Corso di Laurea in Informatica	Analisi Matematica	Test di esempio 06 dicembre 2019

1. $\displaystyle\int_0^2 e^{x^2}(4x^3+x)\,dx =$

(a) $\dfrac{5e^2+3}{2}$ (b) $185e^4-1$ ▶ (c) $\dfrac{13e^4+3}{2}$ (d) $4e^2$

2. $$\lim_{n\to+\infty} \frac{3^n\, n!}{n^n} =$$

▶ (a) $+\infty$ (b) $\dfrac{3}{e}$ (c) 1 (d) 0

3. Sia $y(x)$ la soluzione del problema di Cauchy $\begin{cases} y' = e^{-y}\log x \\ y(3) = \log 3 + \log\log 3. \end{cases}$ Allora $y(4) =$

(a) $\log 4 + \log\log 4$ ▶ (b) $\log(4\log 4 - 1)$ (c) $\log\left(\dfrac{1}{4} + 9\log 3\right)$ (d) $9\log 3$

4. Sia $y(x)$ la soluzione del problema di Cauchy $\begin{cases} y' = \dfrac{2x+\sin x}{y^2} \\ y(0) = 3. \end{cases}$ Allora $y\left(\dfrac{\pi}{2}\right) =$

(a) $\sqrt[3]{\dfrac{\pi^2+8}{2}}$ ▶ (b) $\sqrt[3]{\dfrac{3\pi^2}{4}+30}$ (c) $\sqrt[3]{\dfrac{3\pi^2}{4}}$ (d) $\dfrac{-4+2\pi^2}{\pi^2}$

5. $\displaystyle\lim_{n\to+\infty} \frac{n!\,e^{n\log n}}{(2n)!} =$

(a) $\dfrac{e}{4}$ (b) $+\infty$ (c) 1 ▶ (d) 0

6. Sia $y(x)$ la soluzione del problema di Cauchy $\begin{cases} y'' - 10y' + 25y = 0 \\ y(0) = -3 \\ y'(0) = -19. \end{cases}$ La funzione $y(x)$

(a) non è limitata né superiormente né inferiormente
▶ (b) è limitata superiormente ma non inferiormente
(c) è limitata
(d) è limitata inferiormente ma non superiormente

7. La successione $a_n = \dfrac{\left(\log(n^2+2)\right)^{3n}}{(n+1)^{\frac{n}{4}}}$

▶ (a) ha massimo
(b) è debolmente crescente
(c) non ha limite
(d) non è limitata

8. Una primitiva della funzione $f(x) = \dfrac{\cos(2\log x)}{x}$ è

(a) $\dfrac{\sin(2\log x)}{x^2}$
(b) $\dfrac{-2\sin(2\log x) - \cos(2\log x)}{x^2}$
▶ (c) $\cos(\log x)\sin(\log x)$
(d) $\dfrac{-2\sin(2\log x)}{x}$

9. La successione $a_n = \dfrac{3^n - 100n^2 + n + 1}{n!}$

(a) non ha né massimo né minimo
(b) ha minimo ma non ha massimo
(c) ha massimo ma non ha minimo
▶ (d) ha sia massimo che minimo

10. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 x\,dx =$

(a) -3 (b) $-\dfrac{1}{4}$ ▶ (c) $\dfrac{2}{3}$ (d) $\dfrac{12\pi-\pi^3}{24}$

1) $\displaystyle\int_0^2 e^{x^2}(4x^3+x)\,dx$

$\displaystyle\int e^{x^2}(4x^3+x)\,dx$ sostituzione $x^2=t$ $\quad \frac{dt}{dx}=2x$

$dt=2x\,dx$

$\displaystyle = \int e^{x^2}(4x^2+1)\,\underbrace{x\,dx} =$

per parti

$\displaystyle = \int e^t(4t+1)\frac{dt}{2} = \frac{1}{2}\left(e^t(4t+1)-\int e^t 4\,dt\right)$

$\displaystyle = \frac{1}{2}\left(e^t(4t+1)-4e^t\right)=\frac{1}{2}e^t(4t-3)=\frac{1}{2}e^{x^2}(4x^2-3)$

$\displaystyle\int_0^2 e^{x^2}(4x^3+x)\,dx = \left[\frac{1}{2}e^{x^2}(4x^2-3)\right]_0^2 =$

$\displaystyle = \frac{1}{2}e^4(4\cdot4-3)-\frac{1}{2}e^0(0-3)=\frac{1}{2}e^4\cdot 13+\frac{1}{2}\cdot 3=$

