Oss: $o(|x|^\alpha) - o(|x|^\alpha)$

$$= o(|x|^\alpha).$$

Es: $x^2 = o(x)$, $x^3 = o(x)$

$$x^2 - x^3 \neq 0.$$

$$x^2 - x^3 = o(x).$$

sappiamo che

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\Rightarrow \lim_{x \to 0} \frac{\sin x}{x} - 1 = 0$$

$$\Rightarrow \lim_{x \to 0} \frac{\sin x - x}{x} = 0$$

$$\Rightarrow \sin x - x = o(x)$$

$$\Rightarrow \quad \sin x = x + o(x).$$

per $x \to 0$

______ . ______

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$\Rightarrow \lim_{x \to 0} \frac{1 - \cos x}{x^2} - \frac{1}{2} = 0$$

ott 13-16:08

$$\lim_{x\to 0} \frac{1-\cos x - \frac{x^2}{2}}{x^2} = 0$$

$$\Rightarrow 1-\cos x - \frac{x^2}{2} = o(x^2)$$

$$\Rightarrow \cos x = 1 - \frac{x^2}{2} + o(x^2)$$

$$x \to 0$$

$$\lim_{x\to 0} \frac{tg\,x}{x} = \lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 1 = 1$$

fate i conti come prima

$$\Rightarrow \quad tg\,x = x + o(x).$$

sempre dai limiti notevoli ricaviamo

$$e^x = 1 + x + o(x)$$

$$\log(1+x) = x + o(x)$$

Es: $(\mathrm{tg}\, x)^2 = ?$

$$\mathrm{tg}\, x = x + o(x)$$

$$(\mathrm{tg}\, x)^2 = (x + o(x))^2 =$$

$$= x^2 + 2x\, o(x) + (o(x))^2 =$$

$$= x^2 + o(x^2) + o(x^2)$$

$$= x^2 + o(x^2)$$

ott 13-16:33

Es:
$$\lim_{x\to 0} \frac{\cos(\sin^2 x) - 1}{x^4}$$

$$\sin x = x + o(x)$$

$$\cos(\sin^2 x) - 1 =$$

$$= \cos\left[(x + o(x))^2\right] - 1 =$$

$$= \cos\left(x^2 + o(x^2)\right) - 1 = (*)$$

$$\cos t = 1 - \frac{t^2}{2} + o(t^2)$$

se $t \to 0$

la applico con $t = x^2 + o(x^2)$

lo posso fare?

Sì perché se $x \to 0$

$\Rightarrow x^2 + o(x^2) \to 0$

ott 13-16:41

$$\circledast = \cancel{1} - \frac{\big(\overbrace{x^2 + o(x^2)}^{t}\big)^2}{2} + o\Big(\big(\underbrace{x^2 + o(x^2)}_{t}\big)^2\Big) - \cancel{1}$$

$$= - \frac{x^4 + 2x^2 o(x^2) + (o(x^2))^2}{2} + o(\cdots)$$

$$= - \frac{x^4 + o(x^4) + o(x^4)}{2} + o(\cdots)$$

$$= - \frac{x^4 + o(x^4)}{2} + o\big((x^4 + o(x^4))\big)$$

$$= -\frac{x^4}{2} - \frac{o(x^4)}{2} + o(x^4) =$$

$$= -\frac{x^4}{2} + o(x^4)$$

è $o(x^4)$

$$\Rightarrow \lim_{x\to 0} \frac{\cos(\sin^2 x)-1}{x^4} =$$

$$= \lim_{x\to 0} \frac{-\frac{x^4}{2} + o(x^4)}{x^4} = -\frac{1}{2} + \lim_{x\to 0} \frac{o(x^4)}{x^4}$$

