$$\begin{cases} y' = \dfrac{-2y}{x} + \dfrac{1}{x^2} \\ y(-1) = 3 \end{cases} \qquad y(-2) = ?$$

$$a(x) = -\frac{2}{x} \qquad b(x) = \frac{1}{x^2}$$

$$A(x) = \int -\frac{2}{x}\,dx = -2\log|x|$$

$$\int e^{-A(x)} b(x)\,dx = \int e^{2\log|x|} \frac{1}{x^2}\,dx$$

$$= \int |x|^2 \cdot \frac{1}{x^2}\, dx = \int 1\, dx = x + c$$

$$y(x) = e^{A(x)} \left(\int e^{-A(x)} b(x)\, dx + c \right) =$$

$$= e^{-2\log|x|} (x + c) = \frac{1}{x^2} (x + c)$$

$$\left(e^{\log|x|}\right)^{-2} = |x|^{-2} = \frac{1}{x^2}$$

trovo c $\quad y(-1) = 3$

$$3 = y(-1) = \frac{1}{(-1)^2}(-1+c)$$

$$3 = -1 + c \Rightarrow c = 4$$

$$y(x) = \frac{1}{x^2}(x+4)$$

$$y(-2) = \frac{1}{4}(-2+4) = \frac{1}{4} \cdot 2 = \frac{1}{2}.$$

dic 14-11:23

$$\lim_{n\to\infty} \sqrt[n]{\frac{2n}{3n^2+1}} =$$

$$= \lim_{n\to\infty} \sqrt[n]{\frac{n\cdot 2}{n^2\left(3+\frac{1}{n^2}\right)}} = \lim \sqrt[n]{\frac{1}{n}} \cdot \sqrt[n]{\underbrace{\frac{2}{3+\frac{1}{n^2}}}_{\to 2/3}}$$

$$= 1 \cdot 1$$

$$\sqrt[n]{n^\alpha} \to 1 \quad \forall \alpha .$$

$$a_n = n\left(\log(n^2+1) - 2\log n\right)\sin n$$

$$n\left(\log(n^2+1) - \log(n^2)\right)\sin n =$$

$$= n\log\left(\frac{n^2+1}{n^2}\right)\sin n =$$

$$= n\log\left(1+\frac{1}{n^2}\right)\sin n =$$

$$= n\left(\frac{1}{n^2} + o\left(\frac{1}{n^2}\right)\right)\sin n =$$

$$= \underbrace{\left(\frac{1}{n} + o\left(\frac{1}{n}\right)\right)}_{0} \underbrace{\sin n}_{\text{limitato.}} \longrightarrow 0$$

dic 14-11:29

$$a_n = \log(3n^2+1) + (-1)^n \log(n^2+2)$$

n pari:

$$\log(3n^2+1) + \log(n^2+2) \to +\infty$$

n dispari

$$\log(3n^2+1) - \log(n^2+2) =$$

$$= \log\left(\frac{3n^2+1}{n^2+2}\right) \to \log 3 .$$

$\{a_n\}$ non ha limite.

dic 14-11:31

$$a_n = (-1)^n \sqrt[n]{2n}.$$

$$\sqrt[n]{2n} \longrightarrow 1$$

n pari $\quad a_n = \sqrt[n]{2n} \longrightarrow 1$

n dispari $\quad a_n = -\sqrt[n]{2n} \longrightarrow -1$

non ha limite ma è limitata.

$$\begin{cases} y' = 1 + \dfrac{2y}{x} \\ y(1) = 3 \end{cases}$$

calcolare $y(2)$.

