Esame di ALGEBRA LINEARE - anno accademico 2016/2017

Corso di laurea in Ingegneria ______

Prova scritta del 20 / 12 / 2017

TEMPO A DISPOSIZIONE: 120 minuti

(Cognome) ______ (Nome) MARCO (Numero di matricola) ______

PRIMA PARTE

PUNTEGGIO : risposta mancante = 0 ; risposta esatta = +1 risposta sbagliata = -1

calcoli e spiegazioni non sono richiesti

- $z = \sqrt{3} - i \implies z^3 =$ $\boxed{-8i}$ • $z = -2-5i \implies z^{-1} =$ $\boxed{\frac{-2+5i}{29}}$

- Dati W e Z i seguenti sottospazi di $\mathbb{R}^3$:

$$W = \{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \ : \ x_1 + x_2 - x_3 = 0 \} , \ Z = \langle \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} \rangle.$$

Determinare una base di $W \cap Z$: $\boxed{\left\{ \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} \right\}}$

- $A = \begin{pmatrix} 1 & 2 & 1 & 1 & 2 \\ 1 & 2 & 1 & 1 & 2 \\ 1 & 2 & 1 & 1 & 2 \end{pmatrix} \implies \operatorname{rg}(A) =$ $\boxed{1}$ $\dim(Ker(\mathcal{L}_A)) =$ $\boxed{4}$

- $\det \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -5 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & 0 \end{pmatrix} =$ $\boxed{-5}$ • $A = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \implies m.g.(6) =$ $\boxed{1}$

- $A = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \implies A$ è triangolarizzabile (su $\mathbb{R}$) | vero | ~~falso~~ |

- Data $A = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ determinare il coefficiente di posto $(2,3)$ della matrice A^{-1} : $\boxed{-1}$

- Sia $f : \mathbb{R}^2 \to \mathbb{R}^2$ lineare. Sapendo che $f \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $f \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$, allora $f \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

20 - 12 - 2017

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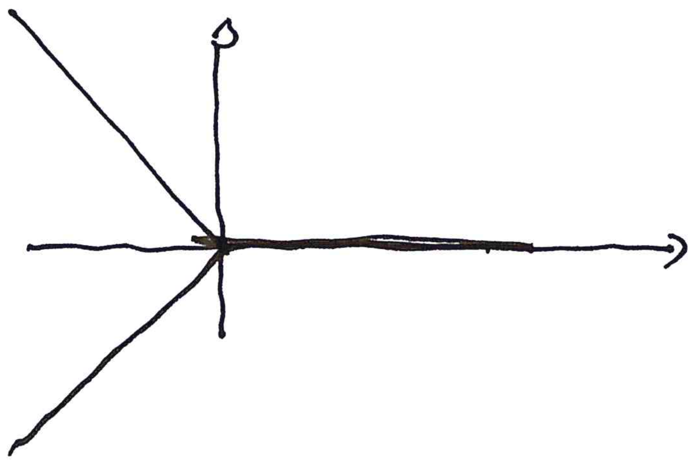
$$\begin{cases} z^3 = |z|^3 \\ |e^{iz}| = e^3 \end{cases}$$

1° eq: $\quad z = \rho \cdot e^{i\vartheta} \qquad |z| = \rho$

$$z^3 = |z|^3 \iff \rho^3 \cdot e^{i3\vartheta} = \rho^3$$

$$\iff \begin{cases} \rho^3 = \rho^3 \\ 3\vartheta = 0 + 2k\pi \end{cases}$$

SOL. DISTINTE: $\begin{cases} \rho \in \mathbb{R}^+ \text{ qualsiasi} \\ \vartheta = \dfrac{2k\pi}{3} \qquad k = 0, 1, 2 \end{cases}$



2ª eq: $\left| e^{iz} \right| = e^{3}$

$z = x + iy$

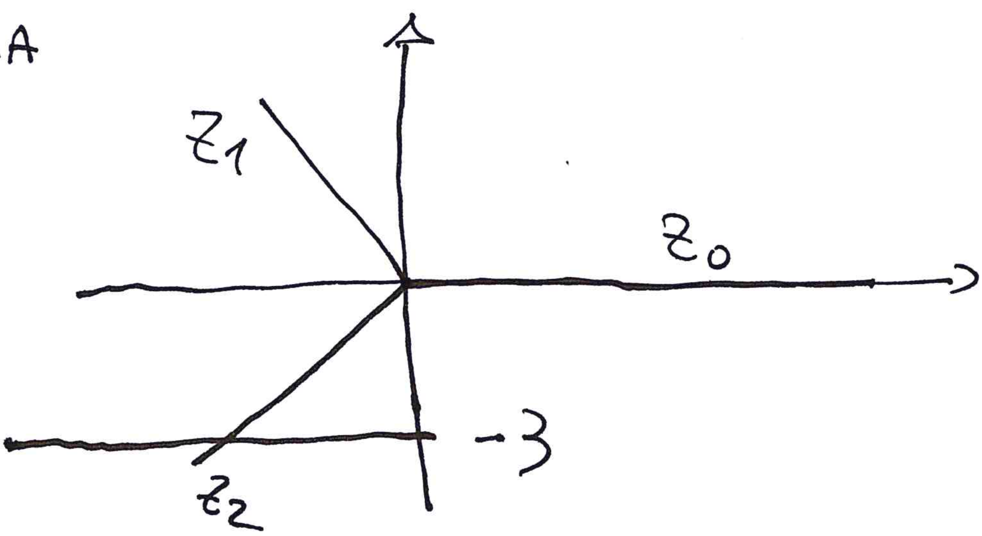
$e^{iz} = e^{ix - y} \Rightarrow \left| e^{iz} \right| = \left| e^{ix-y} \right|$