$\displaystyle = \frac{13e^4+3}{2}$

2) $\displaystyle\lim_{n\to\infty}\frac{3^n n!}{n^n}$ criterio del rapporto

$\displaystyle \frac{a_{n+1}}{a_n} = \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \frac{3^{n+1}}{3^n}\cdot\underbrace{\frac{(n+1)!}{n!}}_{n+1}\cdot\frac{n^n}{(n+1)^{n+1}} =$

$\displaystyle = 3\,\cancel{(n+1)}\cdot\frac{n^n}{(n+1)^{n+\cancel{1}}} = \frac{3n^n}{(n+1)^n} = 3\left(\frac{n}{n+1}\right)^n =$

$\displaystyle = 3\left[\left(\frac{n+1}{n}\right)^n\right]^{-1} = 3\left[\left(1+\frac{1}{n}\right)^n\right]^{-1} \longrightarrow 3\cdot e^{-1} = \frac{3}{e} > 1$

$$\Rightarrow \lim_{n\to\infty} a_n = +\infty .$$

Oss: $$\left(\frac{n}{n+1}\right)^n = e^{n\log\left(\frac{n}{n+1}\right)} = e^{n\log\left(\frac{n+1-1}{n+1}\right)} =$$

$$= e^{n\log\left(1-\frac{1}{n+1}\right)} = e^{n\left(-\frac{1}{n+1}+o\left(\frac{1}{n}\right)\right)} \longrightarrow e^{-1} .$$

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3) $$\begin{cases} y' = e^{-y}\log x \\ y(3) = \log 3 + \log\log 3 \end{cases}$$

è a variabili separabili $\frac{dy}{dx} = e^{-y}\log x$

$$\int e^y \, dy = \int \log x \, dx + c$$

$$e^y = x\log x - x + c$$ trovo c sostituendo $x=3$, $y = \log 3 + \log\log 3$

$$e^{\log 3 + \log\log 3} = 3\log 3 - 3 + c$$

$$e^{\log 3} \cdot e^{\log\log 3} = 3\log 3 - 3 + c$$

$$3\cancel{\log 3} = 3\cancel{\log 3} - 3 + c \Rightarrow c = 3$$

$$\Rightarrow e^y = x\log x - x + 3$$

$$y = \log(x\log x - x + 3)$$

$$y(4) = \log(4\log 4 - 4 + 3) = \log(4\log 4 - 1) .$$

4) $\begin{cases} y' = \dfrac{2x + \sin x}{y^2} \\ y(0) = 3 \end{cases}$

variabili separabili

$$\frac{dy}{dx} = \frac{2x+\sin x}{y^2} \Rightarrow \int y^2\, dy = \int 2x + \sin x\, dx + c$$

$$\Rightarrow \frac{y^3}{3} = x^2 - \cos x + c \qquad y(0)=3$$

$$\Rightarrow \frac{27}{3} = 0 - \cos 0 + c \Rightarrow 9 = -1 + c \Rightarrow c = 10$$

$$\Rightarrow \frac{y^3}{3} = x^2 - \cos x + 10$$

$$\Rightarrow y = \sqrt[3]{3\left(x^2 - \cos x + 10\right)}$$

$$y\left(\frac{\pi}{2}\right) = \sqrt[3]{3\left(\frac{\pi^2}{4} - \cos\frac{\pi}{2} + 10\right)} = \sqrt[3]{\frac{3\pi^2}{4} + 30}$$

5) $\lim\limits_{n\to\infty} \dfrac{n!\, e^{n\log n}}{(2n)!}$

$e^{n\log n} = \left(e^{\log n}\right)^n = n^n$

$$\frac{n!\, e^{n\log n}}{(2n)!} = \frac{n!\, n^n}{(2n)!}$$

criterio del rapporto

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!\,(n+1)^{n+1}}{(2n+2)!} \frac{(2n)!}{n!\, n^n} = \frac{(n+1)!}{n!} \frac{(n+1)^{n+1}}{n^n} \cdot \frac{(2n)!}{(2n+2)!} =$$

$$= (n+1) \frac{(n+1)^n (n+1)}{n^n} \cdot \frac{1}{(2n+2)(2n+1)} =$$

$$= \underbrace{\boxed{\frac{(n+1)^2}{(2n+2)(2n+1)}}}_{\to \frac{1}{4}} \cdot \boxed{\left(\frac{n+1}{n}\right)^n} \to e \qquad \longrightarrow \quad \frac{e}{4} < 1$$

$$\Rightarrow \lim_{n\to\infty} a_n = 0 .$$

6) $\begin{cases} y'' - 10y' + 25y = 0 \\ y(0) = -3 \\ y'(0) = -19 \end{cases}$

polinomio caratteristico $\lambda^2 - 10\lambda + 25 = 0$

$(\lambda-5)^2 = 0 \qquad \lambda = 5$ radice doppia.

