$$y'' + 3y' = e^{7x} + x^2$$

omogenea

$$y'' + 3y' = 0 \qquad \lambda^2 + 3\lambda = 0$$

$$\lambda(\lambda + 3) = 0 \qquad \lambda = 0 , \ \lambda = -3$$

$$y_0 = c_1 + c_2 e^{-3x}$$

$$f_1(x) = e^{7x}$$

risolvo $y'' + 3y' = e^{2x}$

soluzione particolare

$\alpha = 2 \quad \beta = 0 \qquad \alpha + i\beta = 2$

non è radice del polin. caratt.

$\Rightarrow m = 0$.

$\bar{y} = A e^{2x}$

$\bar{y}' = 2A e^{2x}$

$\bar{y}'' = 4A e^{2x}$

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sostituisco in $y'' + 3y' = e^{2x}$

$$4Ae^{2x} + 3 \cdot 2Ae^{2x} = e^{2x}$$

$$10Ae^{2x} = e^{2x} \qquad A = \frac{1}{10}$$

$$\Rightarrow \bar{y} = \frac{1}{10} e^{2x} .$$

soluzione particolare di

$y'' + 3y' = x^2$ $\leftarrow$ grado $(P) = 2$

$\alpha = 0$, $\beta = 0$ $\Rightarrow \alpha + i\beta = 0$

che è radice del polinom. caratt.

$\Rightarrow m = 1$.

$$\Rightarrow \bar{y} = x\,(\underbrace{Ax^2 + Bx + c}_{r(x)\ \text{di grado } 2.})$$

$$= Ax^3 + Bx^2 + Cx$$

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$$\bar{y}' = 3Ax^2 + 2Bx + C$$

$$\bar{y}'' = 6Ax + 2B$$

sostituire in $y'' + 3y' = x^2$

$$6Ax + 2B + 3(3Ax^2 + 2Bx + c) = x^2$$

$$9Ax^2 + (6A + 6B)x + 2B + 3C = x^2$$

$$\begin{cases} 9A = 1 \\ 6A + 6B = 0 \\ 2B + 3C = 0 \end{cases} \qquad \begin{aligned} A &= 1/9 \\ 6B &= -6A \\ B &= -A = -\frac{1}{9} \end{aligned}$$

$$3C = -2B \Rightarrow C = -\frac{2}{3}B =$$

$$= -\frac{2}{3}\left(-\frac{1}{9}\right) = \frac{2}{27}$$

$$\Rightarrow \bar{y} = Ax^3 + Bx^2 + Cx =$$

$$= \frac{1}{9}x^3 - \frac{1}{9}x^2 + \frac{2}{27}x \quad .$$

somma le due soluzioni particolari per il principio di sovrapposizione soluzione completa

$$y(x) = c_1 + c_2 e^{-3x} + \frac{1}{10} e^{9x}$$

$$+ \frac{1}{9} x^3 - \frac{1}{9} x^2 + \frac{2}{27} x \quad .$$

Es: $f(x) = x\, e^{\frac{1}{1+\log x}}$.

insieme di definizione.

$x>0$ per $\log x$.

$1+\log x \neq 0 \qquad \log x \neq -1 \qquad x \neq \frac{1}{e}$

dominio $\left(0, \frac{1}{e}\right) \cup \left(\frac{1}{e}, +\infty\right)$

dic 9-09:31

$$\lim_{x\to 0^+} x\, e^{\frac{1}{1+\lg x}} =$$

$$= 0 \cdot e^{\frac{1}{1+(-\infty)}} = 0 \cdot e^{\frac{1}{-\infty}} =$$

$$= 0 \cdot e^{0} = 0 .$$

$$\lim_{x\to \frac{1}{e}^-} x\, e^{\frac{1}{1+\lg x}} = \frac{1}{e} \cdot e^{\frac{1}{0^-}} =$$