$= \underbrace{\left| e^{ix} \right|}_{1} \cdot \left| e^{-y} \right| = e^{-y}$

cioè

$\left| e^{iz} \right| = e^{3} \iff \begin{cases} x \text{ qualsiasi} \\ y = -3 \end{cases}$

SOL. SISTEMA



$\begin{cases} z_0 = \rho \cdot e^{i0} \\ y = -3 \end{cases} \Rightarrow \emptyset$

$\begin{cases} z_1 = \rho \cdot e^{i\frac{2}{3}\pi} \\ y = -3 \end{cases} \to \emptyset$

$$\begin{cases} z_2 = \rho \cdot e^{i\frac{4}{3}\pi} \\ y = -3 \end{cases} \Rightarrow$$

$$z_2 = \rho \cdot \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right)$$

$$y(z_2) = -\rho \frac{\sqrt{3}}{2} = -3$$

$$\Rightarrow \rho = 2\sqrt{3}$$

CONCLUSIONE :

SOL : $$Z = 2\sqrt{3}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

$$= -\sqrt{3} - i\,3$$

2

$$A_t = \begin{pmatrix} 2 & 1 & 3 \\ t & t & 4 \\ 0 & 2 & t \end{pmatrix}$$

$$\det(A_t) = t^2 + 6t - 16$$

$t \neq 2, -8 \qquad \det \neq 0$

$\Rightarrow rg(A_t) = 3$; dim Ker $= 0$

$t = 2 \qquad M = \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix} \qquad \det \neq 0$

$\Rightarrow rg(A_2) = 2$; dim Ker $= 3 - 2 = 1$

$t = -8 \qquad M = \begin{pmatrix} -8 & -8 \\ 0 & 2 \end{pmatrix} \qquad \det \neq 0$

$\Rightarrow rg(A_{-8}) = 2 \qquad$ dim Ker $= 1$

ii) $t \neq -8, 2 \quad \exists !\ \text{SOL.}$

$t = 2$

$rg(A) = rg(A|b) = 2$

$\Rightarrow \exists\ \infty\ \text{SOL.}$

$t = -8$

$rg(A) = 2$

$rg(A|b) = 3 \qquad \left(\det \begin{pmatrix} 2 & 1 & 2 \\ -8 & -8 & 4 \\ 0 & 2 & 4 \end{pmatrix} \neq 0 \right)$

$\Rightarrow$ non $\exists$ SOL.

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iii) $W = \left\langle \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} \right\rangle \qquad \dim W = 1$

$\mathbb{R}^3 = W \oplus \operatorname{Im}(L_A) \Rightarrow rg(A) = 2$

$t=2$

$$\mathrm{Im}(f_A) = \left\langle \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$$ sono lin IND.

$\Rightarrow$ Per $t=2$ è verificata

$t=-8$

$$\mathrm{Im}(f_A) = \left\langle \begin{pmatrix} 2 \\ -8 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} 2 \\ -8 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$$ son lin. DIP.

$\Rightarrow$ Per $t=-8$

$$\mathbb{R}^3 \neq W \oplus \mathrm{Im}(f_A)$$

3 $f : \mathbb{R}^3 \to \mathbb{R}^3$

$\mathrm{Im}(f) = \left\langle \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \right\rangle$; $f^2 = 0$

$f = \mathcal{L}_A$ A matrice 3×3

$\mathrm{rg}(A) = 1$

Possiamo scegliere $A = \begin{pmatrix} 1 & \alpha & \beta \\ 2 & 2\alpha & 2\beta \\ -2 & -2\alpha & -2\beta \end{pmatrix}$

$f^2 = 0 \iff A^2 = 0$

$\iff 1 + 2\alpha - 2\beta = 0$

Ad esempio per $\beta = -\frac{1}{2}$ $\alpha = -1$

$A = \begin{pmatrix} 1 & -1 & -\frac{1}{2} \\ 2 & -2 & -1 \\ -2 & 2 & 1 \end{pmatrix}$

2^{ndo} metodo

$$\mathrm{Im}(f) = \left\langle \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \right\rangle \qquad f^2 = 0$$

$$\Longleftrightarrow \qquad \mathrm{Im}(f) \subset \mathrm{Ker}(f)$$

Scegliamo $\quad \mathrm{Ker}(f) = \left\langle \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

$$A = \begin{pmatrix} 1 & \alpha & 0 \\ 2 & 2\alpha & 0 \\ -2 & -2\alpha & 0 \end{pmatrix} \qquad \left(\Longleftrightarrow \; A \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right)$$

$$A \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Longleftrightarrow \quad 1 + 2\alpha = 0$$

$$\alpha = -\frac{1}{2}$$

$$A = \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{pmatrix}$$

4 9

$$A = \begin{pmatrix} -2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & -1 & 0 & 2 \end{pmatrix}$$

$$P_A(\lambda) = \lambda^2 \cdot (\lambda^2 - 9)$$

RADICI: $\lambda_0 = 0$ $m.a. = 2$

$\lambda_1 = 3$ $m.a. = 1$

$\lambda_2 = -3$ $m.a. = 1$

$$m.g.(0) = 4 - rg(A - 0 \cdot Id) = 4 - rg(A)$$
$$= 2$$

$m.g.(3) = 1$

$m.g.(-3) = 1$

AUTOSPAZIO RELATIVO a $\lambda_0 = 0$

$$V_0 = \left\langle \begin{pmatrix} -1 \\ -3 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$

AUTOSP. relativo a $\lambda_1 = 3$

$$V_1 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 5 \end{pmatrix} \right\rangle$$

AUTOSP. relativo a $\lambda_2 = -3$

$$V_2 = \left\langle \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

A è diagonalizzabile

Poichè $\forall i \quad m.a.(\lambda_i) = m.g.(\lambda_i)$