$$a(x) = \frac{2}{x} \qquad b(x) = 1$$

$$A(x) = \int \frac{2}{x}\,dx = 2\log|x|$$

$$\int e^{-A(x)} b(x)\,dx = \int e^{-2\log|x|} \cdot 1\,dx$$

dic 14-11:36

$$= \int \frac{1}{x^2}\,dx = -\frac{1}{x} + c$$

$$y(x) = e^{2\log|x|}\left(-\frac{1}{x} + c\right) =$$

$$= x^2\left(-\frac{1}{x} + c\right)$$

$$y(1) = 3$$

$$3 = 1(-1 + c) \Rightarrow c = 4$$

$$y(x) = x^2\left(-\frac{1}{x} + 4\right)$$

$$y(2) = 4\left(-\frac{1}{2} + 4\right) = -2 + 16 = 14.$$

dic 14-11:38

$$\lim_{n\to\infty} \frac{3^n + (-4)^n}{3^n + (-6)^n} = \lim_{n\to\infty} \frac{(-4)^n\left[\underbrace{\left(\frac{3}{-4}\right)^n}_{\to 0} + 1\right]}{(-6)^n\left[\underbrace{\left(\frac{3}{-6}\right)^n}_{\to 0} + 1\right]} = \lim \left(\frac{2}{3}\right)^n \to 0$$

Se $-1 < x < 1 \Rightarrow x^n \to 0.$

$$\begin{cases} y' = (3-y)(2-y) \\ y(0) = \frac{5}{2} \end{cases}$$

$$\lim_{x \to +\infty} y(x).$$

variabili separabili.

$$\int \frac{dy}{(3-y)(2-y)} = \int 1 \cdot dx + c$$

$$\frac{A}{3-y} + \frac{B}{2-y} = \frac{A(2-y) + B(3-y)}{(3-y)(2-y)}$$

$$= \frac{2A+3B-(A+B)y}{(3-y)(2-y)}$$

$$\begin{cases} 2A+3B=1 \\ -(A+B)=0 \end{cases} \Rightarrow B=-A$$

$2A-3A=1, \quad -A=1$

$A=-1$

$B=1$

$$\int \frac{dy}{(3-y)(2-y)} = \int \frac{-1}{3-y} + \frac{1}{2-y}\,dy =$$

$$+\log|3-y| - \log|2-y| = \log\left|\frac{3-y}{2-y}\right|$$

$$\log\left|\frac{3-y}{2-y}\right| = x + c \qquad y(0) = \frac{5}{2}$$

$$\log\left|\frac{3-\frac{5}{2}}{2-\frac{5}{2}}\right| = 0 + c$$

$$\log\left|\frac{\frac{1}{2}}{-\frac{1}{2}}\right| = 0 + \cancel{c} \Rightarrow c = 0$$

$$\log\left|\frac{3-y}{2-y}\right| = x \Rightarrow \left|\frac{3-y}{2-y}\right| = e^{x}$$

trovare una soluzione in un intorno di $y = 5/2$.

dic 14-11:49

Se $y = \frac{5}{2} \Rightarrow \frac{3-y}{2-y} = -1 < 0$

$\Rightarrow \left| \frac{3-y}{2-y} \right| = \frac{y-3}{2-y}$ in un intorno di $y = \frac{5}{2}$.

$\Rightarrow \frac{y-3}{2-y} = e^{x}$

$y-3 = e^{x}(2-y)$

$y-3 = 2e^{x} - ye^{x}$

dic 14-11:54

$$y + y e^{x} = 3 + 2e^{x}$$

$$y(1 + e^{x}) = 3 + 2e^{x}$$

$$y = \frac{3 + 2e^{x}}{1 + e^{x}} \longrightarrow 2$$

per $x \to \infty$.

$$\lim_{n\to\infty}\left(1+\frac{3}{\log n}\right)^{n}$$

$$\log(1+t) = t + o(t)$$

$$t \to 0$$

$$t = \frac{1}{\log n}$$

$$= e^{n\log\left(1+\frac{3}{\log n}\right)} = e^{n\left(\frac{3}{\log n}+o\left(\frac{1}{\log n}\right)\right)}$$

$$= e^{\frac{3n}{\log n}+o\left(\frac{n}{\log n}\right)}$$

$$= e \qquad =$$

$$= e^{\frac{3n}{\log n}\left[1 + o(1)\right]} \qquad \infty(1+0)$$

$$= e$$

$$= e^{\infty} = \infty .$$

$$\frac{3^n}{(\log n)^n} = \underbrace{\left(\frac{3}{\log n}\right)^n}_{< \left(\frac{1}{2}\right)^n \to 0} \to 0$$

$$\begin{cases} y' + x\left(1+2e^{x^2}\right)\cos^2 y = 0 \\ y(0) = \frac{\pi}{4} \end{cases}$$

$$y' = -x\left(1+2e^{x^2}\right)\cos^2 y$$

se $\cos^2 y \neq 0$ divido.