0

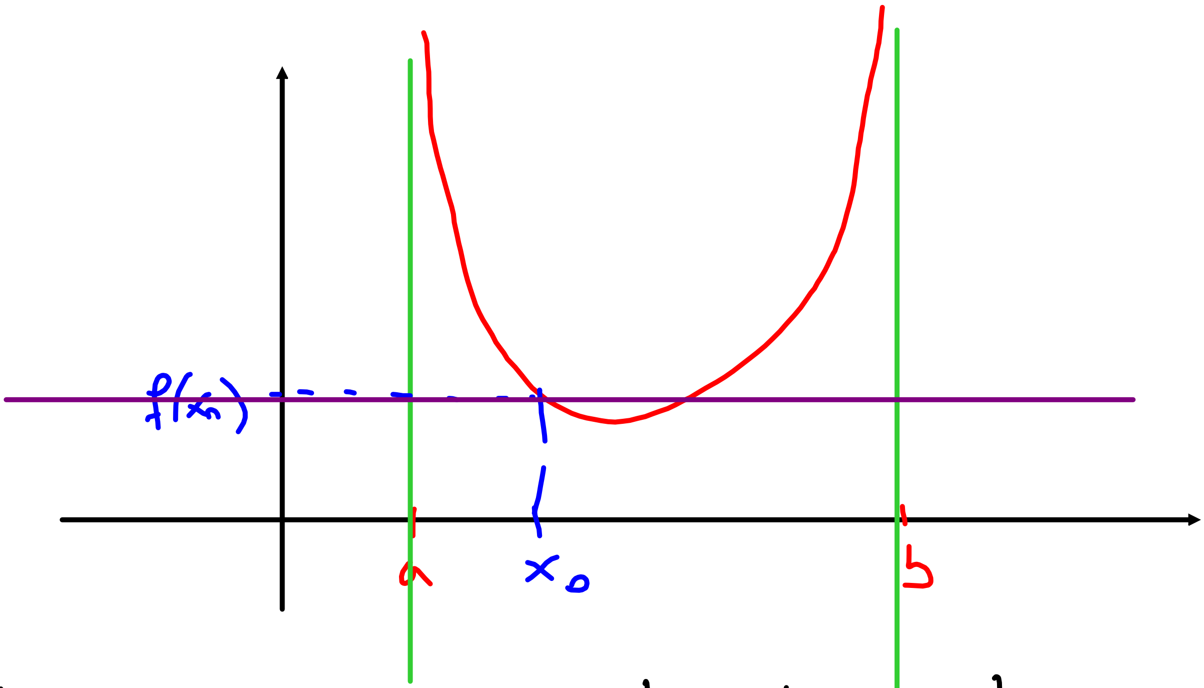
$$= -\frac{1}{2}$$

Prop: $a, b \in \overline{\mathbb{R}}$ $f : (a,b) \to \mathbb{R}$ continua.

Se $\lim_{x \to a^+} f(x) = +\infty$ e

$\lim_{x \to b^-} f(x) = +\infty$

allora f ha minimo.



butto via un piccolo intervallo aperto a destra di a e uno a sinistra di b.

dim: prendo $x_0 \in (a, b)$ qualsiasi.
e considero il valore $f(x_0)$.
Poiché $\lim_{x \to a^+} f(x) = +\infty$
posso trovare a_1 t.c.
$a < a_1 < x_0$ e se
$x \in (a, a_1) \Rightarrow f(x) > f(x_0)$

poichè $\lim_{x \to b^-} f(x) = +\infty$

posso trovare b_1 t.c.

$x_0 < b_1 < b$ e se

$x \in (b_1, b) \Rightarrow f(x) > f(x_0)$.

Dato che f è continua

$\Rightarrow f$ ha minimo in $[a_1, b_1]$

per il teor. di Weierstrass.

Sia $m = \min\limits_{[a_1, b_1]} f$.

