Es: $f(x) = a^x \qquad a > 0$

$f'(x) = ?$

$$a^x = e^{\log(a^x)} = e^{x \cdot \log a}$$

$$f'(x) = e^{x \cdot \log a} \cdot \log a =$$

$$= a^x \cdot \log a$$

ott 20-15:59

Funzioni iperboliche.

coseno iperbolico

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

seno iperbolico

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

$\cosh x$ è pari

$$\cosh(-x) = \frac{e^{-x} + e^{x}}{2} = \cosh x$$

$\sinh x$ è dispari

$$\sinh(-x) = \frac{e^{-x} - e^{x}}{2} = -\sinh x$$

definite in $\mathbb{R}$

$$\lim_{x\to+\infty} \cosh x = \lim_{x\to\infty} \frac{e^x+e^{-x}}{2} =$$

$$= \frac{+\infty+0}{2} = +\infty$$

$$\lim_{x\to-\infty} \cosh x = +\infty \quad (\text{pari})$$

$$\lim_{x\to+\infty} \sinh x = \lim_{x\to+\infty} \frac{e^x - e^{-x}}{2} =$$

$$= \frac{+\infty - 0}{2} = +\infty$$

$$\lim_{x\to-\infty} \sinh x = -\infty \quad (\text{dispari})$$

$$\cosh 0 = \frac{e^0 + e^0}{2} = 1$$

$$\mathrm{sih}\,0 = \frac{e^0 - e^0}{2} = 0$$

$$D(\cosh x) = D\left(\frac{e^x + e^{-x}}{2}\right) =$$

$$= \frac{e^x - e^{-x}}{2} = \sinh x$$

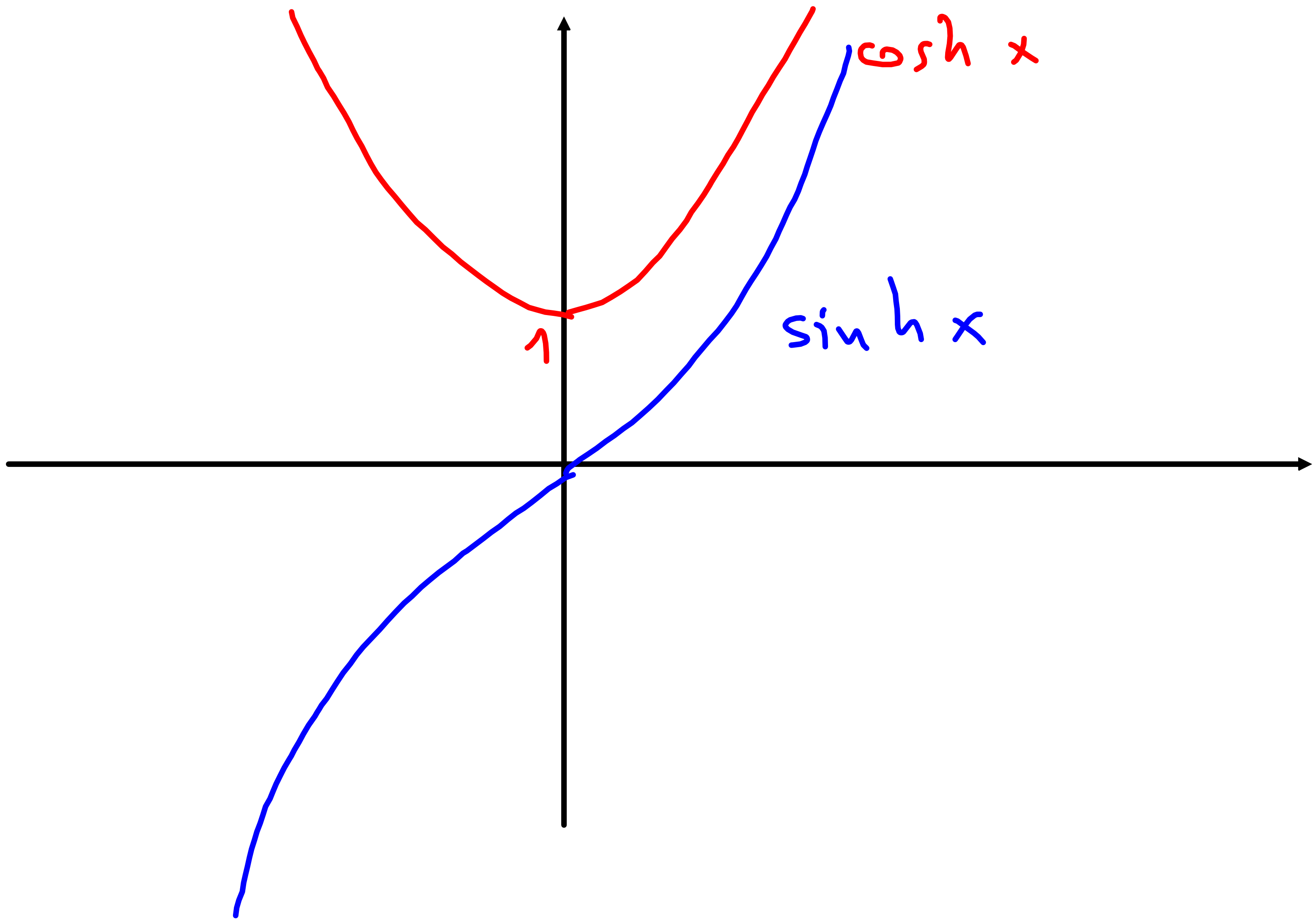
$$D(\sinh x) = D\left(\frac{e^x - e^{-x}}{2}\right) =$$