se $y = \frac{\pi}{4} \Rightarrow \cos^2 \frac{\pi}{4} \neq 0$.

dic 14-12:03

$$\int \frac{dy}{\cos^2 y} = -\int x\left(1+2e^{x^2}\right)dx + c$$

$$\operatorname{tg} y = -\int x + 2xe^{x^2}\, dx + c$$

$$= -\left(\frac{x^2}{2} + e^{x^2}\right) + c$$

$$y(0) = \frac{\pi}{4} \qquad \operatorname{tg}\frac{\pi}{4} = 1$$

$$1 = -(0 + e^{0}) + c$$

$$1 = -1 + c \Rightarrow c = 2$$

$$\operatorname{tg} y = -\frac{x^2}{2} - e^{x^2} + 2$$

$$y = \operatorname{arctg}\left(-\frac{x^2}{2} - e^{x^2} + 2\right).$$

dic 14-12:08

$$\begin{cases} y' = -x\left(1+2e^{x^2}\right)\cos^2 y \\ y(0) = \dfrac{\pi}{2} \end{cases}$$

avete la soluzione costante

$y(x) = \dfrac{\pi}{2}$. infatti $y' = 0$

$$\Rightarrow 0 = -x\left(1+2e^{x^2}\right)\cos^2\left(\frac{\pi}{2}\right) = 0.$$

dic 14-12:10

Es: $f(x) = \frac{3 - \log x}{1 + \log^2 x} - 1$

insieme di definizione.

$\boxed{x > 0}$

$$\lim_{x \to 0^+} \frac{3 - \log x}{1 + \log^2 x} - 1 = \frac{+\infty}{+\infty} - 1 \quad ?$$

dic 14-12:12

$$= \lim_{x\to 0^+} \boxed{\frac{\frac{3}{\log x} - 1}{\frac{1}{\log x} + \log x}} - 1 =$$

$\to 0$ anche per $x \to +\infty$.

$$= \frac{\frac{3}{-\infty} - 1}{\frac{1}{-\infty} - \infty} - 1 = \frac{0-1}{0-\infty} - 1 = 0 - 1 = -1$$

$$\lim_{x\to +\infty} \frac{3-\log x}{1+\log^2 x} - 1 = \frac{3-\infty}{1+\infty} - 1 \quad ?$$

$$= \lim_{x\to\infty} \frac{\frac{3}{\log x} - 1}{\frac{1}{\log x} + \log x} - 1 =$$

$$= \frac{\frac{3}{\infty} - 1}{\frac{1}{\infty} + \infty} - 1 = \frac{0-1}{0+\infty} - 1$$

$$= \frac{-1}{+\infty} - 1 = 0 - 1 = -1.$$

dic 14-12:19

asintoto orizzontale di equazione

$$y = -1.$$

$$f'(x) = \frac{-\frac{1}{x}\left(1+\log^2 x\right) - \left(3-\log x\right) 2\log x \cdot \frac{1}{x}}{\left(1+\log^2 x\right)^2}$$

