

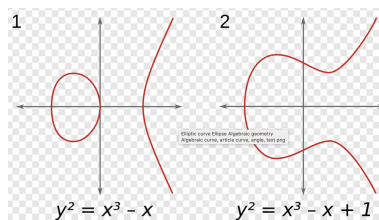
Introduction to Algebraic Geometry - Elementi di Geometria Algebrica

2024/2025

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Introduction

Algebraic Geometry is the branch of Mathematics which studies the relations between polynomial equations and geometric objects.



The aim of the course is to give an introduction to the fundamental notions and instruments used in Algebraic Geometry .

In order to avoid introducing more advanced machinery, such as sheaf theory, cohomology and schemes, we work in the setup of quasi-projective varieties. We introduce fundamental geometrical notions such as irreducible decomposition, regular functions and morphisms, rational functions and maps, products, dimension, tangent space, singularities.

To help the student familiarize with these concepts, we illustrate them with plenty of concrete examples, including classical constructions as the Veronese and Segre varieties and Grassmannians. We also devote some time to the study of plane curves, including linear systems, Bèzout theorem.

The course is meant to serve as an introduction to algebraic geometry both for those who have a cultural interest in the subject (or are just curious to know what it is about) and for those who consider doing research in the area. For the latter, familiarity with quasi-projective varieties and concrete examples will serve as a testing ground for the understanding of the more abstract notions they will encounter in the prosecution of their studies.

Topics

Affine spaces and projective spaces. Plane curves: local geometry, Bézout theorem.

Nullstellensatz. Affine and quasi-projective varieties: Zariski topology, irreducible decomposition, morphisms, rational maps, dimension, tangent spaces and singular points..

Blow-up of a point in the (affine plane).

Plane cubics.

Segre varieties, Veronese varieties, Grassmannians.

Practical information

The course will last 48 hours, and it will take place in the first semester. The exam will be oral. The prerequisites for this course are basic notions of topology, commutative algebra and projective geometry, all taught in the first two years of the mathematics undergraduate program in Pisa. If you want to know more about the course, contact me at marco.franciosi@unipi.it

Website

<http://pagine.dm.unipi.it/~a008702/ega.html>

Literature

- [1] I. R. Shafarevich, Basic Algebraic Geometry 1, (Second edition), Springer (1994).
- [2] M. Reid, Undergraduate Algebraic Geometry, Cambridge University Press (1988).
- [3] F. Flamini, A first course in algebraic geometry and algebraic varieties, Essential Textbooks in Mathematics, World Scientific Publishing Co., (2023).