$$= \frac{e^x + e^{-x}}{2} = \cosh x$$

$$D''(\cosh x) = \cosh x$$

$$D''(\sinh x) = \sinh x$$

ott 20-16:27



ott 20-16:29

$$\lim_{x\to\infty} \frac{\cosh x}{\sinh x} =$$

moltiplico per e^{-x} num. e denom.

$$= \lim_{x\to\infty} \frac{e^{x}+e^{-x}}{\cancel{2}} \cdot \frac{\cancel{2}}{e^{x}-e^{-x}} =$$

$$= \lim_{x\to\infty} \frac{1+e^{-2x}}{1-e^{-2x}} = \frac{1+0}{1-0} = 1$$

$$\lim_{x\to-\infty} \frac{\cosh x}{\sinh x} = -1$$

perché $f(x) = \dfrac{\cosh x}{\sinh x}$ è dispari.

$$(\cosh x)^2 - (\sinh x)^2 =$$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 =$$

ott 20-16:36

$$= \frac{\cancel{e^{2x}} + \cancel{e^{-2x}} + 2 \cdot e^{0}}{4} +$$

$$- \frac{\cancel{e^{2x}} + \cancel{e^{-2x}} - 2 \cdot e^{0}}{4} =$$

$$= \frac{2}{4} + \frac{2}{4} = 1 .$$

Es: funzione continua senza derivata in un punto.

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$$

f è continua in 0?

limitata

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} x \sin \frac{1}{x} = 0$$

$$\shortparallel$$

$$f(0)$$

ott 20-16:40

f è continua in $x_0 = 0$.

È derivabile?

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} =$$

$$= \lim_{x \to 0} \frac{x \sin \frac{1}{x} - 0}{x} = \lim_{x \to 0} \sin \frac{1}{x}$$