$$= \frac{-1-\log^2 x - 6\log x + 2\log^2 x}{x\left(1+\log^2 x\right)^2}$$

dic 14-12:20

$$= \frac{\log^2 x - 6\log x - 1}{x\left(1+\log^2 x\right)^2} > 0 \ ?$$

$$\log x = t \qquad t^2 - 6t - 1 = 0$$

$$t = 3 \pm \sqrt{9+1} = 3 \pm \sqrt{10}$$

$$= \begin{cases} 3+\sqrt{10} \\ 3-\sqrt{10} < 0 \end{cases}$$

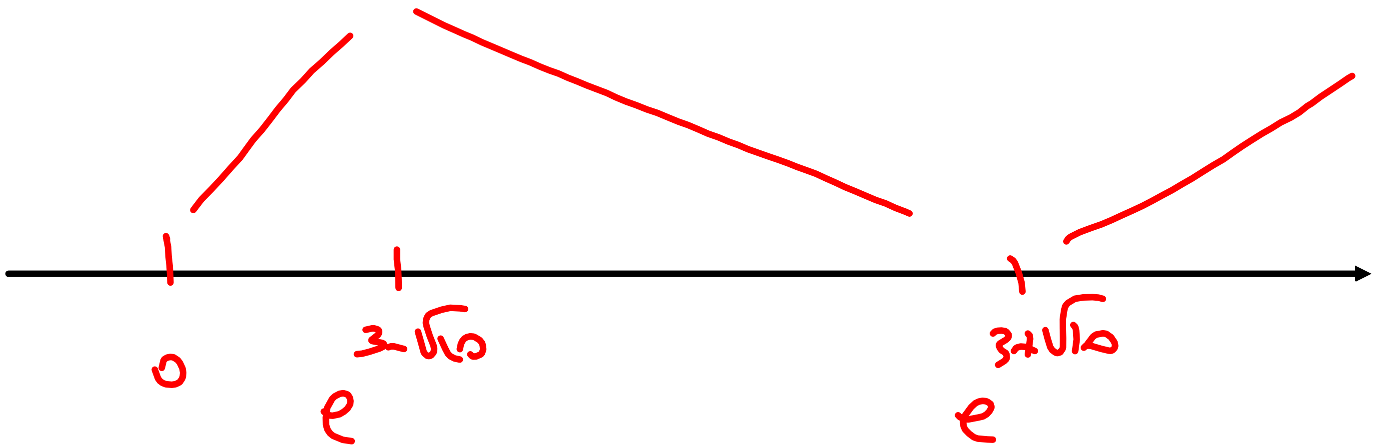
$$t^2 - 6t - 1 > 0 \iff t < 3-\sqrt{10}$$

oppure $t > 3+\sqrt{10}$.

cioè $\log x < 3-\sqrt{10}$

oppure $\log x > 3+\sqrt{10}$.

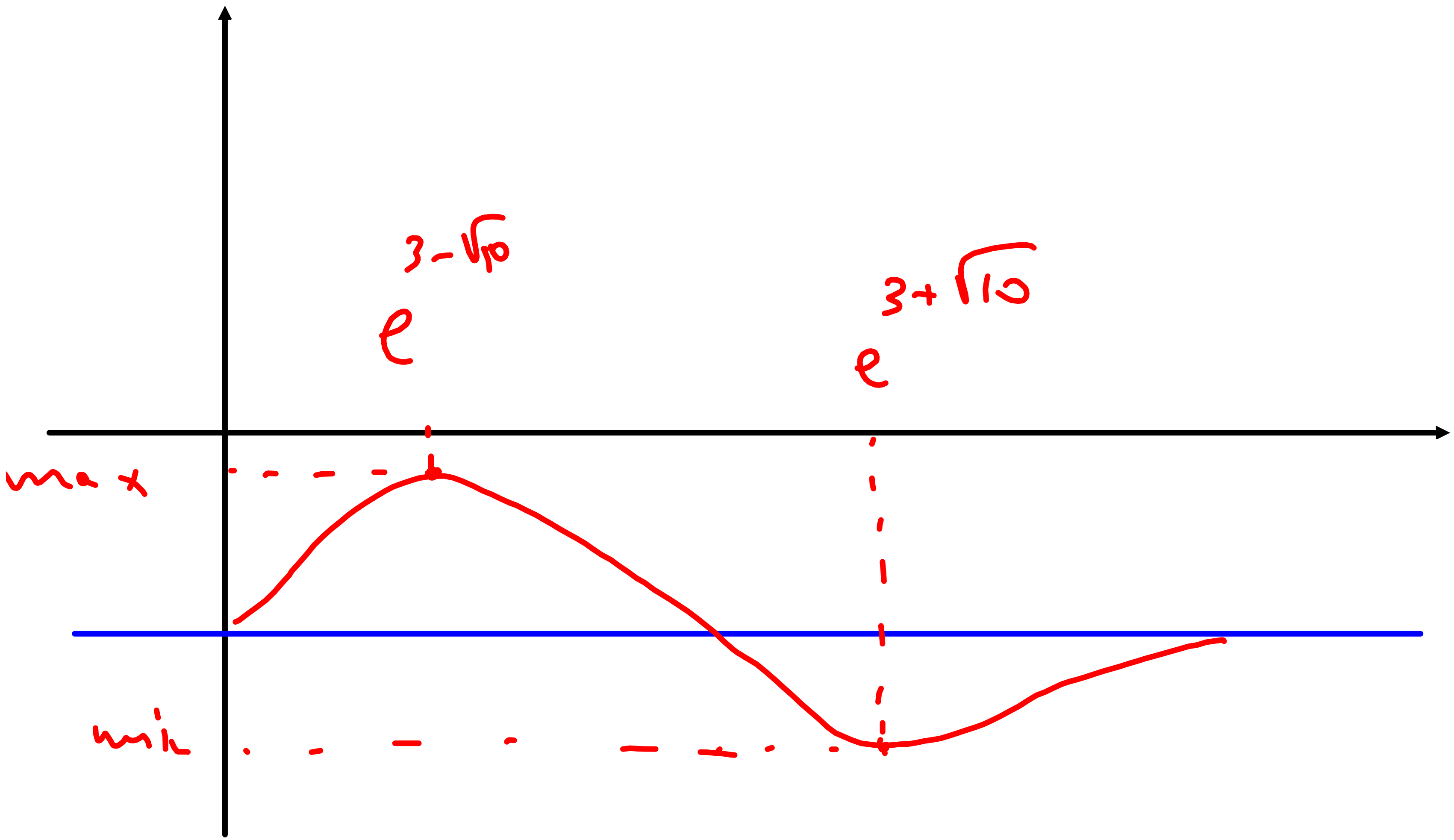
$$x < e^{3-\sqrt{10}} \quad \vee \quad x > e^{3+\sqrt{10}}$$



