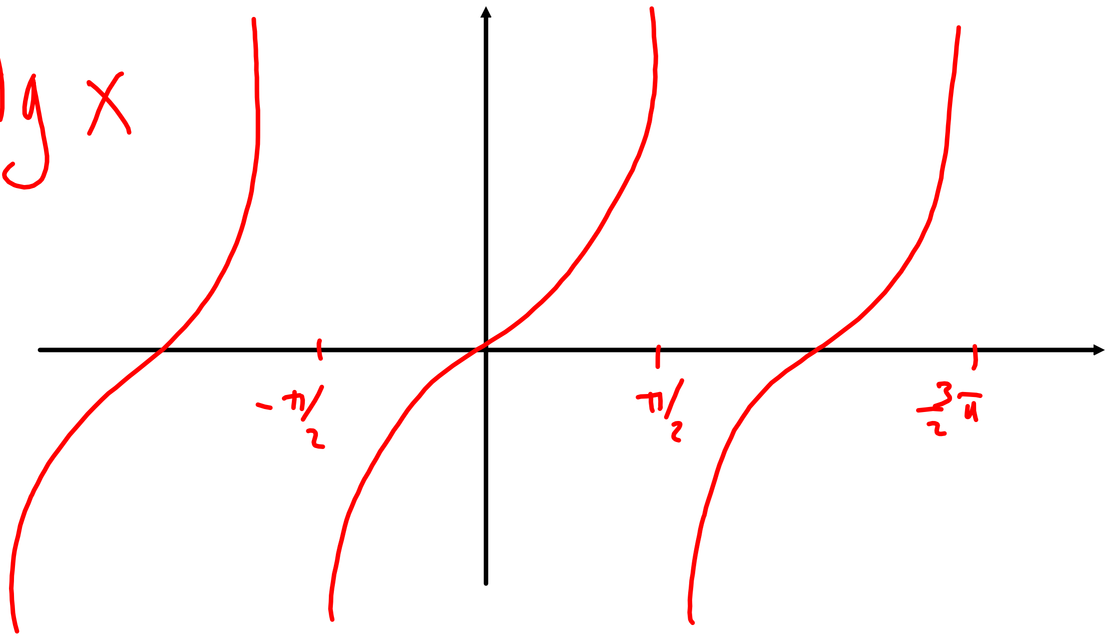
$$\operatorname{tg} x = \frac{\sin x}{\cos x}$$

non è definita se

$\cos x = 0$

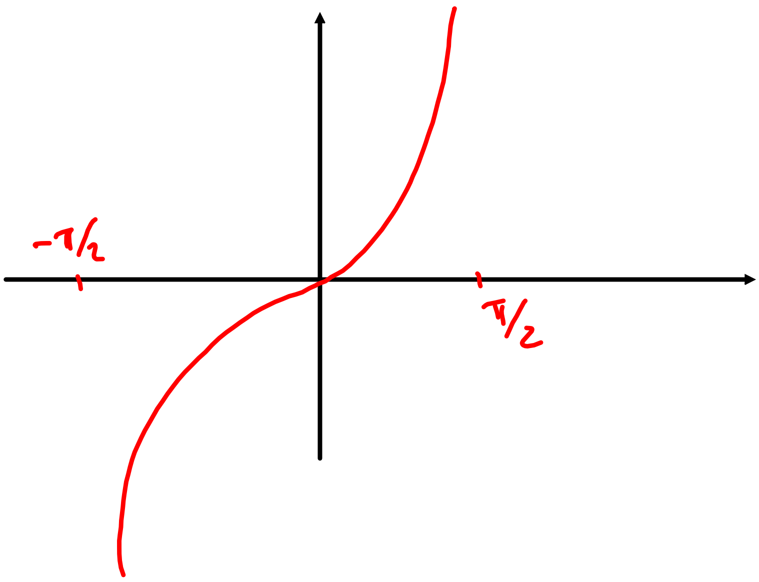
$$x \neq \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$$