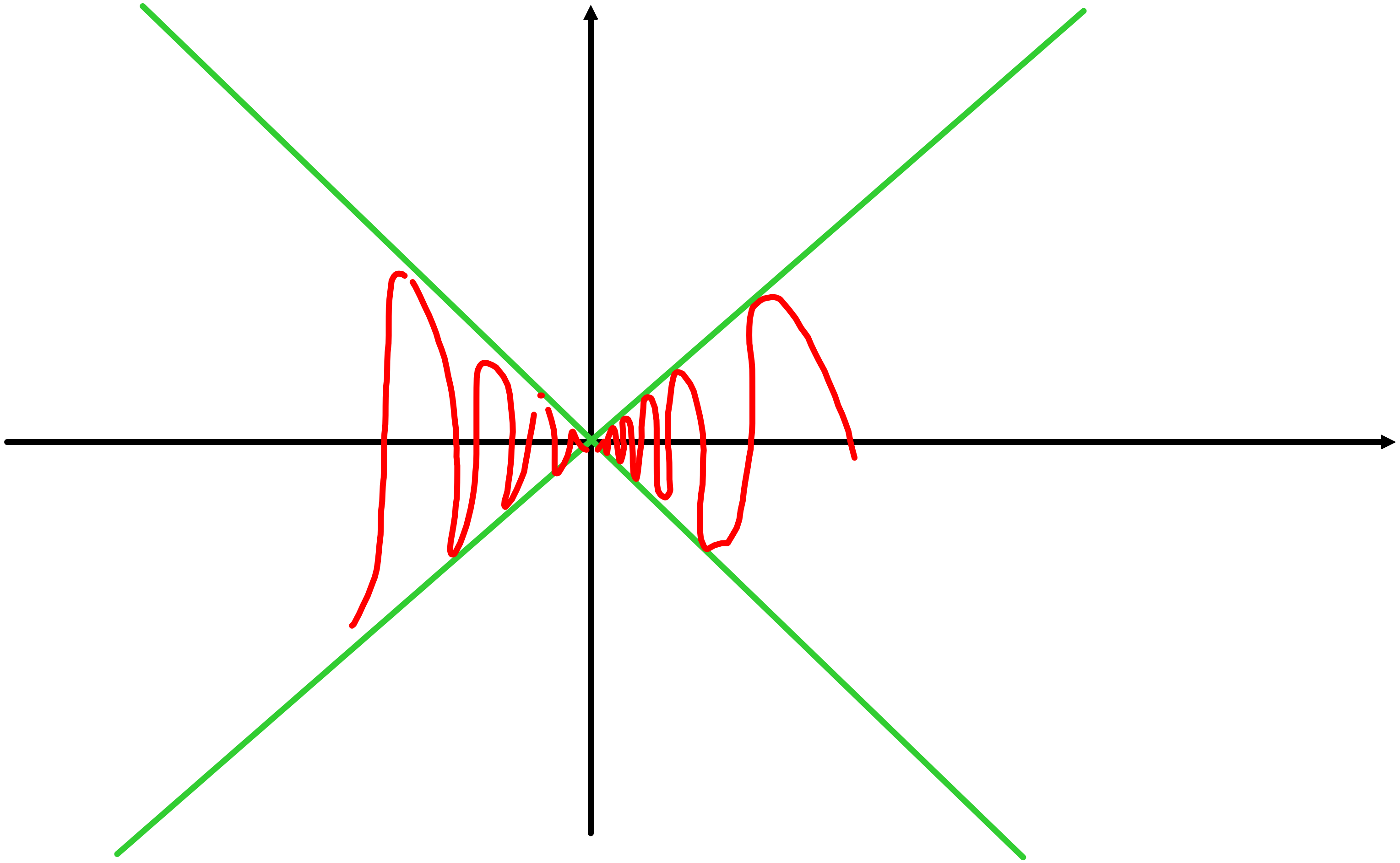
<u>non esiste</u>

$$\lim_{x \to 0^+} \sin\frac{1}{x} = \lim_{t \to +\infty} \sin t \leftarrow \nexists$$

$\frac{1}{x} = t$ se $x \to 0^+ \Rightarrow t \to +\infty$.

quindi f non ha derivata in $x_0 = 0$.

