CN $\boxed{41}$ 6 maggio 2008 / B21

Pb lin di interp:

dati $[a,b]$; $\mathcal{G}$ ssv di $\mathcal{C}(a,b)$ dim finita; k

$L_0, \dots, L_k : \mathcal{G} \to \mathbb{R}$ lin; $y_0, \dots, y_k \in \mathbb{R}$

determ $g \in \mathcal{G}$ t.c. $L_0(g) = y_0, \dots, L_k(g) = y_k$

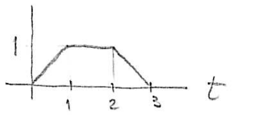
SOLUZIONE: posto $\mathcal{G} = \langle \gamma_1, \dots, \gamma_j \rangle$

cerco $a_1, \dots, a_j \in \mathbb{R}$ t.c. $g = a_1\gamma_1 + \dots + a_j\gamma_j$

verifichi $L_0(g) = y_0, \dots, L_k(g) = y_k$

ovvero t.c.
$$\underbrace{\begin{bmatrix} L_0(\gamma_1) & \cdots & L_0(\gamma_j) \\ \vdots & & \\ L_k(\gamma_1) & \cdots & L_k(\gamma_j) \end{bmatrix}}_{\in\, \mathbb{R}^{(k+1)\times j}} \underbrace{\begin{bmatrix} a_1 \\ \vdots \\ a_j \end{bmatrix}}_{\in\, \mathbb{R}^{j}} = \underbrace{\begin{bmatrix} y_0 \\ \vdots \\ y_k \end{bmatrix}}_{\in\, \mathbb{R}^{k+1}}$$

Es: $\varphi : \mathbb{R} \to \mathbb{R}$ t.c.



$\varphi_0(t) = \varphi(t)$

$\varphi_1(t) = \varphi(t-1)$; $\mathcal{G} = \langle \varphi_0, \varphi_1, \varphi_2 \rangle$

$\varphi_2(t) = \varphi(t-2)$

1) disegnare grafico $\varphi_0, \varphi_1, \varphi_2$; 2) verif che: $\alpha_0\varphi_0 + \alpha_1\varphi_1 + \alpha_2\varphi_2 = 0 \quad \forall t$
$\Rightarrow \alpha_0 = \alpha_1 = \alpha_2 = 0$
(ovvero $\varphi_0, \varphi_1, \varphi_2$ lin indip)

3) determ $g \in \mathcal{G}$ t.c. $g(1) = 3$, $g(3) = -1$, $g(4) = 0$.

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* CAMPIONAMENTO e RICOSTRUZIONE *

def (f di camp, f di ricostruz):

k int ≥ 0; $t_0, \dots, t_k \in [a,b]$ distinti

- $c : \mathcal{C}([a,b], \mathbb{R}) \to \mathbb{R}^{k+1}$ t.c. $c(f) = (f(t_0), \dots, f(t_k))^T$
  ↳ f di campionamento (agli istanti $\underbrace{t_0, \dots, t_k}$)
  "ist di campionam"

  Oss: c è lin e non invert! ← $\exists f_1 \neq f_2 : c(f_1) = c(f_2)$

- $r : \mathbb{R}^{k+1} \to \mathcal{C}([a,b], \mathbb{R})$
  f di ricostruz (rel a c) SE
  - lineare
  - $\forall y \in \mathbb{R}^{k+1}$, $c(r(y)) = y$