Se $x \in [a_1, b_1] \Rightarrow f(x) \geq m$

Se $x \in (a, a_1) \Rightarrow$

$f(x) > f(x_0) \geq m$ perché $x_0 \in [a_1, b_1]$

Allo stesso modo se $x \in (b_1, b)$

$\Rightarrow f(x) > f(x_0) \geq m$

quindi

$$f(x) \geq m \quad \forall\, x \in (a,b).$$

$$\Rightarrow \quad m = \min_{(a,b)} f \quad .$$

$\square$

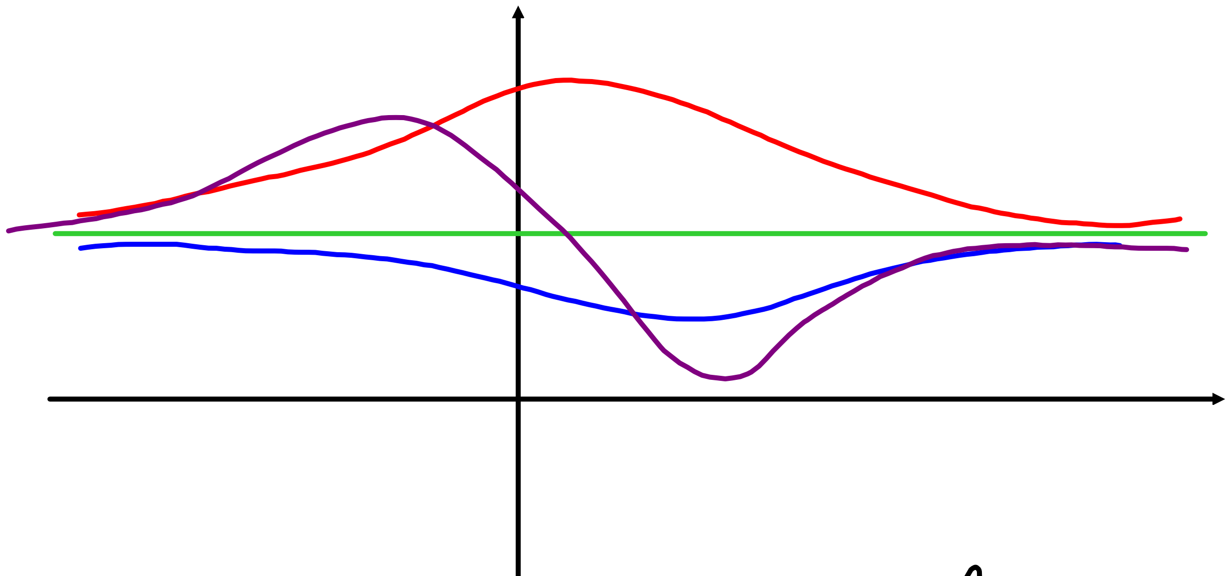
ott 13-17:10

Ovviamente se

$$\lim_{x \to a^+} f(x) = -\infty$$

$$\lim_{x \to b^-} f(x) = -\infty$$

$\Rightarrow f$ ha massimo.



$f:\mathbb{R}\to\mathbb{R}$, $\lim\limits_{x\to+\infty} f(x)=\ell$

$\lim\limits_{x\to-\infty} f(x)=\ell$, $\ell\in\mathbb{R}$, f continua

<u>Prop</u>: $a, b \in \overline{\mathbb{R}}$ $f : (a, b) \to \mathbb{R}$

Sia $\lim_{x \to a^+} f(x) = l_1$ e

$\lim_{x \to b^-} f(x) = l_2$.

Se $\exists\ x_0$ t.c. $f(x_0) \geq \max\{l_1, l_2\}$

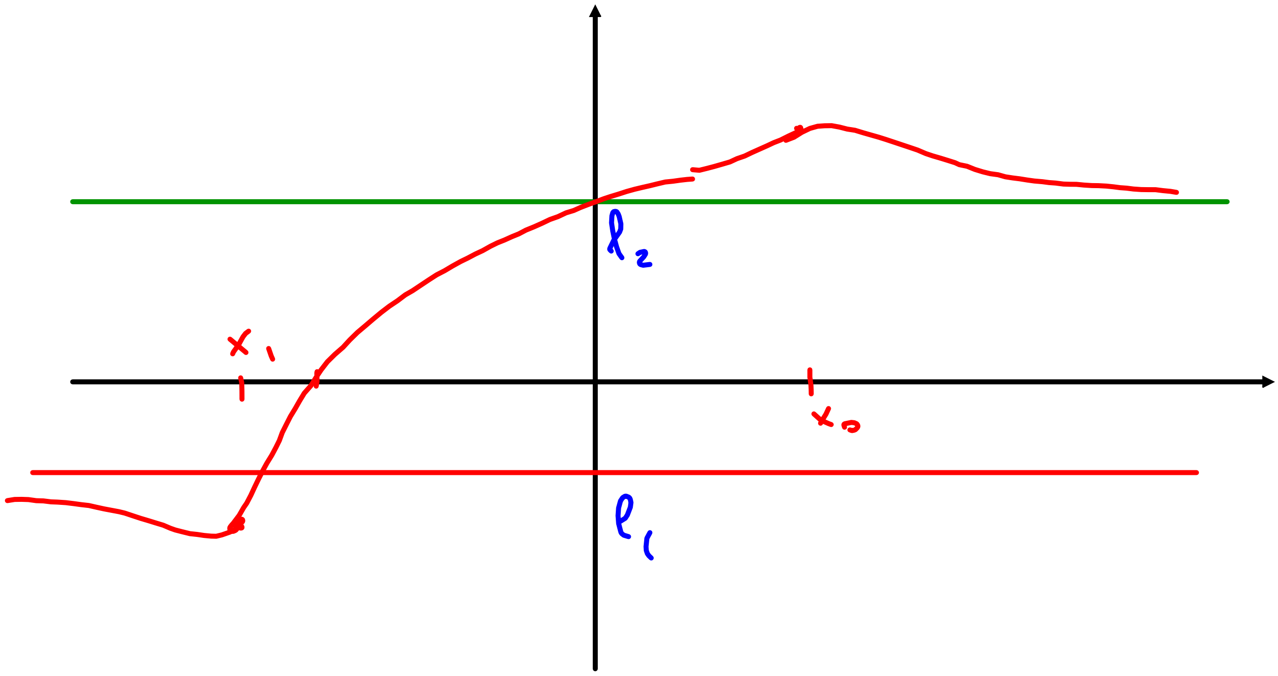
$\Rightarrow$ f ha massimo

ott 13-17:16

Se $\exists\ x_1$ t.c.