ott 20-16:44



$$-x \leq x \sin\frac{1}{x} \leq x$$

$x > 0$

ott 20-16:46

Es: $\boxed{D \text{ arctg}}$

$f(x) = \text{tg}\, x \qquad f'(x) = 1 + (\text{tg}\, x)^2$

$f^{-1}(y) = \text{arctg}\, y \quad , \quad y = \text{tg}\, x$

$$\left(f^{-1}(y)\right)' = \frac{1}{f'\left(f^{-1}(y)\right)} =$$

ott 20-16:49

$$= \frac{1}{1+\left[tg\left(f^{-1}(y)\right)\right]^2} =$$

$$= \frac{1}{1+\left(tg\left(arctg\, y\right)\right)^2} = \frac{1}{1+y^2}$$

ott 20-16:52

$$D(\operatorname{arctg} x) = \frac{1}{1+x^2}$$

$$f(x) = \operatorname{arctg} x$$

$$f(x) = f(0) + f'(0)(x-0) + o(x-0)$$

$$f(0) = \operatorname{arctg} 0 = 0$$

$$f'(0) = \frac{1}{1+0^2} = 1$$

$$\operatorname{arctg} x = x + o(x) \quad \text{per } x \to 0$$

ott 20-16:54

Es: $f(x) = (1+x)^{\alpha} \quad \alpha \in \mathbb{R}$

$x > -1$

$f'(x) = \alpha (1+x)^{\alpha-1}$

$f(0) = 1 \quad , \quad f'(0) = \alpha$

$f(x) = f(x_0) + f'(x_0)(x-x_0) + o(x-x_0)$

$x_0 = 0 \quad f(x) = (1+x)^{\alpha}$

$(1+x)^{\alpha} = 1 + \alpha x + o(x)$ per $x \to 0$

ott 20-16:56

Es:

$$\sqrt{1+x} = (1+x)^{1/2} =$$

$$= 1 + \frac{x}{2} + o(x) \qquad \alpha = \frac{1}{2}$$

$$\sqrt[3]{1+x} = (1+x)^{1/3} \qquad \alpha = \frac{1}{3}$$

$$= 1 + \frac{x}{3} + o(x)$$

ott 20-17:00

Es: $\lim_{x\to\infty}\left[\sqrt{x^2+6}-\sqrt{x^2+1}\right]x$

$= (\infty-\infty)\infty$

$$\frac{\left(\sqrt{x^2+6}-\sqrt{x^2+1}\right)\left(\sqrt{x^2+6}+\sqrt{x^2+1}\right)}{\sqrt{x^2+6}+\sqrt{x^2+1}}\cdot x$$

$$= \frac{\cancel{x^2}+6-(\cancel{x^2}+1)}{\sqrt{x^2+6}+\sqrt{x^2+1}} \cdot x =$$

$x \to \infty$

$$= \frac{5x}{\sqrt{x^2+6}+\sqrt{x^2+1}} =$$

$$= \frac{5}{\frac{\sqrt{x^2+6}}{x}+\frac{\sqrt{x^2+1}}{x}} = \frac{5}{\frac{\sqrt{x^2+6}}{|x|}+\frac{\sqrt{x^2+1}}{|x|}}$$

$x > 0$

$$|x| = \sqrt{x^2}$$

$$= \frac{5}{\frac{\sqrt{x^2+6}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}}} = \frac{5}{\sqrt{\frac{x^2+6}{x^2}} + \sqrt{\frac{x^2+1}{x^2}}}$$

$$= \frac{5}{\sqrt{1+\underbrace{\frac{6}{x^2}}_{\to 0}} + \sqrt{1+\underbrace{\frac{1}{x^2}}_{\to 0}}} = \frac{5}{2}$$

ott 20-17:07

$$\lim_{x \to -\infty} \left[\sqrt{x^2+6} - \sqrt{x^2+1}\right] x =$$

$x = -|x|$ perché $x < 0$

$$\Rightarrow \lim_{x \to -\infty} = -\frac{5}{2}.$$

Soluzione alternativa

$$\left(\sqrt{x^2+6}-\sqrt{x^2+1}\right)x=$$

$$=\left(|x|\sqrt{1+\frac{6}{x^2}}-|x|\sqrt{1+\frac{1}{x^2}}\right)x$$

$$=x|x|\left[1+\frac{1}{2}\overset{t}{\left(\frac{6}{x^2}\right)}+o\left(\frac{1}{x^2}\right)-\left(1+\frac{1}{2}\overset{t}{\left(\frac{1}{x^2}\right)}+o\left(\frac{1}{x^2}\right)\right)\right]$$