se $x < \frac{1}{e} \Rightarrow \log x < \lg \frac{1}{e} = -1$

$\Rightarrow 1 + \lg x < 0$

dic 9-09:34

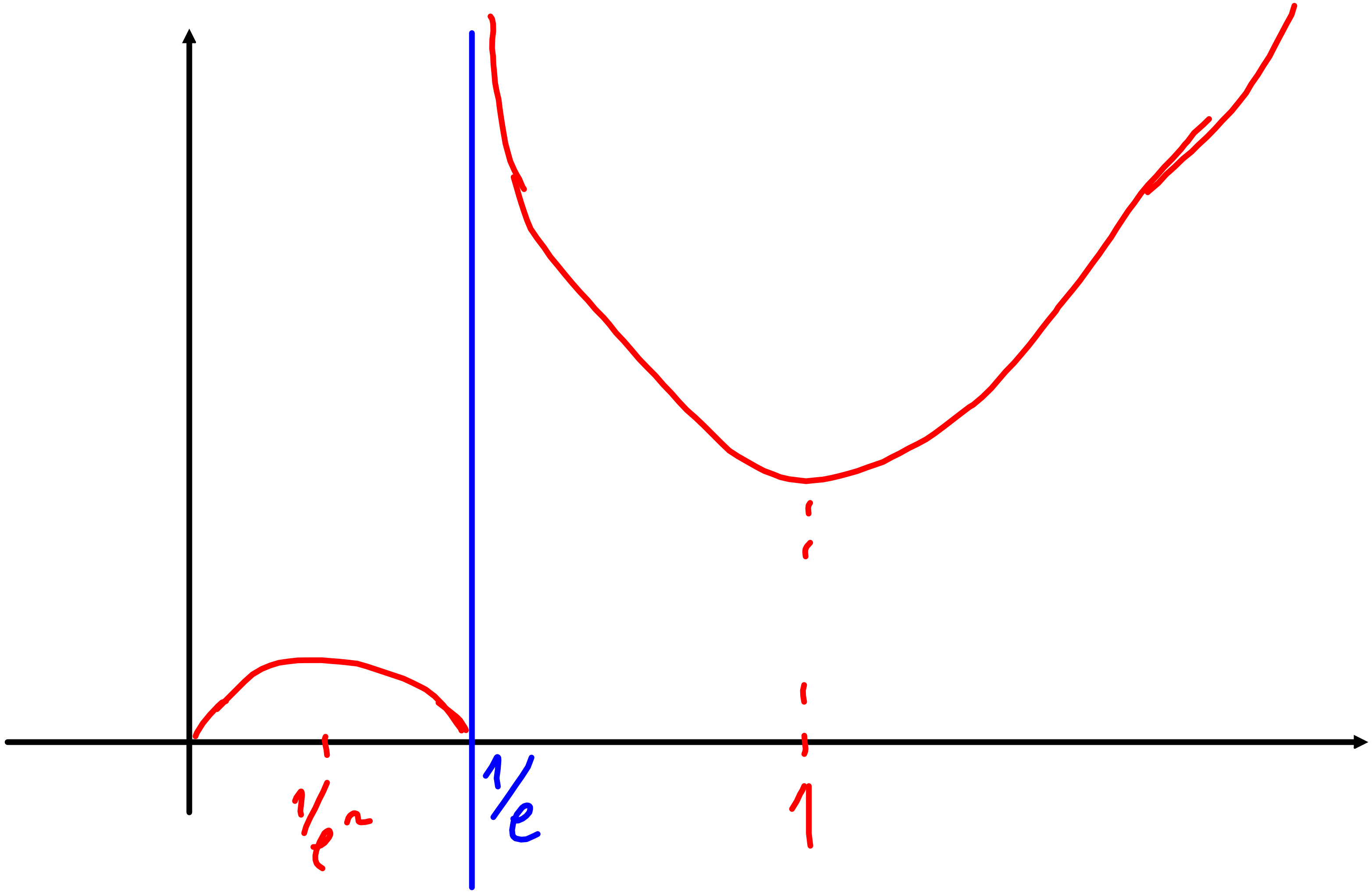
$$= \frac{1}{e} \cdot e^{-\infty} = \frac{1}{e} \cdot 0 = 0$$

$$\lim_{x \to \frac{1}{e}^+} x\, e^{\frac{1}{1+\log x}} = \frac{1}{e} \cdot e^{\frac{1}{0^+}} =$$

$$= \frac{1}{e} \cdot e^{+\infty} = +\infty .$$

$$\lim_{x \to +\infty} x\, e^{\frac{1}{1+\lg x}} = (+\infty) \cdot e^{\frac{1}{1+\infty}} =$$

$$= (+\infty)\, e^{0} = +\infty$$



dic 9-09:15

$f(x) > 0 \quad \forall x \in$ dominio.