tg x

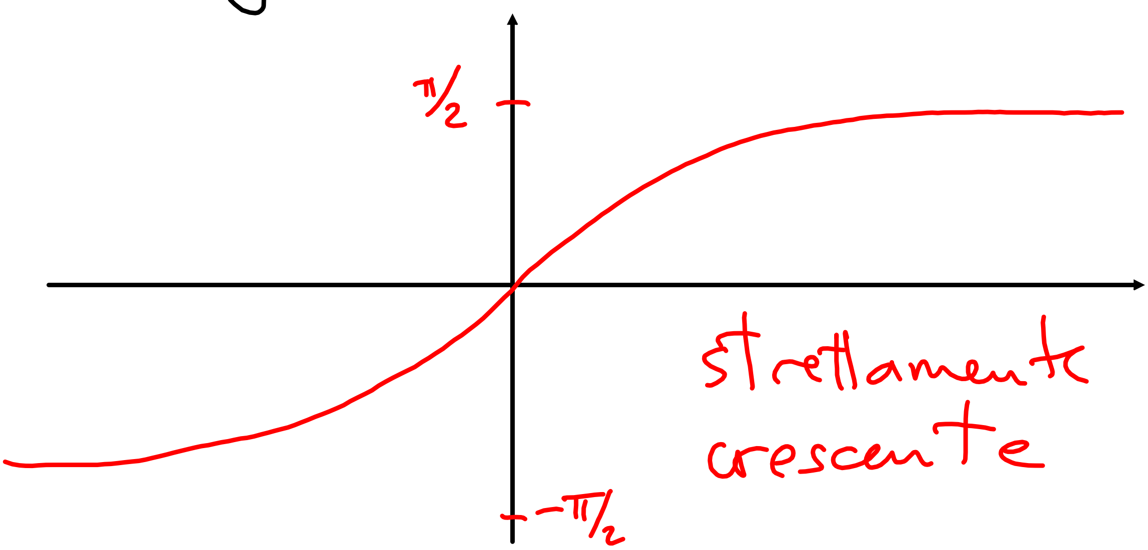


$$f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow \mathbb{R}$$

è invertibile



$$\operatorname{arctg} : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$



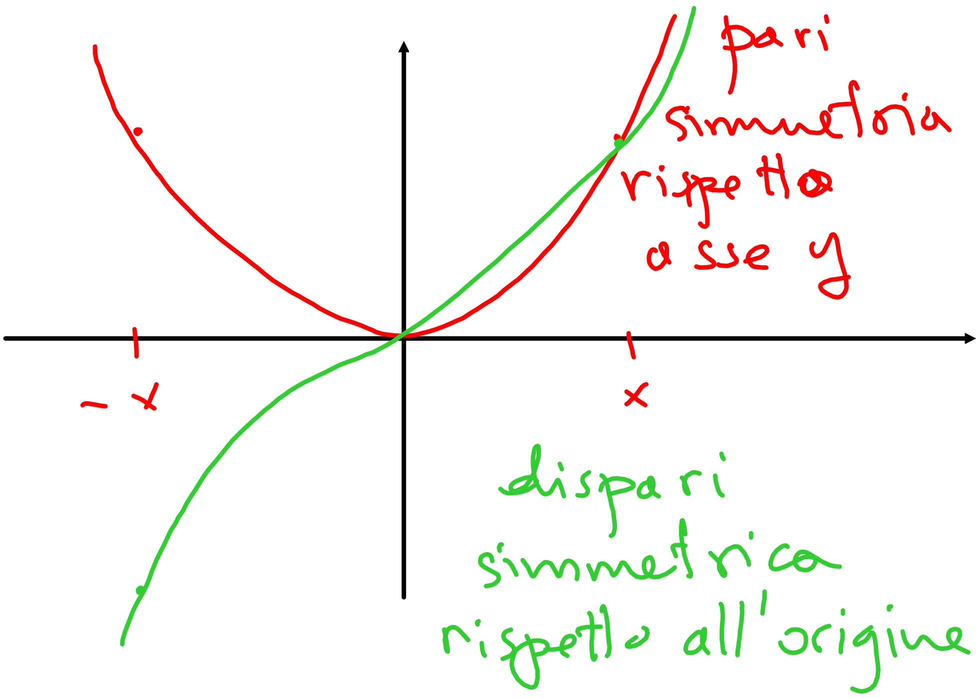
Def: $f : \mathbb{R} \to \mathbb{R}$

f si dice pari se

$f(-x) = f(x) \quad \forall x \in \mathbb{R}$

si dice dispari

se $f(-x) = -f(x) \; \forall x \in \mathbb{R}$


pari
simmetria
rispetto
asse y
-x
x
dispari
simmetrica
rispetto all'origine

$$f(x) = x^2$$

$$f(-x) = (-x)^2 = (-1)^2 x^2 = x^2$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \shortparallel$$

f è pari $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad f(x)$

$$f(x) = x^3$$

$$f(-x) = (-x)^3 = (-1)^3 x^3 = -1 \cdot x^3$$

$$= -x^3 = -f(x)$$

f è dispari