$$(1+t)^\alpha=1+\alpha t+o(t)$$

ott 20-17:10

$$= x|x|\left(\frac{3}{x^2} - \frac{1}{2}\cdot\frac{1}{x^2} + o\left(\frac{1}{x^2}\right)\right) =$$

$$= x|x|\left(\frac{5}{2}\cdot\frac{1}{x^2} + o\left(\frac{1}{x^2}\right)\right)$$

se $x \to +\infty \Rightarrow x\cdot|x| = x^2$

$$\frac{5}{2} + o(1) \to \frac{5}{2}$$

se $x \to -\infty \Rightarrow x|x| = -x^2$

$$-\frac{5}{2} + o(1) \to -\frac{5}{2}.$$

Es: $\lim_{x \to \infty} \left[\sqrt[3]{x^2+8x} - \sqrt[3]{x^2} \right] x^{1/3}$

$$\left[(x^2+8x)^{1/3} - x^{2/3} \right] x^{1/3} =$$

$$= \left[\left[x^2 \left(1 + \frac{8}{x} \right) \right]^{1/3} - x^{2/3} \right] x^{1/3} =$$

$$= \left[x^{2/3} \left(1 + \frac{8}{x} \right)^{1/3} - x^{2/3} \right) x^{1/3} =$$

ott 20-17:17

$$= x^1\left(\left(1+\frac{8}{x}\right)^{1/3}-1\right) =$$

$$(1+t)^\alpha = 1+\alpha t + o(t)$$

$$\alpha = \frac{1}{3} \qquad t = \frac{8}{x}$$

$$= x\left(\cancel{1}+\frac{1}{3}\frac{8}{x}+o\left(\frac{1}{x}\right)\cancel{-1}\right) \to \frac{8}{3}$$

Oss: $o(1)$ vuol dire che è una quantità che tende a 0.

Es:

$$\lim_{x\to\infty} x \cdot o\left(\frac{1}{x}\right) =$$

$$= \lim_{x\to\infty} o(1) = 0 .$$

$$x\, o\left(\frac{1}{x}\right) = \frac{o\left(\frac{1}{x}\right)}{\frac{1}{x}} \longrightarrow 0$$

ott 20-17:25

Prop: $A \subset \mathbb{R}$, $f : A \to \mathbb{R}$
f debolmente crescente in A.
Se f è derivabile in un punto $x_0 \in A$ allora

$$f'(x_0) \geq 0.$$

nel caso f deb. decrescente

$$\Rightarrow f'(x_0) \leq 0.$$

ott 20-17:29

dim: $f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

se f è deb. crescente

numeratore e denominatore

sono concordi in segno

$\Rightarrow f'(x_0) \geq 0$ $\square$

ott 20-17:31

Oss: Se f è strettamente crescente posso dedurre solo che $f'(x_0) \geq 0$ e non che $f'(x_0) > 0$.

Es: $f(x) = x^3$.

è strettamente crescente

ott 20-17:34

ma $f'(x) = 3x^2 \geq 0$

non vale $f'(x) > 0$

perché $f'(0) = 0$.