$$f(x_1) \leq \min\{l_1, l_2\}$$

$\Rightarrow f$ ha minimo.



ott 13-17:19

<u>Es</u>: $f: (0,1) \to \mathbb{R}$

$$f(x) = \frac{1}{x - x^2}$$

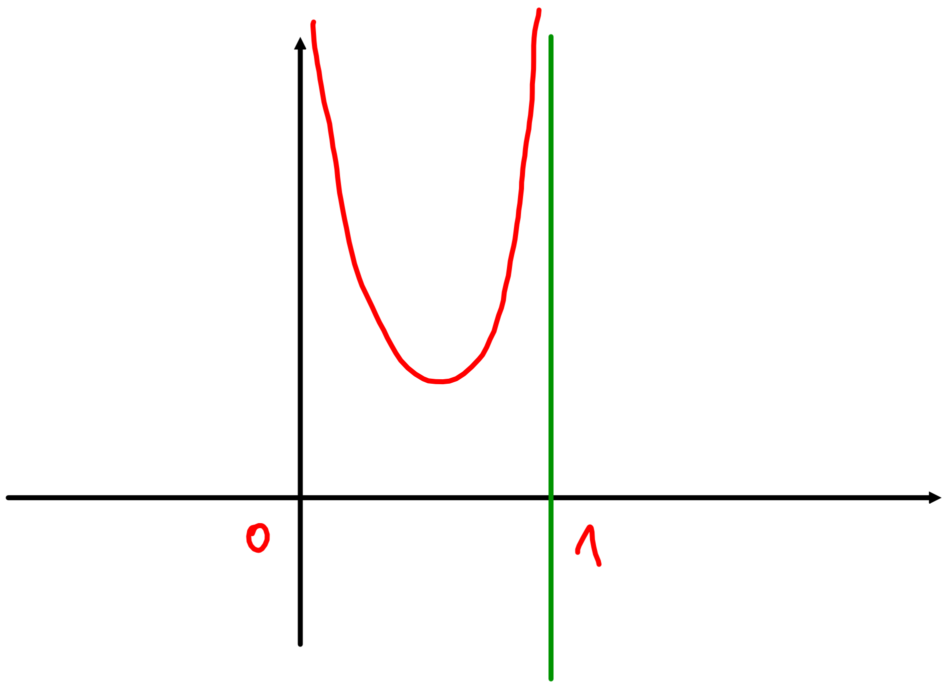
$$\frac{1}{x - x^2} = \frac{1}{x(1-x)}$$

$$\lim_{x \to 0^+} \frac{1}{x(1-x)} = \frac{1}{0^+ \cdot 1} = \frac{1}{0^+} = +\infty$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{1}{x(1-x)} =$$

$$= \frac{1}{1(1-1^-)} = \frac{1}{1 \cdot 0^+} = \frac{1}{0^+} = +\infty$$

ott 13-17:25



f ha minimo.