$\sup(f) = +\infty$

$\inf(f) = 0$ perché $f > 0 \ \forall x$.

e $\lim_{x \to 0^+} f(x) = 0$.

$\min(f)$ non esiste.

perché dovrebbe essere $\min(f) =$
$= \inf(f) = 0$ e invece $f > 0$.

asintoto obliquo ?

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\cancel{x}\, e^{\frac{1}{1+\lg x}}}{\cancel{x}} =$$

$$= e^{\frac{1}{1+\infty}} = e^{0} = 1 = m.$$

$$\lim_{x\to\infty} f(x) - mx =$$

$$= \lim_{x\to\infty} x\, e^{\frac{1}{1+\lg x}} - x =$$

$$= \lim x\left[e^{\frac{1}{1+\log x}} - 1\right]$$

$$t = \frac{1}{1+\log x}$$

se $x \to +\infty$

$$\to t \to \frac{1}{\infty} = 0$$

$$e^{t} = 1 + t + o(t).$$

$$x\left[e^{\frac{1}{1+\log x}} - 1\right] = x\left[\cancel{1} + \frac{1}{1+\log x} + o\left(\frac{1}{1+\log x}\right) - \cancel{1}\right]$$

$$= \frac{x}{1+\log x}\left[1+o(1)\right] \to +\infty$$

$\to +\infty$

$q = +\infty$

$\Rightarrow$ non c'è asintoto obliquo.

$$f'(x) = e^{\frac{1}{1+\log x}} + \cancel{x}\, e^{\frac{1}{1+\log x}} \cdot \frac{-\frac{1}{\cancel{x}}}{(1+\log x)^2}$$

$$= e^{\frac{1}{1+\log x}} \left[1 - \frac{1}{(1+\log x)^2} \right]$$

$$f'(x) > 0 \iff 1 - \frac{1}{(1+\log x)^2} > 0$$

$$1 > \frac{1}{(1+\log x)^2} \iff (1+\log x)^2 > 1$$

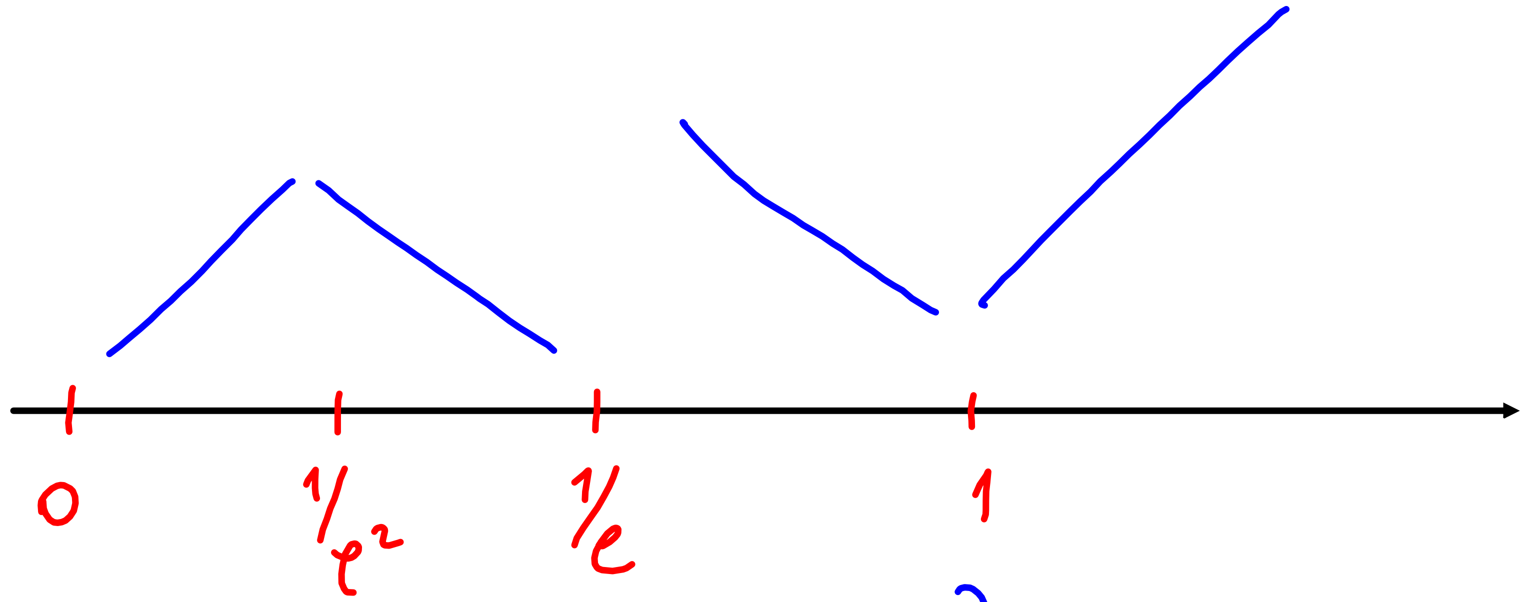
$$\cancel{1} + \log^2 x + 2\log x > \cancel{1}$$