<u>Def</u>: $A \subset \mathbb{R}$, $f: A \to \mathbb{R}$

un punto $x_0 \in A$ si dice punto di minimo locale (o relativo) se $\exists\, \mathcal{U} \in \mathcal{I}(x_0)$ t.c. $f(x) \geq f(x_0)$ $\forall x \in A \cap \mathcal{U}$

ott 20-17:37

si dice di minimo locale stretto se

$$f(x) > f(x_0)$$

$$\forall x \in U \cap A \setminus \{x_0\}$$

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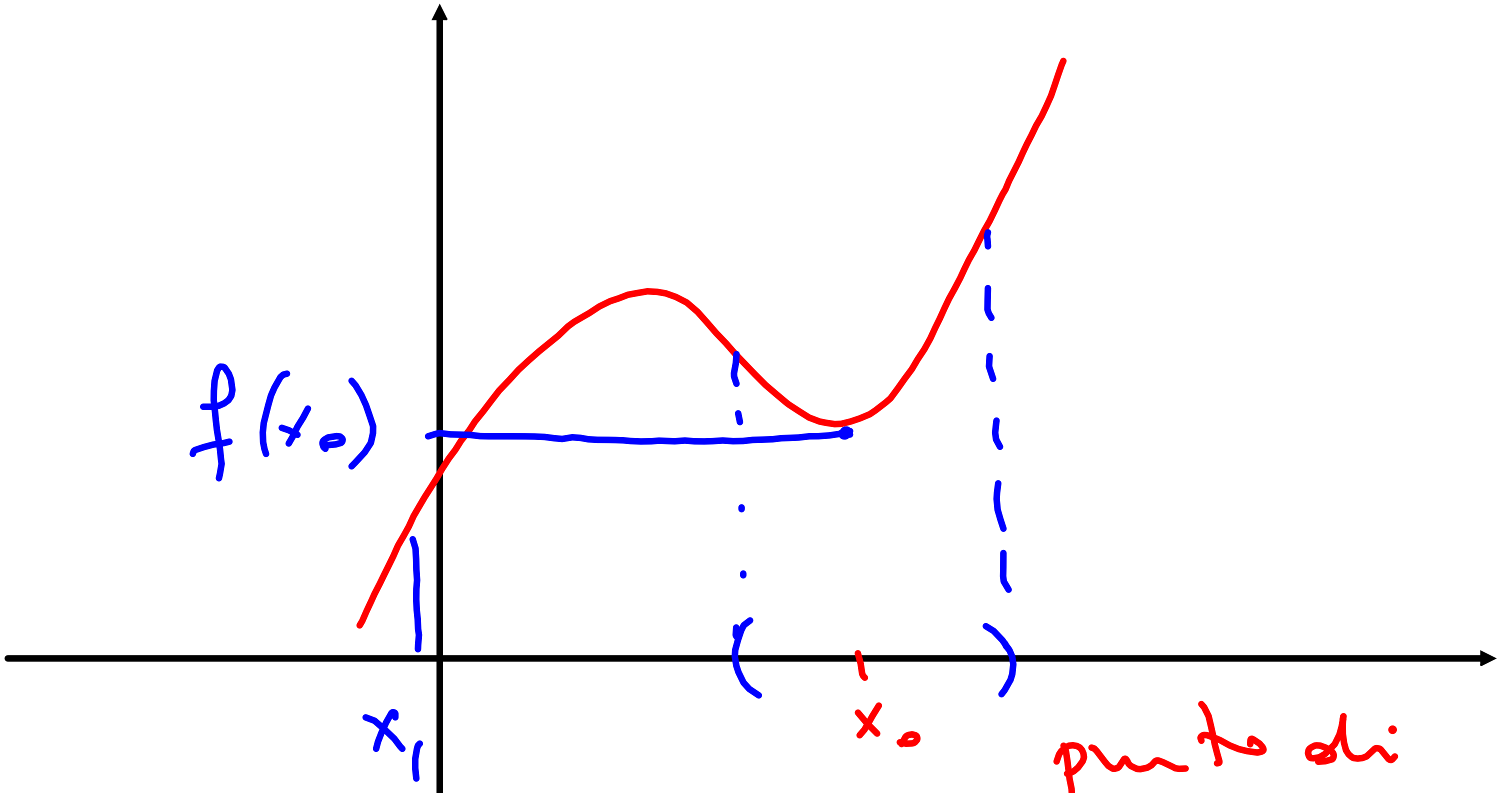
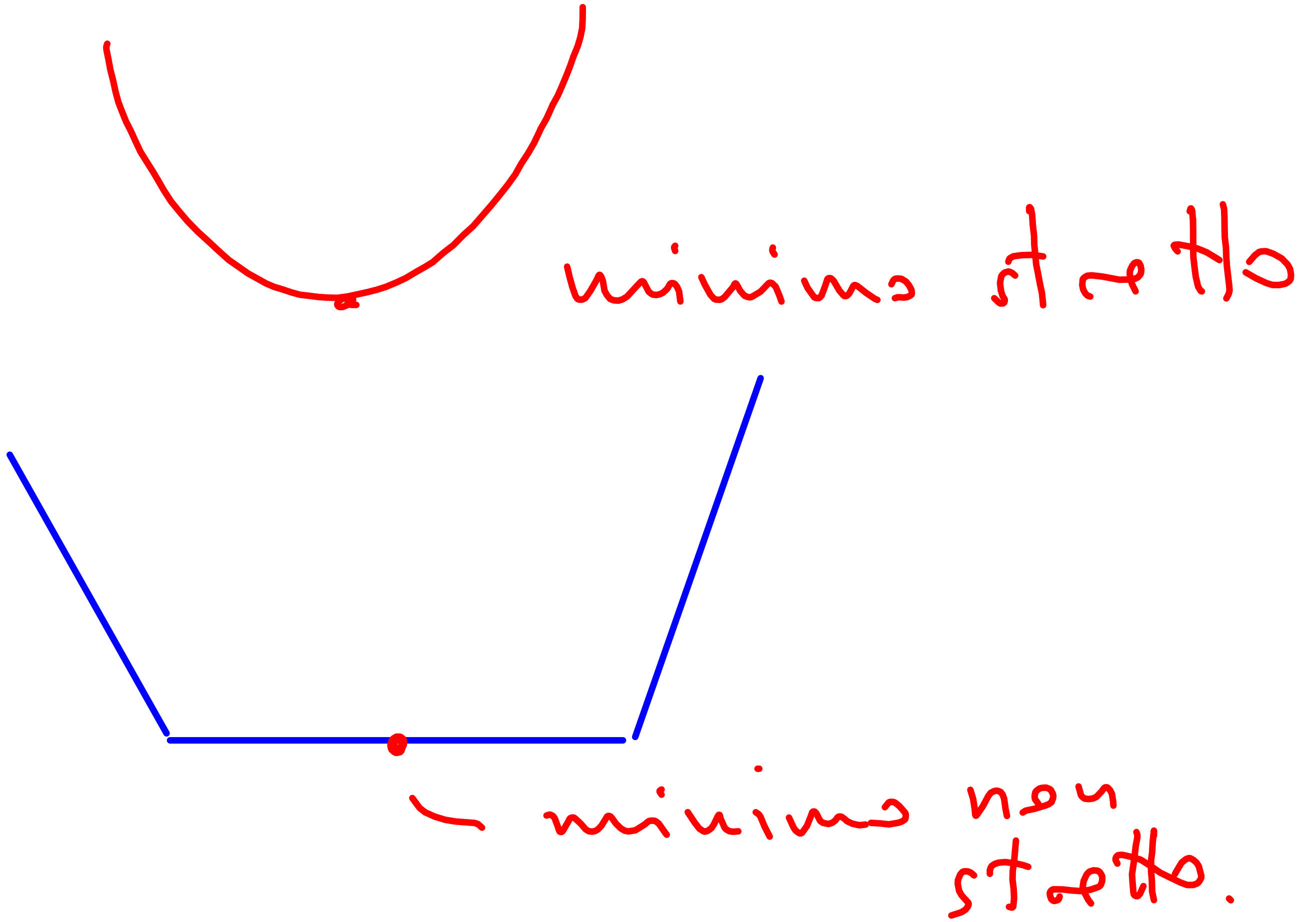


punto di minimo locale

non è un minimo globale perché $f(x_1) < f(x_0)$

ott 20-17:39



ott 20-17:42