$x = e^{3-\sqrt{10}}$ è punto di max locale

$x = e^{3+\sqrt{10}}$ è punto di minimo locale.

dic 14-12:27



dic 14-12:29

$$\max f = f\left(e^{3-\sqrt{10}}\right) =$$

$$= \frac{3-(3-\sqrt{10})}{1+(3-\sqrt{10})^2} = \frac{\sqrt{10}}{1+9-6\sqrt{10}+10}$$

$$= \frac{\sqrt{10}}{20-6\sqrt{10}}.$$

$$\min f = f\left(e^{3+\sqrt{10}}\right) =$$

$$= \frac{3-(3+\sqrt{10})}{1+(3+\sqrt{10})^2} = \frac{-\sqrt{10}}{20-6\sqrt{10}} \; .$$

$$\lim_{n\to\infty} \frac{(2n)!\, 2^{2n}}{n^{2n}}$$

criterio del rapporto.

$$\frac{a_{n+1}}{a_n} = \frac{[2(n+1)]! \cdot 2^{2(n+1)}}{(n+1)^{2(n+1)}} \cdot \frac{n^{2n}}{(2n)!\, 2^{2n}}$$

$$= \frac{(2n+2)!\ 2^{\cancel{2n}+2}}{(n+1)^{2n+2}} \cdot \frac{n^{2n}}{(2n)!\ \cancel{2^{2n}}} =$$

$$= \frac{(2n+2)(2n+1)\ n^{2n} \cdot 4}{(n+1)^2\ (n+1)^{2n}}$$

$$\frac{n^{2n}}{(n+1)^{2n}} = \left(\frac{n}{n+1}\right)^{2n}$$

$$\left(\frac{n}{n+1}\right)^{2n} = e^{2n\log\left(\frac{n}{n+1}\right)} =$$

$$= e^{2n\log\left(1-\frac{1}{n+1}\right)} =$$

$$= e^{2n\left(-\frac{1}{n+1}+o\left(\frac{1}{n+1}\right)\right)} =$$

$$= e^{-\frac{2n}{n+1} + o\left(\frac{2n}{n+1}\right)} \longrightarrow e^{-2+0}$$

$$= \frac{1}{e^2} \, .$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{e^2} \cdot 4 \cdot \lim_{n\to\infty} \boxed{\frac{(2n+2)(2n+1)}{(n+1)^2}} \longrightarrow 4$$

$$= \frac{16}{e^2} > 1 \Rightarrow \lim_{n\to\infty} a_n = +\infty \, .$$