$$y = c_1 e^{5x} + c_2 x e^{5x}$$

$$y(0) = c_1 e^0 + c_2 \cdot 0 \cdot e^0 = c_1$$

$$y'(x) = 5c_1 e^{5x} + c_2 e^{5x} + 5c_2 x e^{5x}$$

$$y'(0) = 5c_1 + c_2$$

$$\Rightarrow \begin{cases} c_1 = -3 \\ 5c_1 + c_2 = -19 \end{cases} \Rightarrow \quad 5(-3) + c_2 = -19$$

$$\Rightarrow c_2 = -19 + 15 = -4$$

$$\Rightarrow y = -3e^{5x} - 4xe^{5x}$$

$$\lim_{x\to-\infty} y(x) = 0 \qquad \lim_{x\to+\infty} y(x) = -\infty$$

y è limitata superiorm. ma non inferiormente.

7) $a_n = \dfrac{(\log(n^2+2))^{3n}}{(n+1)^{n/4}}$

criterio della radice

$$\sqrt[n]{a_n} = \frac{(\log(n^2+2))^3}{(n+1)^{1/4}} \longrightarrow 0$$

$$\frac{(\log n)^\alpha}{n^\beta} \longrightarrow 0 \qquad \forall\, \alpha, \beta > 0.$$

$$\log(n^2+2) = \log\left(n^2\left(1+\frac{2}{n^2}\right)\right) = \log(n^2) + \log\left(1+\frac{2}{n}\right)$$

$$= 2\log n + \log\left(1+\frac{2}{n}\right)$$

Oss: $0^\infty = 0$

$$0^\infty = e^{\infty \log 0} = e^{\infty(-\infty)} = e^{-\infty} = 0$$

$\sqrt[n]{a_n} \to 0 \Rightarrow \lim\limits_{n\to\infty} a_n = 0 \Rightarrow \{a_n\}$ ha massimo

perché $a_n > 0$.

8) $f(x) = \dfrac{\cos(2\log x)}{x}$ primitiva?

$\displaystyle\int \frac{\cos(2\log x)}{x}\,dx$ sostituzione $\log x = t$

$$\frac{dt}{dx} = \frac{1}{x} \Rightarrow dt = \frac{dx}{x}$$

$$\int \cos(2t)\,dt = \frac{\sin(2t)}{2} = \frac{\sin(2\log x)}{2} =$$

$$= \frac{\cancel{2}\cos(\log x)\sin(\log x)}{\cancel{2}}$$

9) $$a_n = \frac{3^n - 100n^2 + n + 1}{n!}$$

$\lim_{n\to\infty} a_n = 0$ per sapere se ha max o min devo sapere se a_n ha valori positivi o negativi.

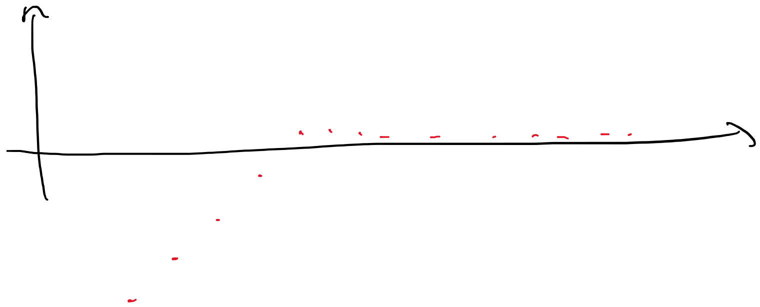
$n! > 0$ $\quad 3^n - 100n^2 + n + 1 \geq 0$?

per n abbastanza grande è sicuramente > 0

perché $\lim 3^n - 100n^2 + n + 1 = +\infty$.

è negativa per qualche n?

$n = 1 \quad 3^1 - 100 \cdot 1 + 1 + 1 = -95 < 0$



ha sia max che min.

10) $$\int_0^{\pi/2} \cos^3 x\,dx$$

$$\int \cos^3 x \, dx = \int \cos^2 x \cos x \, dx =$$

$$\int (1-\sin^2 x) \underbrace{\cos x \, dx}$$

$\sin x = t \qquad \frac{dt}{dx} = \cos x$

$dt = \cos x \, dx$

$$= \int 1 - t^2 \, dt = t - \frac{t^3}{3} =$$

$$= \sin x - \frac{\sin^3 x}{3}$$

$$\int_0^{\pi/2} \cos^3 x \, dx = \left[\sin x - \frac{\sin^3 x}{3} \right]_0^{\pi/2} = \sin \frac{\pi}{2} - \frac{\sin^3 \pi/2}{3} - 0 + 0$$

$$= 1 - \frac{1}{3} = \frac{2}{3} .$$