$$\log x\,(\log x + 2) > 0$$

$$\boxed{\begin{array}{l} t = \log x \\ t^2 + 2t > 0 \\ t > 0 \text{ oppure } t < -2 \end{array}}$$

$$\log x > 0 \text{ oppure } \log x < -2$$

$$x > 1 \text{ oppure } x < \frac{1}{e^2}$$



f è crescente in $\left[0, \frac{1}{e^2}\right]$ e in $[1, +\infty)$

decrescente in $\left[\frac{1}{e^2}, \frac{1}{e}\right)$ e in $\left(\frac{1}{e}, 1\right]$.

$x = \frac{1}{e^2}$ è punto di max locale

$x = 1$ è punto di minimo locale.

Es: $f(x) = \int_0^{x} \frac{\sin t}{t}\, dt$

calcolare $\lim_{x \to 0} \frac{2f(x) - f(2x)}{x - f(x)}$.

$$\lim_{t\to 0} \frac{\sin t}{t} = 1$$

per essere precisi la funzione integranda è

$$\begin{cases} \dfrac{\sin t}{t} & \text{se } t \neq 0 \\ 1 & \text{se } t = 0 \end{cases}$$

$\Rightarrow$ è continua.

$\Rightarrow f(x)$ è <u>derivabile</u>.

dic 9-10:08

$$\lim_{x\to 0} f(x) = 0 = \left(\int_0^0 \frac{\sin t}{t}\,dt\right)$$

$$\lim_{x\to 0} \frac{2f(x) - f(2x)}{x - f(x)} = \frac{0-0}{0-0} = \frac{0}{0}.$$

de l'Hôpital.

$$\lim_{x\to 0} \frac{2f'(x) - f'(2x)\cdot 2}{1 - f'(x)}.$$

$$f'(x) = \frac{\sin x}{x} \cdot$$

$$f'(2x) = \frac{\sin(2x)}{2x} \cdot$$

$$\lim_{x \to 0} \frac{2\,\frac{\sin x}{x} - \cancel{2} \cdot \frac{\sin(2x)}{\cancel{2}x}}{1 - \frac{\sin x}{x}} =$$

$$= \lim_{x \to 0} \frac{2 \sin x - \sin(2x)}{x - \sin x} =$$

$$= \lim_{x \to 0} \frac{2\left(x - \frac{x^3}{6} + o(x^4)\right) - \left(2x - \frac{\overset{(2x)^3}{8x^3}}{6} + o(x^4)\right)}{x - \left(x - \frac{x^3}{6} + o(x^4)\right)}$$

$$= \lim_{x \to 0} \frac{-\frac{2}{6}x^3 + \frac{8}{6}x^3 + o(x^4)}{\frac{x^3}{6} + o(x^4)} =$$

$$= \lim_{x \to 0} \frac{x^3 + o(x^4)}{x^3/6 + o(x^4)} = 6.$$

Es: $a_n = \sqrt{n}\left(\dfrac{1}{\sqrt{n-2}} - \dfrac{1}{\sqrt{n+3}}\right)$

$n \geq 3$

min, max, sup, inf.