Es: $f(x) = e^{-x^2}$

$f : \mathbb{R} \to \mathbb{R}$

$$\lim_{x \to +\infty} e^{-x^2} = e^{-\infty} = 0^+$$

intendo $\lim_{y \to -\infty} e^{y}$

f è pari

$$f(-x) = e^{-(-x)^2} =$$

$$= e^{-x^2} = f(x).$$

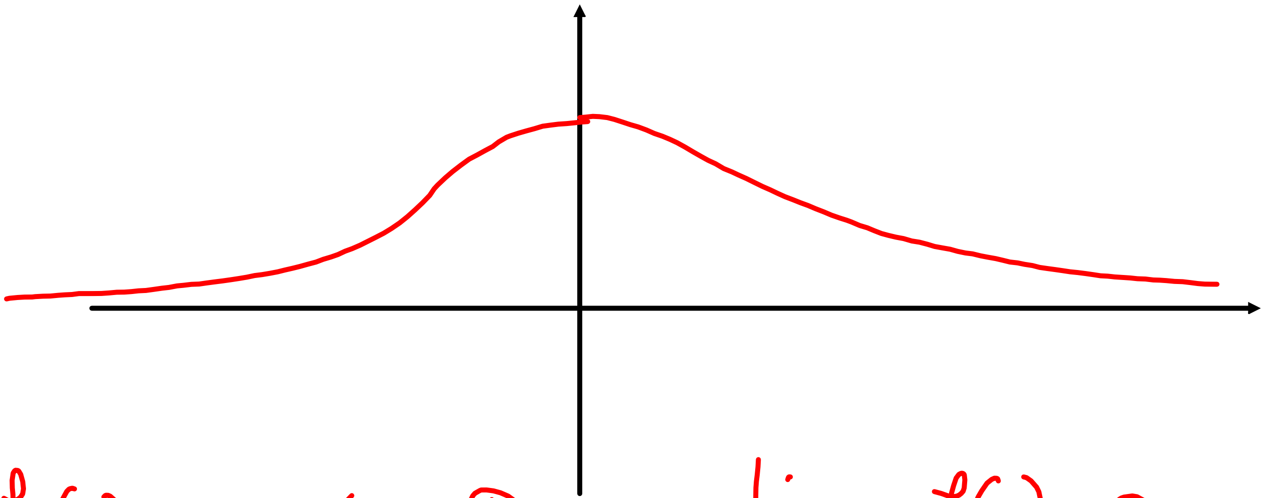
$$\Rightarrow \lim_{x\to -\infty} e^{-x^2} = \lim_{x\to +\infty} e^{-x^2}$$

$$= 0^+$$

$$f(0) = e^{-0} = 1 > 0$$

ott 13-17:30

$\Rightarrow$ f ha massimo.



$f(x) > 0 \ \forall x \in \mathbb{R}$ e $\lim_{x \to \pm\infty} f(x) = 0$

$\Rightarrow \inf_{\mathbb{R}} f = 0$

ott 13-17:31

se f avesse min allora
sarebbe $\min f = \inf f = 0$

$\Rightarrow \exists\ x_0 \in \mathbb{R} : f(x_0) = 0$

cioè $e^{-x_0^2} = 0$ impossibile.
l'esponenziale è sempre > 0.

ott 13-17:34

Es: $f(x) = x \log x$

$f : (0,1] \longrightarrow \mathbb{R}$

è continua.

$$\lim_{x \to 0^+} x \log x = 0^-$$

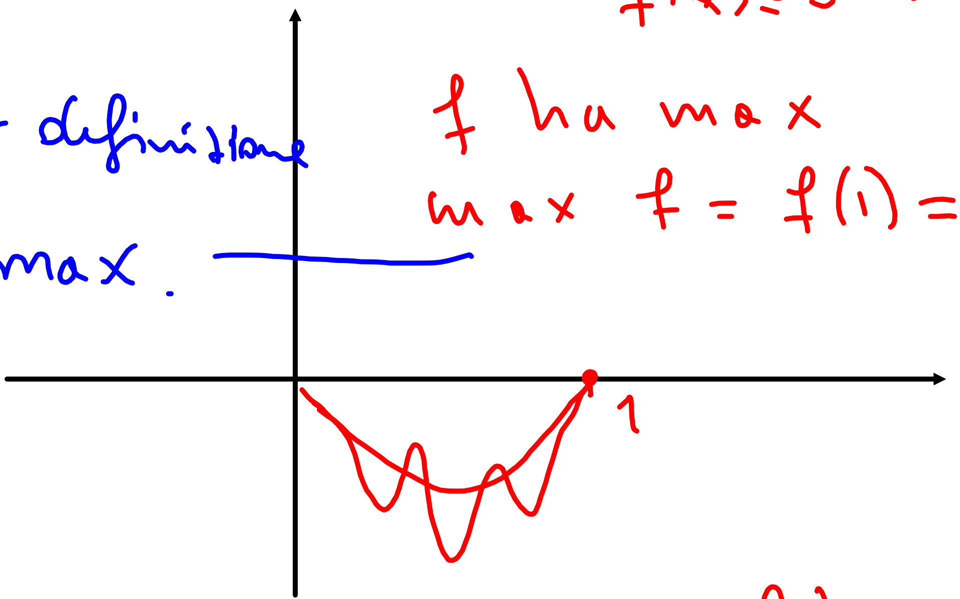
$$\lim_{x \to 1^-} f(x) = f(1) = 1 \cdot \log 1 = 0$$

$f(x) \leq 0 \quad \forall x \in (0,1]$

per definizione di max.

f ha max

$\max f = f(1) = 0$



$f\left(\frac{1}{2}\right) = \frac{1}{2}\log\left(\frac{1}{2}\right) < 0 \quad \Rightarrow f$ ha minimo