Esame di ALGEBRA LINEARE - anno accademico 2019/2020
Corso di laurea in Ingegneria Gestionale
Prova scritta del 8/1/2020
TEMPO A DISPOSIZIONE: 120 minuti

(Cognome) (Nome) (Numero di matricola)

PRIMA PARTE

PUNTEGGIO : risposta mancante = 0 ; risposta esatta = +1 risposta sbagliata = -1

calcoli e spiegazioni non sono richiesti

• Calcolare i^{26} : = $\boxed{-1}$ • $z = 2+i \implies z^{-1} = \boxed{\frac{2}{5} - \frac{i}{5}}$

Dati W e Z i seguenti sottospazi di $\mathbb{R}^3$:

$$W = \{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \ : \ x_1 + x_2 - x_3 = 0\} \ , \ Z = \langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} , \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \rangle .$$

• $\dim(W+Z) = \boxed{3}$ • Determinare una base di W

$$\boxed{\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}}$$

• $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix} \implies \mathrm{rg}(A) = \boxed{2} \quad \dim(Ker(\mathcal{L}_A)) = \boxed{1}$

• $\det \begin{pmatrix} 1 & -2 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 1 \\ 1 & -3 & 1 & 3 \end{pmatrix} = \boxed{8}$ • $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \implies m.g.(1) = \boxed{1}$

• $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$

• Sia $f : \mathbb{R}^2 \to \mathbb{R}^2$ lineare. Sapendo che $f \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $f \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, allora $f \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

8 - 1 - 2020

Traccia Sol.

1 $\begin{cases} (z-i)^2 = -i\,(\bar{z}+i) \\ |z| > |z-2i| \end{cases}$

1ª eq: $z - i = w$

$$w^2 = -i\,\overline{w}$$

$$w = \rho \cdot e^{i\vartheta} \Rightarrow \rho^2 \cdot e^{i2\vartheta} = \rho \cdot e^{-i\vartheta + i\frac{3}{2}\pi}$$

$$\Rightarrow \begin{cases} \rho^2 = \rho \\ 2\vartheta = -\vartheta + \frac{3}{2}\pi + 2k\pi \end{cases}$$

SOL. DISTINTE: $\rho = 0 \to w = 0 \to z_4 = i$

$$\begin{cases} \rho = 1 \\ \vartheta = \frac{1}{2}\pi + \frac{2k\pi}{3} \quad k = 0, 1, 2 \end{cases}$$

$w_0 = i \longrightarrow z_0 = 2i$

$w_1 = -\frac{\sqrt{3}}{2} - i\frac{1}{2} \longrightarrow z_1 = -\frac{\sqrt{3}}{2} + \frac{i}{2}$

$w_2 = -\frac{\sqrt{3}}{2} + i\frac{1}{2} \longrightarrow z_2 = \frac{\sqrt{3}}{2} + \frac{i}{2}$

2° eq:

$$|z| = \sqrt{x^2+y^2} > \sqrt{x^2+(y-2)^2} = |z-2i|$$

$$\iff 0 > 4-4y$$

$$\iff y > 1$$

SOL. SISTEMA: $2i$

(2) (3)

i) $A_t = \begin{pmatrix} t & 0 & -1 \\ 2 & t & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$ 4×3

considero M = minore ottenuto eliminando riga 2

$M = \begin{pmatrix} t & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$ $\det(M) = t+1$

$\Rightarrow t \neq -1$ $rg(A) = 3$
$\dim(\ker) = 0$

$t = -1$: $A = \begin{pmatrix} -1 & 0 & -1 \\ 2 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$

colonna 3 = colonna 1 + colonna 2
colonne 1,2 lin. IND. $\Rightarrow$ $rg = 2$
$\dim(\ker) = 1$

ii)

$$(A_t|b) = \begin{pmatrix} t & 0 & -1 & 1 \\ 2 & t & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 0 \end{pmatrix}$$

$$\det(A_t|b) = -3t^2+3$$

$\Rightarrow$ $t \neq 1,-1$ $\quad rg(A_t|b) = 4$

$\Rightarrow$ $rg(A_t|b) = 4 > 3 \geq rg(A_t)$

non $\exists$ SOLUZIONE

$t=-1$ $\quad rg(A_t|b) = 3 > rg(A_t) = 2$

non $\exists$ sol.

$t=1$ $\quad 4 > rg(A_t|b) \geq rg(A_t) = 3$

$\Rightarrow rg = 3$ $\Rightarrow$ $\exists$ SOLUZIONE

iii) $\mathbb{R}^4 = \mathrm{Im}(\mathcal{L}_A) \oplus W$

$$\left(\dim(W) = 1 \right) \Rightarrow \begin{cases} \dim(\mathrm{Im}(\mathcal{L}_A)) = 3 \\ \mathrm{Im}(\mathcal{L}_A) \cap W = \{0_V\} \end{cases}$$

OPPURE

$$\Longleftrightarrow \quad \left\{ \begin{pmatrix} t \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ t \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

BASE di $\mathbb{R}^4$

$$\det \begin{pmatrix} t & 0 & -1 & 0 \\ 2 & t & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{pmatrix} = (t-2)\cdot(t+1)$$

$t \neq -1, 2$ si ha somma diretta

3 $\quad \operatorname{Im}(f) = \left\langle \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right\rangle \qquad f^2 = 0$

$$\iff \begin{cases} A = \begin{pmatrix} s & t \\ -2s & -2t \end{pmatrix} \\ \& \\ A^2 = A \cdot A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{cases}$$

Poniemo $s = 1$

$$A = \begin{pmatrix} 1 & t \\ -2 & -2t \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \iff \begin{cases} 1 - 2t = 0 \\ t - 2t^2 = 0 \\ -2 + 4t = 0 \\ -2t + 4t^2 = 0 \end{cases}$$

$$\iff t = \frac{1}{2}$$

$$A = \begin{pmatrix} 1 & \frac{1}{2} \\ -2 & -1 \end{pmatrix}$$

4

$$A = \begin{pmatrix} -1 & 0 & -2 & 0 \\ -2 & 0 & -3 & -1 \\ 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P_A(\lambda) = +\lambda^2(\lambda-1)^2$$

AUTOVALORI: 0 m.a. $= 2$ m.g. $= 1$

1 m.a. $= 2$ m.g. $= 1$

A è triang.le A non è diag.le

AUTOSPAZI:

$$V_0 = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle$$

$$V_1 = \left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix} \right\rangle$$