$$\lim_{n\to\infty} \sqrt{n}\left(\frac{1}{\sqrt{n-2}} - \frac{1}{\sqrt{n+3}}\right) =$$

$$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n-2}} - \frac{\sqrt{n}}{\sqrt{n+3}} = 1 - 1 = 0$$

dic 9-10:20

$a_n > 0$?

$$\cancel{\sqrt{n}}\left(\frac{1}{\sqrt{n-2}} - \frac{1}{\sqrt{n+3}}\right) > 0$$

$$\frac{1}{\sqrt{n-2}} > \frac{1}{\sqrt{n+3}} \iff \sqrt{n+3} > \sqrt{n-2}$$

sempre vero.

$\Rightarrow a_n > 0 \quad \forall n.$

$\Rightarrow \{a_n\}$ ha massimo.

dic 9-10:24

$a_n > 0 \quad \forall n. \quad , \quad \inf(a_n) = 0$

$\Rightarrow \{a_n\}$ non ha minimo

perché dovrebbe essere 0.

$$\lim_{x \to 1} \frac{2 \sin(x-1)}{x^2-1} \quad \frac{e^{x^2-1} - 1}{x-1}$$

$x - 1 = t$

$x^2 - 1 = (x-1)(x+1) = t(t+2)$

$t(t+2)$

$$\lim_{t \to 0} \frac{2 \sin t}{t(t+2)} \cdot \frac{e \qquad -1}{t} =$$

$t(t+2)$

$$= \lim_{t \to 0} \underbrace{\frac{2 \sin t}{t}}_{\to 1} \cdot \frac{e \qquad -1}{t(t+2)} =$$

$$\lim_{t\to 0} \frac{e^{t(t+2)} - 1}{t(t+2)} =$$

$$= \lim_{z\to 0} \frac{e^{z} - 1}{z} = 1. \qquad z = t(t+2)$$

$$\Rightarrow \lim (\ldots) = 2 \cdot 1 \cdot 1 = 2.$$

dic 9-10:33

Es. $y'' - 4y' + 4y = e^{2x}$.

omogenea.

$y'' - 4y' + 4y = 0$

$\lambda^2 - 4\lambda + 4 = 0 \qquad (\lambda - 2)^2 = 0 \quad \boxed{\lambda = 2}$

$y_0 = c_1 e^{2x} + c_2 x e^{2x}$.

soluzione particolare

$\alpha = 2, \beta = 0 \rightarrow \alpha + i\beta = 2$

$p(x) \equiv 1$.

$\text{grado}(p) = 0$.

dic 9-10:34

2 è radice del polin. caratter.
con molteplicità 2 $\Rightarrow m = 2$.

$$\bar{y}(x) = A x^2 e^{2x}$$

$$\bar{y}'(x) = 2Ax e^{2x} + Ax^2 \cdot 2 e^{2x}$$

$$\bar{y}''(x) = 2Ae^{2x} + 2Ax \cdot 2e^{2x} +$$

$$+ 4Ax e^{2x} + 2Ax^2 \cdot 2e^{2x} =$$

$$= 2Ae^{2x}\left[2x^2 + 4x + 1\right]$$

sostituisco in

$$y'' - 4y' + 4y = e^{2x}$$

$$2A\cancel{e^{2x}}\left[\cancel{2x^2} + 4x + 1\right] - 4\cdot 2A\cancel{e^{2x}}\left[\cancel{x^2} + x\right] + \cancel{4Ax^2}\,\cancel{e^{2x}} = \cancel{e^{2x}}$$

$$\begin{cases} 4A - 8A + 4A = 0 \\ 8A - 8A = 0 \\ 2A = 1 \end{cases}$$

$$\boxed{A = \frac{1}{2}}$$

dic 9-10:40

$$\bar{y} = \frac{1}{2} x^2 e^{2x}$$

$$\Rightarrow y(x) = c_1 e^{2x} + c_2 x e^{2x} + \frac{x^2}{2} e^{2x} .$$

dic 9-10:43